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PSTAT 172A

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Pricing Insurance and Setting a Security Loading

This is an actuarial report for Javier Garcia's insurance company. This report analyzes a whole life insurance policy on (60) for which the benefit of $1,000 is paid at the end of the year of death. Here are some preliminaries before we begin:

- The effective annual interest rate is 6% (i = 0.06)
- The effective annual discount rate (d) = i / (1 + i) = 0.05660377358 or ≈ 0.0566
- The mortality rates to calculate the fully discrete whole life insurance policy and premiums are based on [the following table]

1. The net single premium for a whole life insurance policy on (60) for which the benefit of $1,000 is paid at the end of the year of death policy should be approximately $345.60.

$$P^{ns} = 1000 \cdot A_{60}$$

$$P^{ns} = 1000 \cdot \sum_{k=0}^{119-60} v^{k+1} \cdot {}_kp_{60} \cdot q_{60+k}$$

$$P^{ns} = 1000 \cdot (0.34556)$$

$$\boxed{P^{ns} = 345.56}$$

2. The net annual premium for a whole life insurance policy on (60) for which the benefit of $1,000 is paid at the end of the year of death policy should be approximately $29.90.

$$P^{na} \cdot \ddot{a}_{60} = 1000 \cdot A_{60}$$

$$P^{na} = \frac{1000 \cdot A_{60}}{\ddot{a}_{60}}$$

$$\ddot{a}_{60} = \frac{1 - A_{60}}{d} = \frac{1 - (0.34556)}{0.05660377538} = 11.56177333$$

$$\boxed{P^{na} = \frac{345.56}{11.56177333} = 29.88814865}$$

3. For a group of 2,500 identical insureds, with a probability of a loss less than or equal to 0.025, the single premium for a whole life insurance policy on (60) for which the benefit of $1,000 is paid at the end of the year of death should be approximately $353.57.

$$L^{(i)} = 1000 \cdot v^{K^{(i)}+1} - P^{ns}$$ (net loss at issue random variable for single premium of whole life insuance policy)

$$L = L^{(1)} + \cdots + L^{(2500)}, \quad n = 2500$$

$$\mathbb{E}[L] = 2500 \cdot (1000 A_{60} - P^{nsp})$$

$$Var(L) = (2500) \cdot (1000)^2 \cdot ({}^2A_{60} - (A_{60})^2)$$

$$P(L>0) = 0.025$$

$$P\left(\frac{L-\mathbb{E}[L]}{\sqrt{Var(L)}} > 0\right) = 0.025$$

$$P\left(\frac{L-\mathbb{E}[L]}{\sqrt{Var(L)}} > \frac{0-\mathbb{E}[L]}{\sqrt{Var(L)}}\right) = 0.025$$

$$P\left(Z > \frac{-2500(1000A_{60}-P^{\varepsilon ns})}{\sqrt{2500}\cdot 1000\cdot\sqrt{({}^2A_{60}-(A_{60})^2)}}\right) = 0.025$$

$$1-\phi\left(\frac{-\sqrt{2500}(1000A_{60}-P^{\varepsilon ns})}{1000\cdot\sqrt{({}^2A_{60}-(A_{60})^2)}}\right) = 0.025$$

$$\frac{-\sqrt{2500}(1000A_{60}-P^{\varepsilon ns})}{1000\cdot\sqrt{({}^2A_{60}-(A_{60})^2)}} = \phi^{-1}(1-0.025)$$

$$\sqrt{2500}(P^{\varepsilon ns}-1000A_{60}) = \phi^{-1}(0.975)\cdot 1000\cdot\sqrt{({}^2A_{60}-(A_{60})^2)}$$

$$P^{\varepsilon ns}-1000A_{60} = \frac{\phi^{-1}(0.975)\cdot 1000\cdot\sqrt{({}^2A_{60}-(A_{60})^2)}}{\sqrt{2500}}$$

$$P^{\varepsilon ns} = \frac{\phi^{-1}(0.975)\cdot 1000\cdot\sqrt{({}^2A_{60}-(A_{60})^2)}}{\sqrt{2500}} + 1000A_{60}$$

$$P^{\varepsilon ns} = \frac{(1.96)\cdot 1000\cdot\sqrt{(0.16114)-(0.34556)^2}}{\sqrt{2500}} + 1000(0.34556)$$

$$P^{\varepsilon ns} = (8.007581034) + 345.56$$

$$\boxed{P^{\varepsilon ns} = 353.567581}$$

4. For a group of 2,500 identical insureds, with a probability of a loss less than or equal to 0.025, the annual premium for a whole life insurance policy on (60) for which the benefit of $1,000 is paid at the end of the year of death should be approximately $30.96.

$$L^{(i)} = 1000\cdot v^{K^{(i)}+1} - P^{na}\cdot \ddot{a}_{\overline{K^{(i)}+1|}} \quad \left(\begin{array}{l}\text{net loss at issue random variable for annual}\\ \text{premium of whole life insurance policy}\end{array}\right)$$

$$L = L^{(1)} + \cdots + L^{(2500)}, \quad n=2500$$

$$\mathbb{E}[L] = 2500\cdot(1000A_{60} - P^{na}\cdot\ddot{a}_{60}) = 2500\cdot\ddot{a}_{60}(P^{\varepsilon na} - P^{na})$$

$$Var(L) = 2500\cdot(1000+P^{\varepsilon na}/d)^2\cdot({}^2A_{60}-(A_{60})^2)$$

$$P(L > 0) = 0.025$$

$$P\left(\frac{L - \mathbb{E}[L]}{\sqrt{Var(L)}} > 0\right) = 0.025$$

$$P\left(\frac{L - \mathbb{E}[L]}{\sqrt{Var(L)}} > \frac{0 - \mathbb{E}[L]}{\sqrt{Var(L)}}\right) = 0.025$$

$$P\left(Z > \frac{2500 \cdot \ddot{a}_{60}\,(P^{\varepsilon na} - P^{na})}{\sqrt{2500} \cdot (1000 + P^{\varepsilon na}/d) \cdot \sqrt{({}^2A_{60} - (A_{60})^2)}}\right) = 0.025$$

$$1 - \Phi\left(\frac{\sqrt{2500} \cdot \ddot{a}_{60}\,(P^{\varepsilon na} - P^{na})}{(1000 + P^{\varepsilon na}/d) \cdot \sqrt{({}^2A_{60} - (A_{60})^2)}}\right) = 0.025$$

$$\frac{\sqrt{2500} \cdot \ddot{a}_{60}\,(P^{\varepsilon na} - P^{na})}{(1000 + P^{\varepsilon na}/d) \cdot \sqrt{({}^2A_{60} - (A_{60})^2)}} = \Phi^{-1}(1 - 0.025)$$

$$\sqrt{2500} \cdot \ddot{a}_{60}\,(P^{\varepsilon na} - P^{na}) = \Phi^{-1}(0.975) \cdot (1000 + P^{\varepsilon na}/d) \cdot \left(\sqrt{{}^2A_{60} - (A_{60})^2}\right)$$

$$P^{\varepsilon na} - \frac{(1.96 \cdot \sqrt{{}^2A_{60} - (A_{60})^2} \cdot P^{\varepsilon na})}{\sqrt{2500} \cdot \ddot{a}_{60} \cdot d} = \frac{(1.96 \cdot \sqrt{{}^2A_{60} - (A_{60})^2} \cdot 1000)}{\sqrt{2500} \cdot \ddot{a}_{60}} + P^{na}$$

$$P^{\varepsilon na}\left(1 - \frac{(1.96 \cdot \sqrt{{}^2A_{60} - (A_{60})^2})}{\sqrt{2500} \cdot \ddot{a}_{60} \cdot d}\right) = \frac{(1.96 \cdot \sqrt{{}^2A_{60} - (A_{60})^2} \cdot 1000)}{\sqrt{2500} \cdot \ddot{a}_{60}} + P^{na}$$

$$P^{\varepsilon na} = \left(\frac{(1.96 \cdot \sqrt{{}^2A_{60} - (A_{60})^2} \cdot 1000)}{\sqrt{2500} \cdot \ddot{a}_{60}} + P^{na}\right) \Big/ \left(1 - \frac{(1.96 \cdot \sqrt{{}^2A_{60} - (A_{60})^2})}{\sqrt{2500} \cdot \ddot{a}_{60} \cdot d}\right)$$

$$P^{\varepsilon na} = \frac{\left(\frac{(1.96) \cdot \sqrt{(0.16114) - (0.34556)^2} \cdot 1000}{\sqrt{2500} \cdot (11.56177333)} + (29.88814865)\right)}{\left(1 - \frac{(1.96) \cdot \sqrt{(0.16114) - (0.34556)^2}}{\sqrt{2500} \cdot (11.56177333) \cdot (0.05660377358)}\right)}$$

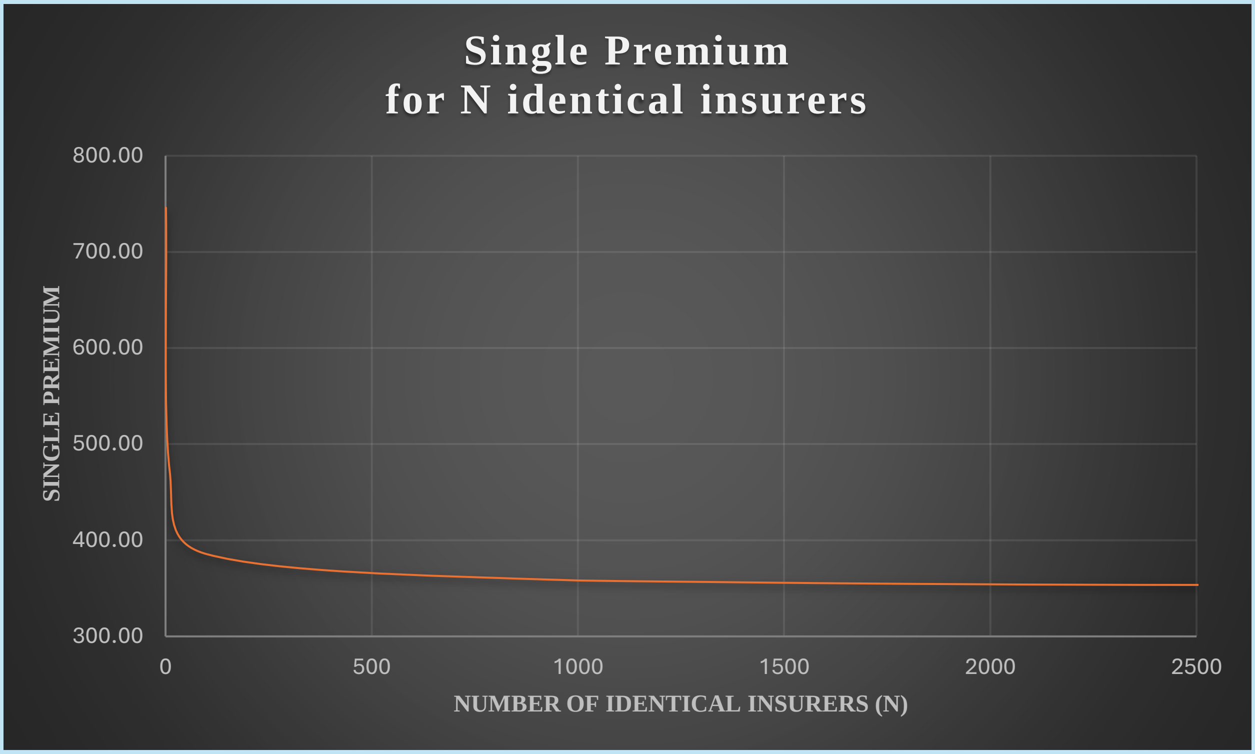
$$\boxed{P^{\varepsilon na} = 30.95955388}$$

5. As the number of identical insureds (N) increases under the premium calculations in (3) and (4), the single and annual premiums converge to the net premium calculations in (1) and (2). In the single premium case, the net premium is a constant, 345.56, which is added by the risk loading expenses which decline as the number of identical insureds (N) increases. Similarly, in the annual premium case, the net premium is a constant, 29.88814865, added by the risk loading expenses. However, the sum of the net premium and loading expenses are which are divided by the difference between the premium and premium expenses, ultimately the risk loading expenses decline as the number of identical insureds (N) increases. The risk loading declines in both cases due to the diversification in the mortality risk of the pool of insureds. Therefore, as the number of identical insureds (N) increases, the risk loading declines, and the premiums calculated in (3) and (4), converge to the net premiums calculated in (1) and (2).

6. The following formula represents the single premium as a function of the number of insureds calculated in (3):

$$P^{Ens} = \frac{(1.96)\cdot 1000 \cdot \sqrt{(0.16114)-(0.34556)^2}}{\sqrt{N}} + 1000(0.34556)$$

the following chart represents the results as (N) identical insureds increase:

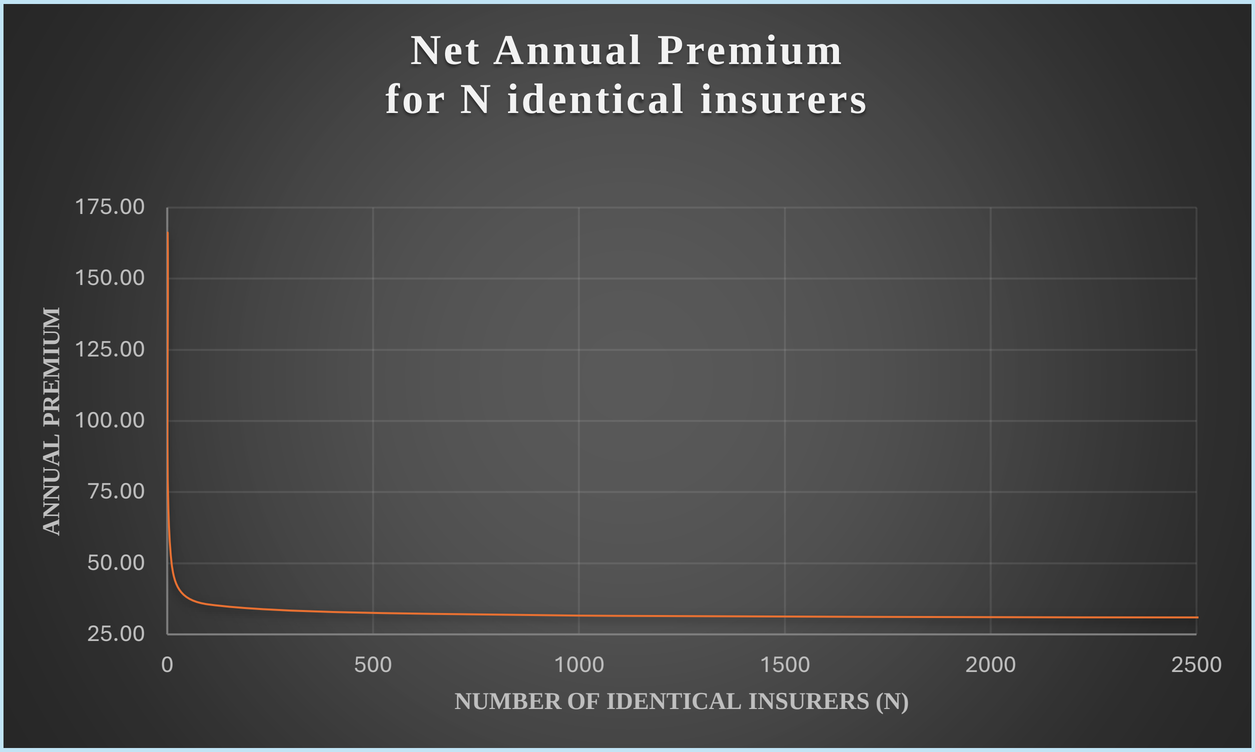


As discussed in (5), as the number of insured approaches infinity, the single premium converges to the net single premium calculated in (1).

The following formula represents the annual premium as a function of the number of insureds calculated in (4):

$$P^{Ena} = \frac{\left(\frac{(1.96)\cdot\sqrt{(0.16114)-(0.34556)^2}\cdot 1000}{\sqrt{N}\cdot(11.56177333)} + (29.88814865)\right)}{\left(1 - \frac{(1.96)\cdot\sqrt{(0.16114)-(0.34556)^2}}{\sqrt{N}\cdot(11.56177333)\cdot(0.05660377358)}\right)}$$

the following chart represents the results as (N) identical insureds increase:



As discussed in (5), as the number of insureds approaches infinity, the annual premium converges to the net annual premium calculated in (1).

7. Assuming 10 years have passed, and 2,050 lives are remaining from the original pool of insureds. Then for the annual premium case, Javier Garcia's insurance company should have approximately $219.70 per policy in its reserve to have a 98% probability of not losing money.

$$L_{x,n}^{(i)} = v^{K^{(i)}(x+n)+1} - P_x^{\varepsilon} \cdot \ddot{a}_{\overline{K^{(i)}(x+n)+1|}} \quad \left(\begin{array}{l}\text{net loss at issue random variable for annual premium} \\ \text{of whole life insuance policy for a life age } (x) \\ \text{after } (n) \text{ years}\end{array}\right)$$

$$L_{x,n}^{(i)} = L_{60,10}^{(1)} + \cdots + L_{60,10}^{(m)}$$

Let the reserve be denoted by R_n = reserve per policy

$$\mathbb{E}[L_{60/10}^{2050}] = 2050(1000 A_{60+70} - P_{60}^{\varepsilon} \cdot \ddot{a}_{60+10})$$

$$= -2050\, \ddot{a}_{70} (P_{60}^{\varepsilon} - 1000P)$$

$$\mathrm{Var}(L_{60,10}^{2050}) = 2050 \cdot (1000 + P_{60}^{\varepsilon}/d)^2 \cdot ({}^2A_{70} - (A_{70})^2)$$

$$P(L > 2050 \cdot R_n) = 0.02$$

$$P\left(\frac{L - \mathbb{E}[L_{60,10}]}{\sqrt{\mathrm{Var}(L_{60,10})}} > \frac{2050 \cdot R_n - \mathbb{E}[L_{60,10}]}{\sqrt{\mathrm{Var}(L_{60,10})}}\right) = 0.02$$

$$P\left(Z > \frac{2050 \cdot R_n - 2050 \cdot \ddot{a}_{70} \cdot (P_{60}^{\varepsilon} - 1000 \cdot P)}{\sqrt{2050} \cdot (1000 + P^{\varepsilon}/d) \cdot \sqrt{{}^2A_{70} - (A_{70})^2}}\right) = 0.02$$

$$P\left(Z > \frac{2050 \cdot (R_n - \ddot{a}_{70} \cdot (P_{60}^{\varepsilon} - 1000 \cdot P))}{\sqrt{2050} \cdot (1000 + P_{60}^{\varepsilon}/d) \cdot \sqrt{{}^2A_{70} - (A_{70})^2}}\right) = 0.02$$

$$1 - \Phi\left(\frac{\sqrt{2050} \cdot (R_n - \ddot{a}_{70} \cdot (P_{60}^{\varepsilon} - 1000 \cdot P))}{(1000 + P_{60}^{\varepsilon}/d) \cdot \sqrt{{}^2A_{70} - (A_{70})^2}}\right) = 0.02$$

$$\frac{\sqrt{2050} \cdot (R_n - \ddot{a}_{70} \cdot (P_{60}^{\varepsilon} - 1000 \cdot P))}{(1000 + P_{60}^{\varepsilon}/d) \cdot \sqrt{{}^2A_{70} - (A_{70})^2}} = \Phi^{-1}(0.98)$$

$$\sqrt{2050} \cdot (R_n - \ddot{a}_{70}(P_{60}^{\varepsilon} - 1000 \cdot P) = (2.054) \cdot (1000 + P_{60}^{\varepsilon}/d) \cdot \sqrt{{}^2A_{70} - (A_{70})^2}$$

$$R_n - \ddot{a}_{70}(P_{60}^{\varepsilon} - 1000 \cdot P) = \frac{(2.054) \cdot (1000 + P_{60}^{\varepsilon}/d) \cdot \sqrt{{}^2A_{70} - (A_{70})^2}}{\sqrt{2050}}$$

$$R_n = \frac{(2.054) \cdot (1000 + P_{60}^{\varepsilon}/d) \cdot \sqrt{{}^2A_{70} - (A_{70})^2}}{\sqrt{2050}} + (1000 \cdot A_{70} - P_{60}^{\varepsilon} \cdot \ddot{a}_{70})$$

$$R_n = \frac{(2.054) \cdot (1000 + \frac{29.88814865}{0.05660377358}) \cdot \sqrt{0.27039 - (0.48028)^2}}{\sqrt{2050}} + 480.28 - \left(\frac{1000 A_{60}}{\ddot{a}_{60}} \cdot \frac{1 - 0.48028}{0.05660377358}\right)$$

$$\boxed{R_n = 219.6708183}$$

8. A benefit of charging a portfolio-level premium versus a net premium based on the equivalence principle is the probability of a large aggregate loss is smaller compared to a net premium. As discussed in (5), as the number of identical insureds increases the risk loading declines due to the diversification in the mortality risk of the pool of insureds.

Hence, a portfolio-level premium is better equipped to counteract a large aggregate loss due to the diverse mortality within the pool compared to a net premium. **However, one drawback of charging a portfolio-level premium versus a net premium based on the equivalence principle is the lack of precise risk**. If the portfolio accrues high-risk insureds over time, then in the case of Javier's Garcia insurance company, they will not have sufficient funds in its reserve if the portfolio-level premium being charged does not accurately reflect the potential risk of the insureds within the pool over time. **Yet, accounting for a net premium for every insured leads to another benefit of charging a portfolio-level premium versus a net premium based on the equivalence principle, the costs of labor.** Charging a portfolio-level premium reduces the amount of time and energy spent on underwriting policies compared to individual net premiums since insurers do not have to calculate every insured's mortality risk, rather a portfolio-level premium allows insurers to charge a premium that is already set by a portfolio, depending on if the insured meets the criteria of the portfolio of course. **Lastly, another drawback of charging a portfolio-level premium versus a net premium based on the equivalence principle is unfair premium rates.** Touching back on the first drawback**,** a portfolio-level premium being charged might not accurately reflect the potential risk of the insureds within the pool, therefore some will pay higher premium rates, while others pay lower premium rates, displeasing some insureds while pleasing others. Ultimately, charging a portfolio-level premium allows insurance companies to mitigate large aggregate losses and costs of labor, while net premiums allow insurance companies to precisely evaluate each policy risk and offer accurate premiums.