Macroeconomic Effects of Entrepreneurial Gender Gaps*

by Javier Gonzalez†

August, 2020

Abstract

Several gender gaps have shrunk over past decades in developed countries. However, a gender gap in entrepreneurship sill persists. In fact, the difference between men and female entrepreneurs have risen in recent decades. Fewer female entrepreneurs, in quantity and quality, can decrease aggregate output and productivity. The entrepreneurial gap can be caused by four labor market distortions: (i) a gender profit gap, (ii) the presence of household production, (iii) a capital restriction for female entrepreneurs and a (iv) wage gap. Using a span-and-control model, this thesis measures the macroeconomic effects of each distortion and identifies which ones have negative effects. A 20% household productivity increase leads to a 6.3% reduction of GDP, while a profit gap of 20% can decrease GDP in 3%. In contrast, the capital and wage gaps have little and no impact on macroeconomic variables in the general equilibrium. Overall, these findings suggest that childcare policies and reducing a gender profit gap can increase aggregate output and productivity.

*M.A. Thesis from the Universidad Adolfo Ibañes in Chile.

†I am gratefull to my main supervisor Francisco Parro for his support and patience. I will like to thank Juan Pedro Eberhard and Marcos Gomes who provided invaluable feedback. Lastly, special thanks to Catalina Garcia who provided excelent editorial services.

1 Introduction

Several gender gaps have shrunk in the last decades. In the United States, the difference between female and male labor force participation rates has decreased from 52% in 1950 to 14% in 2005 (Toossi, 2006). For OECD countries, International Labor Organization (ILO) data show that this difference has changed from 25.8% in 1990 to 17.3% in 2018. The gender wage gap followed a similar trend. According to the OECD, the difference between the median earnings of men and women relative to median earnings of men has decreased from 19.6% in 1995 to 13.4% in 2017. In terms of education, the differences between secondary and tertiary education attainment between male and females has significantly reduced. The gender gap in tertiary education enrollment flipped from 2.9% in 1990 to -2.2% in 2000 for the United States (Parro, 2012) and flipped from 0.7% in 2000 to -6.5% in 2018 for OECD. In addition, women have increased their political participation. In the European Union, women represented 20.5% of the parliament seats in 2002 and this increased to 31.7% in 2019.

Although there has been some progress on bridging labor gender gaps, there remains a significant gender gap in entrepreneurship within developed countries. The report *Closing the Gender Gap* (2012) finds that women represent 25% of business owners across 27 European countries. Furthermore, the proportion of small businesses with a female owner can vary from 20 to 40% in the OECD countries. According to the ILO, the difference between the percentage of self-employed males and females in OECD countries has increased from 4.3% in 1992 to 5% in 2019. In contrast, the managerial sector experienced a slow and steady rise in the participation of women in senior and middle management from 32% in 2010 to 33% in 2019. But the share of firms that has a female in the top management has decreased from 21.4% in 2009 to 17.2% in 2019. In other words, women are sub-represented in entrepreneurship and the gap is getting wider.

A widening of the entrepreneurial gender gap could have negative effects on productivity through an inefficient distribution of talent between women and male entrepreneurs. Hsieh et al. (2019) emphasize that an efficient distribution of talent within industries is important for aggregate productivity and market output. An aggregate production loss could be explained with a reduction of entrepreneurs' quantity and quality. For instance, aggregate production decreases through quality when high skilled entrepreneurs are replaced by low skilled ones. In contrast, aggregate production decreases through quantity if female agents cannot create productive firms. Hence, the sub-representation of women in entrepreneurship can negatively affect aggregate variables because the quantity and quality of entrepreneurs is not efficiently distributed.

This thesis quantitatively measures the long-run macroeconomic effects of gender gaps in entrepreneurship. In particular, it will study the impact of (i) a gender profit gap, (ii) increased household production costs, (iii) a capital glass ceiling, and (iv) a labor wage gap. First, there is a gap between the performance of male and female-run firms. Women's enterprises are over-represented in the least profitable firms within industries and gain 30% fewer profits than their male counterparts (Bird and Sapp, 2004). The literature has not found a clear explanation for this gap, but it has proven that it is not driven by firm-owner characteristics (Hardy and Kagy, 2018) or supply and demand pressures (Delecourt and Ng, 2019). Lower profits can disincentivize potential women entrepreneurs to create new businesses, leading to an disproportionate numbers of male and female business owners.

Second, time and financial costs associated with household production, child-caring activities, restricts a portion of women from participating in the labor market. Women face lower costs for household production (Blau, 2015; Friedman and Becker, 1993) and women tend to allocate more time to household activities than their business (*Closing the Gender Gap* 2012). Additionally, there is extensive research on childcare policies increasing female labor participation, which includes female entrepreneurship (Baker et al., 2008; Lundin et al., 2008; Olivetti and Petrongolo, 2017). This implies that the absence of childcare policies has the potential to prevent for skilled females from becoming workers or entrepreneurs.

Third, women use less financial credit than men entrepreneurs. *Closing the Gender Gap* (2012) reports that 6.3% of female entrepreneurs and 11% of male entrepreneurs used external finance. In the United States, 60% of female entrepreneurs start with less than $5.000 U.S. compared to 42% of male counterparts. In addition, female returns to capital investments are lower than market prices (de Mel et al., 2008). Both facts could be explained by outright discrimination in interest rates and differential access to credit. There's no evidence for interest rates discrimination of micro-firms in Sub-Saharan Africa and some European countries

(Asiedu et al., 2012, 2013; Aterido et al., 2013; Bellucci et al., 2010). Nonetheless, there's evidence that female entrepreneurs face worse credit conditions than men, even when controlling for firm and entrepreneur characteristics (Moro et al., 2017; Aristei and Gallo, 2016). For instance, Agier and Szafarz (2013) examines the Brazilian case and finds a glass ceiling for women business owners in the credit market. In other words, Brazilian women have maximum debt levels lower than similar male entrepreneurs. Additionally, Sauer and Wilson (2016) showed that relaxing female liquidity constraints significantly causes a rise on firm creation. This suggests that credit and liquidity constraints can deter women from participating in the labor market as entrepreneurs.

Fourth, the difference in wages between male and female employees can distort female labor participation decisions. The average wage gap of OECD countries was 13.4% in 2017. That is, the female median earnings is 86.6% of the male median earning. The wage gap could distort the labor market through female workers (labor supply) or entrepreneurs (labor demand). In the supply side, recent studies have shown how gender wage gaps are a barrier for an efficient match between skilled female worker and a productive firm. Morchio and Moser (2020) studied how a gender wage gap in Brazil decreases aggregate output through an inefficient match of productive firms and talented females. On the demand side, Ribes-Giner et al. (2018) argues a combination of high wage gap and low female participation positively correlates with low female entrepreneurship participation. However, lower pay may incentivize female agents, who decide to participate in the labor market, to become entrepreneurs instead of looking for a job.

To measure the macroeconomic effects of these distortions, I compute and compare five different economies using a span-of-control model. The first one is a benchmark economy with no distortions. This economy was calibrated to match the average female labor force participation and the shares of employment per firm size for an average between Sweden, Slovenia and Norway, the countries with the most egalitarian labor markets in the OECD. Lastly I will compare four economies, each including a single labor market distortion, to the benchmark to measure the impact of each distortion in aggregate output and productivity.

There is significant variability in my results. On the one hand, high household productivity and a profit gap can significantly reduce both aggregate output and productivity.

For instance, higher household productivity can reduce aggregate output by 6.3% and decrease productivity, measured in TFP, by 4.3%. On the other hand, the capital gap and the wage gap have a low or non-existing effects in the long-run. Overall, these findings suggest that childcare and profit gap reduction policies can increase aggregate production and productivity.

The rest of the thesis will proceed as follows. Section 2 will discuss the related literature. Section 3 contains the model. Section 4 explains the calibration strategy used. Section 5 will discuss the estimated results using one labor market distortion at a time. Lastly, Section 6 concludes.

2 Related Literature

Most of the economic literature has focused on explaining the entrepreneurial gap. Studies have found what characteristics of women entrepreneurs and their firms could cause an entrepreneurial gender gap (Sauer and Wilson, 2016). Some of these characteristics include: female competitiveness aversion, lower female self-confidence, culture environment, gender peer influences and initial startup orientation (Bönte and Piegeler, 2013; Caliendo et al., 2015; Guzman and Kacperczyk, 2019; Markussen and Røed, 2017; Max and Ballereau, 2013; Shinnar et al., 2012; Thébaud, 2010; Watson et al., 2009). Other studies has focused on understanding out-of-necessity self-employment of both immigrant and native entrepreneurs in developing countries (Greenwood et al., 2005; Jamali, 2009; Poschke, 2013). In contrast, this thesis will be exploring a different question: How does the gender gap impact macroeconomic variables in developed countries? Furthermore, I will measure the effects of an entrepreneurial gender gap on productivity and output in order to understand the magnitude of the impact.

Research on the relationship between entrepreneurial gender gaps and macroeconomic variables is scarce. Cuberes and Teignier (2016) developed a span-of-control model to illustrate how lower number of female managers and workers induces lower average income in developed and developing countries, accounting for out-of-necessity self-employment in the latter. Specifically they exogenously restrict women from participation in the labor market. They estimated a 15% and 18% average income loss for developed and developing countries, respectively. In a recent paper, Cuberes et al. (2019) used the same framework and estimated a 20% income per capita loss in the Balkans and Turkey. Esteve-Volart (2009) used a similar framework and adds human capital accumulation to the model to study the Indian case. She finds that a 10% increase of the ratio of female and male managers leads to a 2% GDP per capita increase. Cuberes and Teignier (2018) expanded their model to account for household production and found an average 17% aggregate output loss for the United States and Europe.

This thesis will differ from previous research in three ways. First, I will disregard any out-of-necessity self-employment to focus on developed countries. Second, previous research has focused on an exogenous restriction that prevents women from participating in the labor market. In contrast, I will introduce four labor market distortions which endogenously changes women's and men's labor decisions.[1] Lastly, the thesis will evaluate the impact of each gender distortion on aggregate output in order to provide an answer the question: Which are the most efficient policies to reduce output loss due to an entrepreneurial gender gap?

3 Theoretical Model

In this section, I introduce the basic model with which I evaluate the macroeconomic effects of an entrepreneurial gender gap. The model contains a two-sector economy (firm and household production) populated by an infinitely lived representative household composed by female and male members, who have heterogenous entrepreneurial abilities.[2] The representative household must decide the optimal sequence of consumption and the amount of capital carried over to the next period. In addition, there are two occupational choice problems that the representative household must solve. First, individuals must decide whether or not to participate in the labor market. Second, in case they participate, they must decide whether to run a business or become a production worker. I first describe their prefer-

[1] Cuberes and Teignier (2018) introduced household production as an endogenous distortion in their model. However, they analysed the cases when an exogenous percentage of women could not enter the labor market (gender gap) and or a percentage of women could not become entrepreneurs (entrepreneurship gap).

[2] Hereafter, I use the terms "abilities" and "skills" indistinguishably.

ences, their endowments, and the production technology. Then, I present how each distortion changes the proportion of workers, entrepreneurs and household producers for female agents. Finally, I discuss the market clearing and the equilibrium of this model.

3.1 Preferences

A single infinitely lived representative household populate the economy. The size of the household is L_t. Initial population size is exogenous an equal to L_0. Three types of agents populate this economy: mothers, childless women, and men. I denote by L_t^f, L_t^s, and L_t^m the number of mothers, childless women, and men, respectively. A fraction θ^f of the population are mothers, a fraction θ^s are childless women, and a fraction θ^m are men. The household preferences are described by a time-additively separable utility function over sequences of per capita consumption (c_t). The representative household gets flow utility from per capita consumption, by an increasing and concave function $(u(c_t))$. Then, the representative household maximizes:

$$U(\{c_0, ..., c_\infty\}) = \sum_{t=0}^{\infty} \beta^t L^t u(c_t), \quad u(c_t) = \frac{c_t^{1-\lambda}}{1-\lambda}, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor for the future and $1/\lambda$ is the intertemporal elasticity of substitution for the per capita consumption.

3.2 Endowments

The household is endowed with a positive (aggregate) stock of capital at date $t = 0$ ($K_0 > 0$). In addition, each household member is endowed with z units of managerial abilities. The distribution of abilities are identical between agent types. Specifically, I denote by $g(z)$ the density function of agents' abilities, with cumulative density function $G(z)$, and support in $Z = [\underline{z}, \overline{z}]$. There is perfect information regarding the entrepreneurial skills of each of the members of the representative household. Each household member is also endowed with one unit of time which is inelastically supplied in the labor market when the individual participates in it, or inelastically allocated to household production when the agent stays at home.

3.3 Production Technology

Each firm, a production unit, produces an homogeneous output.[3] There is significant firm heterogeneity in this model because each agent type with a particular skill z will create a firm with an specific size. In other words, firm and demand sizes will depend on agent types and ability levels.

Production of a single firm for agent type i and skill z is carried out using capital (k_i), labor (n_i), and entrepreneurial skills (z). Specifically, mothers' labor (n_i^f) and childless female labor (n_i^s) are combined in a CES function with an elasticity of substitution given by ρ and a share parameter μ:

$$n_i^w = \left(\mu^{\frac{1}{\rho}}(n_i^f)^{\frac{\rho-1}{\rho}} + (1-\mu)^{\frac{1}{\rho}}(n_i^s)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}. \tag{2}$$

The output n_i^w is then combined with male labor, n_i^m, in another CES function with an elasticity of substitution equal to σ and a share parameter ω:

$$n_i = \left(\omega^{\frac{1}{\sigma}}(n_i^m)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}}(n_i^w)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

This nested CES function n_i aggregates mother's, childless women's, and men's labor is then combined with capital using a Cobb-Douglas aggregator:

$$q_i = k_i^{\alpha}(An_i)^{1-\alpha}, \tag{4}$$

where A reflects a labor augmenting technological parameter.

Lastly, the output q_i is combined with managerial abilities using a Cobb-Douglas aggregator:

$$y_i(z) = z^{1-\zeta} q_i^{\zeta}. \tag{5}$$

where ζ is the span-of-control parameter that governs the diminishing returns to scale in

[3] Hereafter, I include the time index t when is strictly necessary for a clear exposition.

variable factors at the establishment level.[4]

3.4 The problem of a type-z entrepreneur

An agent with entrepreneurial ability z maximizes profits taking wages and the rental price for capital services as given. The static maximization problem of a type-z male agent is

$$\max_{k,n^f,n^s,n^m} \left[y_m(z) - w^f\left(1-\tau_w\right)n_m^f - w^s\left(1-\tau_w\right)n_m^s - w^m n_m^m - Rk_m \right], \tag{6}$$

where R is the rental price for capital services and w^i is the wage of labor type-i for $i \in \{f, s, m\}$. Because of the gender wage gap, male entrepreneurs face potentially distorted female wages. The magnitude of the distortion is modeled with the parameter τ_w, which represents a percentage reduction of female labor prices w^f and w^s. The first order conditions for a type-z male ($i = m$) entrepreneur are:

$$[k] : \zeta\alpha\frac{y_i(z)}{k_i(z)} = R. \tag{7}$$

$$[n^m] : \zeta(1-\alpha)\omega^{\frac{1}{\sigma}}\left(\frac{y_i(z)}{n_i}\right)\left(\frac{n_i}{n_i^m}\right)^{\frac{1}{\sigma}} = w^m. \tag{8}$$

$$[n^f] : \zeta(1-\alpha)(1-\omega)^{\frac{1}{\sigma}}\mu^{\frac{1}{\rho}}\left(\frac{y_i(z)}{n_i}\right)\left(\frac{n_i}{n_i^m}\right)^{\frac{1}{\sigma}}\left(\frac{n_i^w}{n_i^f}\right)^{\frac{1}{\rho}} = w^f\left(1-\tau_w\right). \tag{9}$$

$$[n^s] : \zeta(1-\alpha)(1-\omega)^{\frac{1}{\sigma}}(1-\mu)^{\frac{1}{\rho}}\left(\frac{y_i(z)}{n_i}\right)\left(\frac{n_i}{n_i^m}\right)^{\frac{1}{\sigma}}\left(\frac{n_i^w}{n_i^s}\right)^{\frac{1}{\rho}} = w^s\left(1-\tau_w\right). \tag{10}$$

In contrast, female entrepreneurs (f, s) have to consider a gender profit difference, a gender wage gap and a female capital constraint. The maximization problem for a constrained type-z female agent is

$$\max_{k,n^f,n^s,n^m} \left[y_i(z) - w^f\left(1-\tau_w\right)n_i^f - w^s\left(1-\tau_w\right)n_i^s - w^m n_i^m - Rk_i\left(\bar{k}\right) \right](1-\tau_\pi) \tag{11}$$

The parameter τ_π is a percentage reduction to the profits of a female entrepreneurs at all

[4] Given the vast firm heterogeneity, the aggregate production will be a weighted sum of existing firms by type of agent, represented in equation 27 on Section 5.

skill levels. They also face potentially distorted female labor prices due to the wage gap (τ_w). Female entrepreneurs could face a capital constraint, where they can only demand capital up until a certain point $\left(\bar{k}\right)$. After certain skill threshold, constrained female entrepreneurs will demand $\bar{k}$ independent of their entrepreneurial skill. One interpretation of $\bar{k}$ is a capital glass ceiling for female entrepreneurs. Capital demand of entrepreneurs is defined by:

$$k_i\left(\bar{k}\right) = \begin{cases} k_i(z) & \text{if } k_i(z) < \bar{k} \\ \bar{k} & \text{if } k_i(z) \geq \bar{k} \end{cases} \tag{12}$$

Note that the first order conditions for capital constrained and unconstrained females are different. An unconstrained female entrepreneur's first order conditions should satisfy equations (7) to (10). A constrained female agent's first order conditions should satisfy equation (8) to (10) given the fixed capital maximum $\bar{k}$.

Using equations (7) to (10), I can compute the relative demand of inputs for a type-z entrepreneur. The computed demand level for each of the inputs embodied in the production technology of the model for unconstrained and constrained agents are available in Appendix A and Appendix B, respectively. Therefore, relative input demands only depend on the set of prices $M = \{w^f, w^s, w^m, R\}$, the technological parameters of the economy, and the gender labor market distortions $T = \{\tau_w, \tau_\pi, \bar{k}, \tau_h\}$. [5]

3.5 The occupational choices

The representative household must decide who of their members participate in the labor market and, in case of doing so, how they participate. I assume that the former decision is only relevant for mothers, who can carry out household production. On the one hand, the occupational choice for men and childless women is whether to operate a business or work as a production worker. On the other hand, mothers must also choose whether or not to participate in the labor market. At home, a mother can use their time endowment to produce a household income of size $(1 + \tau_h)\, h(z)$. The parameter τ_h can be interpreted as additional productivity in house production or increased costs of childcare. Moreover, I

[5] Section 3.5 will explain the meaning of τ_h, the household distortion.

assume that h is a continuous function

$$h(z) = z^{B_s} + B_1 \tag{13}$$

where B_s represents the slope of the household production curvature and B_1 is a constant.

I relate the income produced by mothers at home to childcare duties.[6] To model this idea, I assume that that market income and household income are fungible. Nonetheless, the representative household must spend in childcare an amount equal to the size of its household production. The resources spent in childcare are no longer available for consumption. Concretely, the budget constraint of the representative household must satisfy:

$$L^f \underline{h} = L^f \int_{\underline{z}^f}^{\bar{z}^f} (1 + \tau_h)\, h(z) g(z) dz. \tag{14}$$

I now discuss the occupational choice of the agents. Working agents must decide in each period whether to operate a single productive unit or work for a wage. Let $\mathcal{I}^i(z; M; T)$ be an indicator that describes the optimal occupational choice for an agent with abilities of size z that face the market prices given by M and a set of labor distortions denoted by T. Specifically, I set $\mathcal{I}^i(z; M; T) = 1$ if the agent optimally decides to become and entrepreneur and operate a business and $\mathcal{I}^i(z; M; T) = 0$ if the agent decides to work for a wage. An individual that decides to become a entrepreneur must receive in equilibrium a compensation weakly higher that the wage earned by a production worker. Denote by $\pi^i(z; M; T)$ the solution to problem (6) for a type-z male entrepreneur and problem (11) for female entrepreneurs. For male agents,

$$\mathcal{I}^m(z; M; T) = \begin{cases} 1 \text{ if } \pi^m(z; M; T) > w^m \\ 0 \text{ if } \pi^m(z; M; T) \leq w^m. \end{cases} \tag{15}$$

Female agents face a profit gap (τ_π), a wage gap (τ_w), and a capital gap $(\bar{k})$. Then, the

[6] This assumption is quite strong because gender household roles are not fixed. Although women increasingly leaving their maternal roles in the household (Aldrich and Cliff, 2003), a vast majority of female parents comply with their traditional childcaring role.

occupational choice of female agents follow

$$\mathcal{I}^i(z;M;T) = \begin{cases} 1 \text{ if } \pi^i(z;M;T)\,(1-\tau_\pi) > w^i\,(1-\tau_w) \\ 0 \text{ if } \pi^i(z;M;T)\,(1-\tau_\pi) \leq w^i\,(1-\tau_w)\,. \end{cases} \tag{16}$$

for all $i \in \{f, s\}$.

Then, labor market earnings of a male agent i, can be expressed as:

$$e^m(z;M;T) = \mathcal{I}^m(z;M;T)\pi^m(z;M;T) + (1-\mathcal{I}^m(z;M;T))w^m \tag{17}$$

While female labor market earnings are defined by:

$$e^i(z;M;T) = \mathcal{I}^i(z;M;T)\pi^i(z;M;T)\,(1-\tau_\pi) + (1-\mathcal{I}^i(z;M;T))w^i\,(1-\tau_w) \tag{18}$$

for all $i \in \{f, s\}$.

In terms of female labor force participation, market and household income are fungible. Then, in equilibrium, working women must earn an income higher than the household income they can produce at home. Let $\mathcal{P}^f(z;M;T)$ be an indicator for optimal participation choice of a type-z mother that faces a set of market prices M and a set of labor distortions denoted by T. I set $\mathcal{P}^f(z;M;T) = 1$ if she decides to participate in the labor market and $\mathcal{P}^f(z;M;T) = 0$ if the she decides not to do so. Then,

$$\mathcal{P}^i(z;M;T) = \begin{cases} 1 \text{ if } e_t^f(z;M;T) > (1+\tau_{hh})\,h(z) \\ 0 \text{ if } e_t^f(z;M;T) \leq (1+\tau_{hh})\,h(z) \end{cases} \tag{19}$$

where $\mathcal{P}^s(z;M;T) = \mathcal{P}^m(z;M;T) = 1$.

Therefore, the per capita income of the representative household generated by members type i is

$$v^i(z;M;T) = \int_{\underline{z}^i}^{\overline{z}^i} e^i(z;M;T)\mathcal{P}^i(z;M;T)g^i(z)dz, \tag{20}$$

In addition, I can express per-mother income generated from household production as

$$d(z;M;T) = \int_{\underline{z}^f}^{\overline{z}^f} (1+\tau_{hh})\, h(z)(1-\mathcal{P}^f(z;M;T))g^f(z)dz. \tag{21}$$

3.6 The representative household problem

The representative household must choose the sequence of per capita consumption, the sequence of aggregate capital to carry over to the next period, the labor market participation of its mothers, and the occupation of its working members, taking all prices and distortions as given:

$$\max_{\{c_t, K_{t+1}, \{\mathcal{P}_t^f(z)\}_{z\in Z^f}, \{\mathcal{I}_t^i(z)\}_{z\in Z^i}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t \frac{c_t^{1-\lambda}}{1-\lambda}, \tag{22}$$

$$\text{s.t. } C_t + L^f \underline{h} + K_{t+1} = \sum_{i\in\{f,s,m\}} L_t^i v_t^i(z) + L_t^f d_t(z) + R_t K_t + (1-\delta)K_t,$$

$$L_t^f \underline{h} = L_t^f \int_{\underline{z}^f}^{\overline{z}^f} (1+\tau_h)\, h(z) g^f dz,$$

$$K_0 > 0, L_0^f > 0, L_0^s > 0, L_0^m > 0,$$

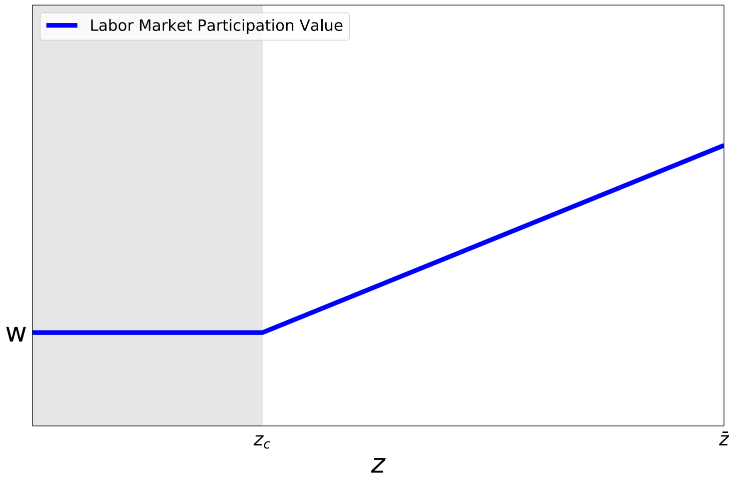
where C_t is aggregate consumption at date t, that is, $C_t = L_t c_t$, and δ is the depreciation rate of capital.[7] The first order conditions for per capita consumption and aggregate capital allow me to derive the Euler Equation:

$$\left[\frac{c_{t+1}}{c_t}\right]^{\lambda} = \beta\left(R_{t+1} + 1 - \delta\right) \tag{23}$$

Notice that equations (15), (16), and (19), which I have already discussed, satisfy the first order conditions for occupational choices and the mothers' decision on whether or not to participate in the labor market.

[7] I omitted the price vector M and distortion vector T to simplify the equations.

Figure 1: Childless women and men entrepreneurial decisions.



3.7 Labor market distortions and occupational choices

The interaction between occupational decisions, the entrepreneurial problem and labor market distortions (T) define the number of entrepreneurs, workers, and household producers in each period. In this section I will explain how agents decide whether to become a worker, entrepreneur, or a stay at home producer in a benchmark economy with no labor market distortions. Then, I will compare the benchmark economy to each labor market distortion.

3.7.1 Benchmark Economy

In the benchmark economy, absent of labor market distortions, the entrepreneurial decisions are driven by the labor market participation value (equation 17 for male and 18 for female agents), their entrepreneurial ability (z), and agent type (i). The market value function is a convex non-decreasing function in z. Figure 1 shows the shape of the market value function and the occupational decisions of childless women and men. The skill cutoff z_c represents the point where childless female or male agents decide to become entrepreneurs. Therefore, workers have skills between $\underline{z}$ and z_c and the rest of the skill space is occupied by childless women and men entrepreneurs.

A mother must decide whether to participate in the labor market or produce in the household. For this reason, the household production function is concave and increasing in z. Figure 2 shows the mothers occupational choices given their skill level. On the one hand,

z_w denotes skill cut-off between mother workers and household producers. On the other hand, z_e is the cutoff between between household producers and mother entrepreneurs. Thus, mother workers occupy the skill space from $\underline{z}$ to z_w, household producers occupy the space between z_w to z_e, and mother entrepreneurs occupy the remaining skill space.

A distortion will change these thresholds and thereby change the proportions of mothers who are entrepreneurs, workers, and household producers.

The next subsection discusses how a gender profit gap, an increase in household productivity, a capital glass ceiling, and a wage gap will change the occupational and labor choices of all agents.

3.7.2 Profit Gap

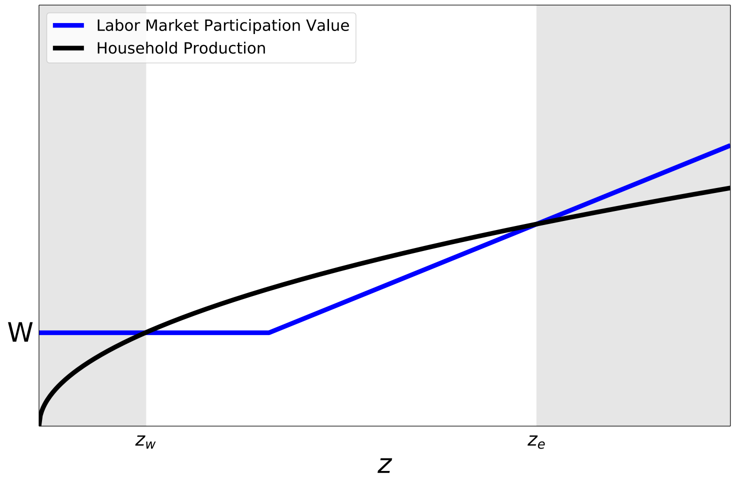
An increase in the profit gap will decrease the type-z female entrepreneur market value.[8] Figure 3 shows how the skill thresholds changes in the presence of a positive profit gap for childless female (Panel A) and mother (Panel B) agents. For childless female agents there is a slight increase of the entrepreneurial skill cut-off ($z_c' > z_c$). This increases the amount of childless workers and decreases the number of childless entrepreneurs. For mother agents the effect depends on the household production function. In order to simplify the analysis, I will only consider the case where a mother with skill z_c decides to not participate in the labor market - this is the case for Figure 3 Panel B. With this in mind, a profit gap increases mothers' skill threshold to become an entrepreneur, thus reducing the number of mother entrepreneurs. The mother's maximum worker skill will remain unchanged if the household production function stays over the new z_c'.

3.7.3 Household Productivity

A positive increase in household productivity (τ_h) distorts mother's occupational choices through two effects. First, this change should reduce the maximum working skill threshold ($z_w' < z_w$), leading to a reduction of mother workers. Second, the amount of mother

[8]Female entrepreneur's first order conditions (7-10) are insensitive to a profit gap. A profit gap will only change female occupational decisions.

Figure 2: Mothers labor market and entrepreneurial decisions.



entrepreneurs decreases because the minimum skill to become a mother entrepreneur increases ($z'_e > z_e$). In other words, more productive households simultaneously reduce the amount of mother workers and mother entrepreneurs. Figure 4 shows the skill threshold movements due to the increase in household productivity.

3.7.4 Capital Glass Ceiling

In the model, the capital restriction acts as a maximum capital level that female entrepreneurs can demand. It directly impacts high skilled women entrepreneurs because they demand the most capital. Women face a capital demand glass ceiling, which I interpret as differential credit conditions. Let z_d be the skill cut-off for female entrepreneurs that are capitally constrained. Unconstrained women share the same entrepreneurial choices as the benchmark. Meanwhile, constrained women located above the skill level z_d have a lower fixed capital demand, which reduces their production levels, labor demand, and subsequently profits.

Figure 5 shows how does capital glass ceiling distorts childless female (Panel A) and mother (Panel B) agents' decisions. Constrained childless female agents will only change their occupational decisions if the profit level is lower than their wage. Constrained mothers will most likely leave the labor market as they are more productive at home than with a capital glass ceiling. Consequently, the net effect is a reduction of mother entrepreneurs with skills between $[z_d, \bar{z}]$.

3.7.5 Wage gap

A wage gap directly affects labor and production decisions. On the one hand, lower female wages decreases female labor as it becomes less profitable. On the other hand, it will reduce entrepreneurs' costs for every agent type, thus making entrepreneurship more profitable and increasing the market value for every entrepreneur. Figure 6 shows how this gap change childless female (Panel A) and mother (Panel B) agents' occupational choices.

Male agents experience an increase in profits due to the lower female labor wages. Conversely, childless agents experience two amplifying effects. First, lower wages will incentivize women near the skill cut-off to create new businesses, which will lower skill cut-off ($z'_c < z_c$). Second, childless entrepreneurs face lower production costs, thus increasing potential profits. The latter effect will further decrease the working skill cut-off. Therefore, there are fewer female workers and more female entrepreneurs compared to the benchmark case.

Mother agents experience similar effects but the household production will defines final amount of workers and entrepreneurs. A decrease in working mothers can be noted by a reduction of their skill cut-off ($z'_w < z_w$). Those working mothers will leave the labor market for household production. Simultaneously, the increase in relative profits will incentivize entrepreneurship, as seen in a decrease of the entrepreneur skill cut-off ($z'_e < z_e$). In this case, a portion of women in household production decide to participate in the labor market as entrepreneurs. Thus, the wage gap decreases the amount of mother workers and increases mother entrepreneurs.

3.8 Market clearing

I now derive the market clearing conditions of the model. The market clearing condition for capital requires that the aggregate capital of the representative household optimally carry over to the next periods equals the aggregate demand for capital by the productive units:

$$K_t(M_t) = \sum_{i \in \{f,s,m\}} L_t^i \int_{\underline{z}^i}^{\overline{z}^i} k_t^i(z; M_t; T)\mathcal{P}_t^i(z; M; T)\mathcal{I}_t^i(z; M_t; T)g^i(z)dz. \tag{24}$$

Figure 3: Profit gap restriction and occupational choices.

Panel A: Childless
Panel B: Mothers

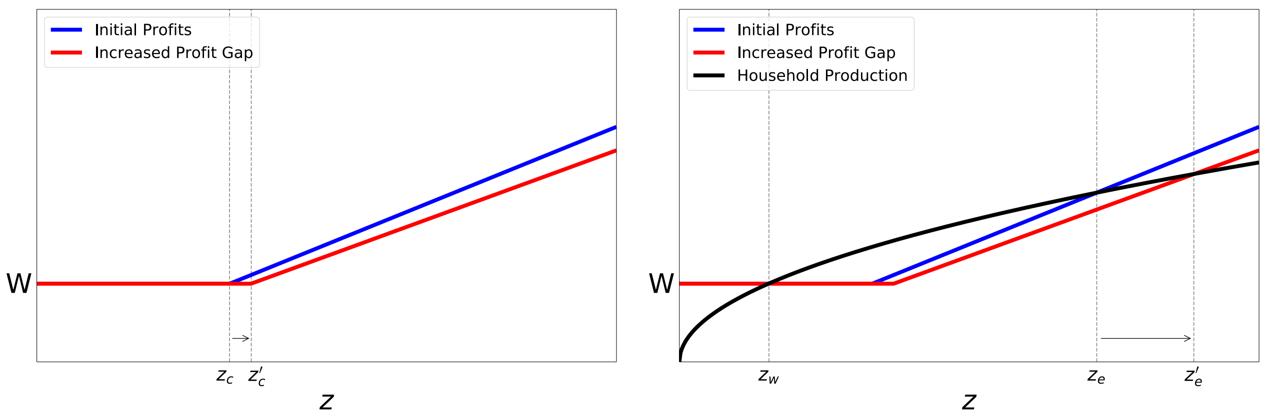


Figure 4: Household productivity and mother's occupational choices.

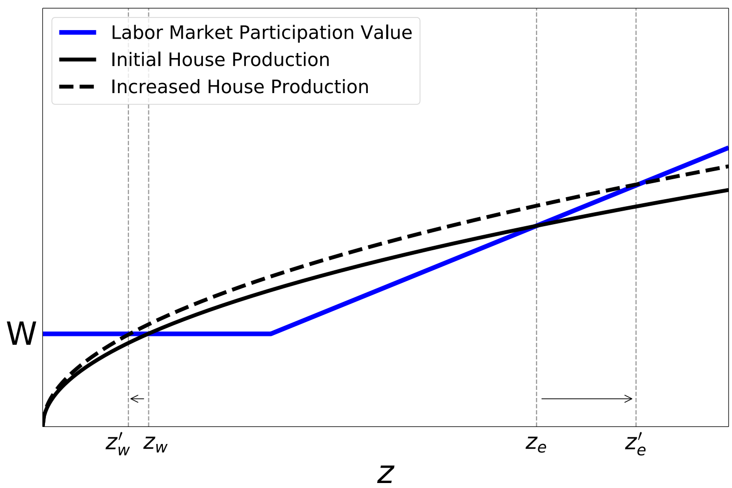
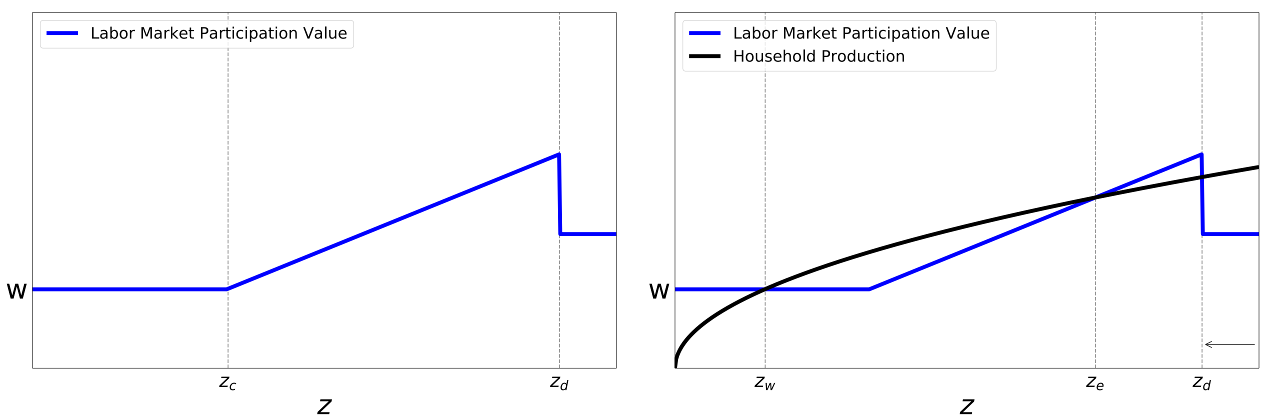


Figure 5: Capital glass ceiling effects on mother's occupational choices.

Panel A: Childless
Panel B: Mothers



The market clearing for the female labor market must equal demand and supply of female labor. The latter is determined by the women who decide to participate in the labor market as production workers. An analogous condition determines the labor market clearing condition for the labor of childless women and mothers. Men labor markets must also clear, so the labor demand for men must equal the available supply. Then,

$$L_t^j \int_{\underline{z}^j}^{\overline{z}_t^j} \mathcal{P}_t^j(z;M_t;T)(1-\mathcal{I}_t^j(z;M_t;T))g^j(z)dz = \sum_{i\in\{f,s,m\}} L_t^i \int_{\underline{z}^i}^{\overline{z}^i} [n_t^j(z;M_t)]^i \mathcal{P}_t^i(z;M_t;T)\mathcal{I}_t^i(z;M_t;T)g^i(z)dz, \quad (25)$$

for $j = \{f,s,m\}$. Finally, the market clearing condition for the unique good produced in this economy is given by:

$$\sum_{i\in\{f,s,m\}} L_t^i \int_{\underline{z}^i}^{\overline{z}^i} y_t^i(z;M_t;T)\mathcal{P}_t^i(z;M;T)\mathcal{I}_t^i(z;M_t;T)g^i(z)dz = C_t(M) + K_{t+1}(M) - (1-\delta)K_t(M). \quad (26)$$

3.9 Equilibrium

I now define the competitive equilibrium of the economy:

Definition 1. *A competitive equilibrium is a set of allocations* $\{c_t, K_{t+1}, \{\mathcal{P}_t^f(z)\}_{z\in Z}, \{\mathcal{I}_t^i(z)\}_{z\in Z}\}_{t=0}^{\infty}$ *for the representative household, a set of prices* $\{w_t^f, w_t^s, w_t^m, R_t\}$*, and a set of labor market distortions* $\{\tau_w, \tau_\pi, \tau_h, \bar{k}\}$ *such that (i) the household problem (22) is solved taking all the prices as given, (ii) the market for capital services clears for all t (equation 24 holds), (iii) the market for labor services clears for all t (equations 25 hold), and (iv) the market for goods clears for all t (equation 26 holds).*

In Appendix C I show step-by-step how to solve the competitive steady state equilibrium of our benchmark economy.

4 Calibration Strategy

To identify the effects of an entrepreneurial gender gap I will calibrate a benchmark economy in the absence of labor market distortions ($\tau_\pi = \tau_w = \tau_h = 0$ and $\bar{k} = \infty$). The model parameters are set to match the observations of Norway, Sweden, and Slovenia which have some of the most gender equal labor markets in the OECD. Averaging these countries should yield an economy with low labor market distortions which I call the benchmark country.

Table 1 summarizes the statistics of each country (columns 1-3), the benchmark country (column 5), and the OECD sample means (column 6). The statistics presented are: (i) an entrepreneurship gender gap, (ii) a profit gender gap, (iii) a wage gap, (iv) a labor force participation ratio between genders, (v) secondary and tertiary education attainment ratios, and (vi) percentage of female parliament seats. The entrepreneurship gap measures the difference between female and male percentages in self-employment extracted from the WorldBank database for 33 countries between 2010 and 2019. The profit gap is the difference in median earnings between self-employed males and self-employed females, relative to the median earnings of the latter. This data is available in the OECD Gender Equality Database for 27 countries between 2009 and 2018. The wage gap measures the difference between median earnings of men and women relative to median earnings of men available in the OECD Gender Equality Database for 37 countries between 2009 and 2018. The labor force participation ratio, extracted from the WorldBank database for 36 countries between 2010 and 2019, compares the females labor participation rate relative to males. Secondary education attainment ratio measures the percentage of females with a secondary education relative to the percentage of males with a secondary education extracted from the WorldBank database for 35 countries between 2010 to 2019. The tertiary education attainment is the ratio of the percentage of females to the percentage of males who completed any type of tertiary education, estimated from the updated database of Barro and Lee for the years 2005, 2010 and 2015 for 150 countries. Lastly, the percentage of female parliament seats was extracted from the European Institute for Gender Equality for 25 European countries between 2009 and 2018.

Table 2 shows the out-of-the-model parameters divided into three categories: population, preferences and production. First, male and female population percentages where calculated using population statistics from the WorldBank for the 40-44 years cohort in 2010. The childless population was extracted from the Human Fertility Database for the same cohort and year. In the model, men (θ_m) represent 51% of the household, while mothers (θ_f)

Figure 6: Wage gap and occupational choices.

Panel A: Childless

Panel B: Mothers

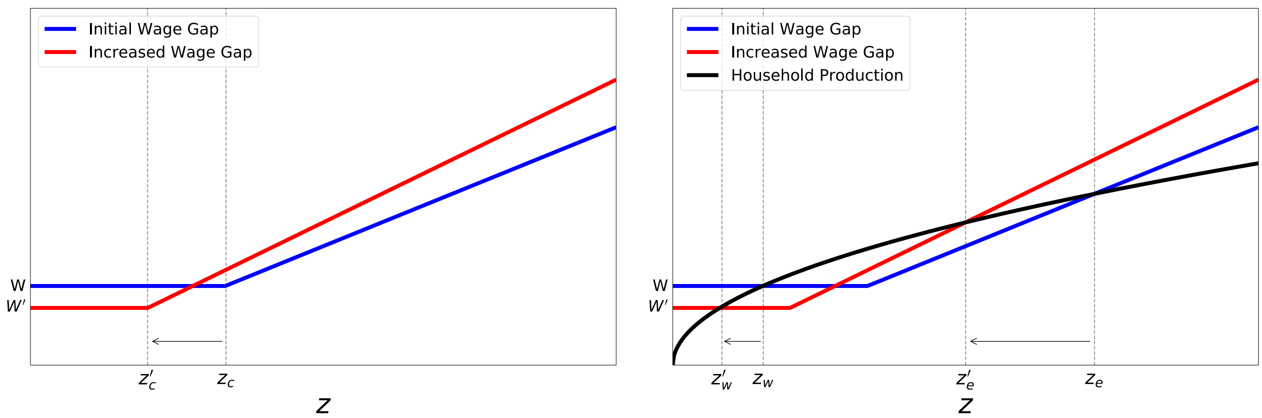


Table 1: Benchmark countries.

Statistic	Norway	Sweden	Slovenia	Benchmark Country	Sample Mean
Entrepreneurship Gap	0.051	0.081	0.059	0.064	0.068
Profit Gap	0.21	0.16	0.32	0.23	0.31
Wage Gap	0.07	0.09	0.03	0.06	0.14
Labor force participation ratio	0.90	0.89	0.82	0.87	0.79
Secondary education attainment ratio	0.98	0.99	0.89	0.95	0.94
Tertiary education attainment ratio	1.14	1.17	1.30	1.20	1.07
Percentage of female parliament seats	0.39	0.45	0.22	0.35	0.29

Notes: Each statistic represents the mean value between the years in the sample. Labor force participation ratio, education completion ratio of secondary school where extracted from the WorldBank Database for the years 2010 to 2019. Tertiary education completion where extracted from Barro & Lee dataset from 2005 to 2015. The entrepreneurship, wage and profit gender gap where extracted from the Gender Equality Database from 2009-2018. Lastly the political female participation where extracted from the Eurostat from the European Institute for Gender Equality (EIGE) from 2009 to 2018.

and childless women (θ_s) represent 43% and 6%, respectively.

The preference parameters are the discount factor (β) and constant relative risk aversion (λ). The discount factor takes a standard literature value of 0.93. In order to correctly estimate the constant relative risk aversion parameter, I chose an intertemporal elasticity of substitution (IES) of 0.3, which is consistent with Havránek (2015) macro estimates. To determine the constant relative risk aversion I took the inverse of the IES which set $\lambda = 3.33$.

For the production parameters, I set the depreciation rate (δ) to 0.04. Consistent with previous estimations, I set the constant elasticity of substitution (CES) between male and female labor (σ) to 3 (Acemoglu et al., 2004). To avoid an arduous econometric analysis, beyond this thesis aims, I set the mother's and childless females' labor elasticity (ρ) to 6. I assume that the substitutability within females should be greater than the substitutability between genders. Consequently, doubling the curvature of the CES function should allow relatively more substitution between mothers' and childless females' labor than between different genders' labor . Both of the CES technology weights (ω, μ) were normalized to 0.5. Lastly, the capital share (α) is set to 0.33, which is consistent with European estimates (Gopinath et al., 2017).

The remaining parameters are the household production slope (B_s) and constant (B_1), the skills distribution, and the span of control parameter (ζ). The household production function directly impacts the female labor force participation in the model. I normalize the slope of the household production function (B_s) to 0.5, since this decreasing slope allows for mothers to participate in the labor market as entrepreneurs. In contrast, a slope higher than 1 would imply that the most skilled mothers would be fully committed to household production. However, the existence of female managers is consistent with a decreasing slope approach. I calibrated the household production constant (B_1) to match the average female labor participation mean between 2010 and 2019 for the chosen countries.

The skill distribution and the span of control parameter (ζ) directly determine firm size distribution and their labor demands. I used the Eurostat's Structural business statistics estimates of the (i) share of employment of firms with 10 to 19 employees, (ii) share of employment of firms with 20 to 49 employees, (iii) share of employment of firms with 50 to 249 employees, (iv) share of employment of firms with 250+ employees as the optimal targets.

Similar to Guner et al. (2008), our estimates show that a handful of large firms account for a disproportionate fraction of employment. Most of the firms in the data (90%) are small (0-9 employees) and demand 28% of total employment. Meanwhile, large firms (50+ employees) represent 2% of firms and demand 50% of total employment. Given this empirical disparity, I followed Guner et al. (2008) and set entrepreneurial skills to follow a log normal distribution with an extreme value in the right tail. Therefore, there are four free parameters: the mean (μ_z) and variance (σ_z) of the log-normal distribution, the extreme value (e_z), and its mass point (m_e). The mass point of the extreme value was insensitive to all calibration experiments, thus I normalized it to 0.000025. In total there are four shares of employment to match four parameters (ζ, μ_z, σ_z, e_z). Table 3 shows the calibrated parameters and the model performance.

5 Results

In this section I compare the benchmark economy to a set of distorted economies in their respective steady states. First, I define aggregate output and aggregate productivity, which are the model measures used to quantify the macroeconomic effects of each gender distortion. Additionally, I introduce four additional variables that help understand the mechanisms behind the aggregate changes. Then, I will describe the general equilibrium effects on these outputs of (i) a 20% female profit reduction, (ii) a 20% household productivity increase, (iii) a capital glass ceiling, and (iv) a 20% female wage reduction.

5.1 Model Outputs

My analysis will focus on aggregate output and productivity to measure the effects of an entrepreneurial gender gap. The total aggregate production is the weighted sum of the entrepreneurs' production given the population parameters (θ_i) and the probability of being type-z ($g(z)$),

$$Y = \sum_{i=\{f,s,m\}} \theta_i \int_{\underline{z}}^{\overline{z}} y_i I_i P_i g(z) dz \tag{27}$$

Table 2: Out-of-the-model Parameters.

Parameter		Value	Notes
Population Parameters			
Mothers	(θ_f)	0.43	World Bank
Childless	(θ_s)	0.06	Human Fertility Database
Men	(θ_m)	0.51	World Bank
Utility			
CRRA	(λ)	3.33	Havránek (2015)
Discount Factor	(β)	0.93	Normalization
Production			
Elasticity substitution male and female	(σ)	3	Acemoglu (2004)
Elasticity substitution mothers and childless	(ρ)	6	Normalization
CES weights	(ω, μ)	0.5	Normalization
Capital share	(α)	0.3	Gopinath et al. (2017)
Depreciation rate	(δ)	0.04	Normalization

Table 3: Calibration and Model Performance

Parameter		Value
Household Production		
Slope	(B_s)	0.5
Intercept	(B_1)	-0.51822
Firm Distribution		
Span of Control	(ζ)	0.725350
Log-normal Average	(μ_z)	-0.00029
Log-normal Standard Deviation	(σ_z)	0.768108
Extreme Value	(e_z)	77.14029
Extreme Value Mass	(m_e)	0.000025

Targets	Data	Model
Labor Participation		
Female rate	0.58	0.57
Employement Shares:		
10 - 19 employees	0.10	0.08
20 - 49 employees	0.11	0.10
50 - 249 employees	0.19	0.14
250+ employees	0.31	0.28

where $i = \{f, s, m\}$ denotes the type of agent in the economy, I_i is the entrepreneurial decision vector, P_i is the labor decision vector, and $g(z)$ is the calibrated skill density function. A entrepreneurial gap reduces aggregate output because it leads some skilled female agents who endogenously decide to stay at home or work rather than create a productive firm.

The Total Factor Productivity (TFP) is computed with a standard Sollow-Residual,

$$TFP = \frac{Y/L}{(K/L)^{\alpha}}$$

where K is the aggregate capital and α is the calibrated capital share.

An entrepreneurship gap decreases aggregate output and productivity through two main channels: fewer overall entrepreneurs (quantity) and less skilled entrepreneurs (quality). First, a lower number of entrepreneurs in the economy leads to lower production levels if the average skill level remains unchanged. Second, lower aggregate production can be caused by a policy that lowers entrepreneurs' skill level across all types, even when no entrepreneur leaves the labor market or becomes a worker.

Changes in the number of firms and in the percentage of female entrepreneurs suggest that the quantity channel is at play. Meanwhile, changes in the average entrepreneur's skill and in the gender entrepreneurship skill ratio[9] suggest that the quality channel is at play. Therefore, I report these four variables in order to help understand the mechanisms behind a reduction of aggregate output and productivity due to a gender distortion.

5.2 Distortions

In this subsection I explain the results of applying each distortion to the calibrated benchmark economy. Each distorted economy has one distortion applied at a time. The first distorted economy introduces a 20% profit reduction for female entrepreneurs at all z levels ($\tau_w = 0.2$). The second one introduces a 20% increase in household productivity at any entrepreneurship skill level ($\tau_h = 0.2$). Higher household productivity could be interpreted as higher costs for childcare services. The third economy introduces a capital maximum

[9]The gender entrepreneurship ratio is the female entrepreneurs' percentage of the female population divided by the male entrepreneurs' percentage of the male population.

Table 4: Key model outcomes.

		Aggregate Output	TFP	Average entr. skills	N° of firms	% female entrepreneurs	Gender entr. ratio
Benckmark		100.00	100.00	100.00	100.00	100.00	100.00
Profit Gap	$(\tau_\pi = 0.2)$	96.95	97.95	98.75	100.06	61.23	60.84
Household Prod Increase	$(\tau_h = 0.2)$	93.65	95.66	123.99	98.91	63.87	63.69
Capital Glass Ceiling	(3rd male quantile)	99.40	99.60	94.96	100.77	70.52	70.44
Wage Gap	$(\tau_w = 0.2)$	100.00	100.00	100.00	100.00	100.00	100.00

demand ($\bar{k}$) for female entrepreneurs. Specifically, female entrepreneurs have a maximum capital demand equal to the third quantile of male entrepreneurs' capital demand. The last economy reduces female wages by 20% ($\tau_w = 0.2$). Table 4 summarizes key outcomes in these distorted economies relative to the benchmark model.

5.2.1 Profit Gap

For the first experiment I considered a profit gap of 20%. That is, female entrepreneurs only obtain 80% of the equilibrium profits for their skill level. This policy distorts both childless females' and mothers' occupational decisions. Lower profits reduces the amount of female entrepreneurs as it increases the entrepreneurial skill thresholds for all female agents (higher z_c and higher z_e). In turn, this decreases the labor demand for all agent types. Additionally, it increases labor supply as working becomes more attractive than being an entrepreneur. At this point the aggregate production decreases because of the lower quantity of female entrepreneurs.

A labor demand decrease combined with a supply increase drives wages down and increases entrepreneurs' profits, thus increasing the output of every firm. The net effect of a profit gap is ambiguous because the initial drop in female entrepreneurs' profits is mitigated by the subsequent reduction in production costs.

Table 4 shows that the aggregate output is 3% less than the benchmark's. Productivity, measured by the TFP, decreases by 2%, suggesting that a profit gender gap has a net negative effect on macroeconomic variables. The gender entrepreneurship ratio is 39% lower than in the benchmark economy. That is, there are a lower number of skilled female firms and a higher number of low skilled male firms. Even though the number of firms stayed roughly the same, the lower average entrepreneur's skills significantly reduces aggregate production and productivity. This implies that the effect of entrepreneurs' quantity is offset by the lower quality of entrepreneurs. In sum, a profit gap significantly reduces aggregate output and aggregate productivity because there are fewer female entrepreneurs, even when the number of low skilled male entrepreneurs increased.

5.2.2 Household productivity increase

The household production, by itself, excludes some mothers from participating in the labor market. Moreover, increasing household productivity excludes a greater number of mothers. This experiment introduces a 20% ($\tau_h = 0.2$) increase in household productivity,[10] which shifts the household production upwards and increases its slope at every point.

As a result, mothers leave the labor force to produce in the house instead, thus decreasing labor demands for all agents and reducing mothers' available labor supply. Initially, aggregate output and productivity suffer a significant decrease.

The distribution of skills ($g(z)$) and their probability mass determine the overall wage effect. The skill are shaped by a log-normal distribution with an extreme value at the right tail. Therefore, the central mass of the distribution is concentrated on the lower skill values, where workers reside. Due to the nature of the distribution, an increase in household productivity should disproportianatly affect mother workers. Specifically, more workers leave the labor market than the reduction in entrepreneurs' labor demand implies. Consider the next condition:

$$G(z_w) - G(z'_w) > G(z'_e) - G(z_e) \tag{28}$$

Let $G(z)$ be the cumulative distribution function of the calibrated skills distribution, z_w and z_e are the benchmark's skill thresholds workers and entrepreneurs, respectively. Lastly, z'_w and z'_e are those same thresholds for the distorted economy. The left hand side of equation 28 is the total mass of mother workers leaving the labor force, while the right hand side is

[10] An alternative interpretation is a 20% increase in the costs of childcare activities.

the total mass of mother entrepreneurs leaving the labor force. Equation 28 holds in the calibrated model, leading to a higher supply reduction than the demand reduction. The joint movement of the two curves, i.e. lower demand and supply, leads to an increase in mothers' wages. Higher mother wages decreases profits for every entrepreneur, amplifying the negative effects of increased household productivity on aggregate output and aggregate productivity.

As shown in Table 4, an increase in household productivity accounts for a 6.3% drop in aggregate production and reduces productivity by 4.4%. The overall effect is driven by the exit of moderately skilled female entrepreneurs and low skilled male entrepreneurs. Average entrepreneur's skill increases by 24% because female and male entrepreneurs, with moderate and low skills respectively, decide to become household producer or workers. The 1% decrease in the number of firms in the economy suggest that the production cost increase is sufficient to drive some agents out of entrepreneurship. Additionally, a 36.1% reduction of female entrepreneurs, compared to 0.27% reduction of male entrepreneurs, indicate that this distortion disproportionately hurts female agents. To summarize, a household production increase decreases aggregate output and productivity because female and male entrepreneurs exit the market.

5.2.3 Capital glass ceiling

In this distortion, a capital glass ceiling $(\bar{k})$ is going to be equal to the third quantile of male entrepreneurs' capital demand. That is, I search for the male entrepreneur at the third quantile of the capital vector and then select his capital demand to be $\bar{k}$. A capital glass ceiling reduces production levels of high skilled female entrepreneurs because they demand the most capital. Consequently, a portion of female entrepreneurs leave the labor market as it becomes less profitable. This reduces the labor demand for each type of agent, driving wages down. The surviving firms, mostly owned by high skilled men entrepreneurs, face lower production costs, thus making entrepreneurship more attractive and incrising production. In the general equilibrium, the initial production loss is later mitigated by lower production costs.

Table 4 shows that aggregate output and productivity decreases by less than 1% compared to the benchmark. There are two forces that explain this net effect. First, a 29% decrease of female entrepreneurs suggests that the exit of high skilled female entrepreneurs reduces the average entrepreneur's skill by 5%. Second, a 0.77% increase in the number of firms supports that the entry of low skilled male and female entrepreneurs mitigates most of the initial negative effect. In other words, the reduced entrepreneurs' quality is mostly offset by an increase in the quantity of entrepreneurs and the productivity of the remaining entrepreneurs.

5.2.4 Wage gap

In this last experiment I reduce female wages by 20% ($\tau_w = 0.2$). Once the distortion sets, the relative cost of women's labor decreases and is followed by an increase on the relative use of female labor. Lower wages also increases total labor and capital demand, increasing profits and total production. In the partial equilibrium, lower wages have a positive impact on output and productivity. However, the labor supply of female labor decreases as entrepreneurship becomes relatively more attractive. The joint behavior of demand and supply inevitably increases wages. This experiment yielded the same aggregate output, productivity, skill's averages, number of firms, and percentage of female entrepreneurs of the benchmark economy, i.e. the distortion did not have any effect. Particularly, equilibrium female wages were exactly $\frac{w_0^i}{(1-\tau_w)}$, where w_0^i is the benchmark's equilibrium wages for mothers' (f) and childless females' (s) labor. This suggests that demand and supply movements perfectly adjusted to the benchmark output but with female wages higher in proportion to the magnitude of the wage gap.[11]

This distortion has a particular property. Both the distortion and the general equilibrium involve changes in the same variable, wages, which was not the case in previous distortions. A wage gap distorts the economy through labor prices, while the general equilibrium clears markets through those same prices. Therefore, any effect of a wage gap can be completely mitigated in the long run when the workers' marginal productivity equals their wage. The long run optimal allocation of labor in an economy with immutable popula-

[11]The model described in this thesis does not internalize the mechanisms in which a gender wage gap can influence aggregate output. For instance, it does not contain an open economy like in Blecker and Seguino (2002b) nor does it have an educational process that Morrison et al. (2007) studied.

tion and skills should yield the same workers' marginal productivity and aggregate output regardless of the wage gap's magnitude.[12]

6 Conclusion

The entrepreneurship gap has increased in the last decades and has brought lower productivity and output. To measure these effects, I constructed a span-and-control model with four labor gender distortions that prevent women from participating in entrepreneurship. First, a profit gap between female and male entrepreneurs can deter skilled women from creating productive firms. Second, household production, or childcare costs, play an important role on females' decision to join the labor market. Higher household costs, which solely affect women, have the potential to significantly reduce the number of female entrepreneurs and workers. Third, different credit conditions between men and women increase budget and liquidity constraints for female entrepreneurs, leading to fewer of them. Lastly, a gender wage gap can disincentivize women from participating in the labor market while incentivizing business creation. Taking into consideration the previous distortions, I calibrated a benchmark model based on three countries with gender equal labor markets. Then, I implemented four experiments and compared them to the benchmark model: a 20% profit gap, a 20% increase in household productivity, a capital glass ceiling, and a 20% wage gap.

Reducing female profits and increasing household productivity produces significant negative effects on both productivity and output. Thus, increased childcare and bridging the profit gap can reduce the negative effects of an entrepreneurial gender gap. Childcare policies like vouchers for private childcare, increased coverage of public childcare facilities, and government subsidies have the potential to decrease mothers' household costs, increase both their labor [13] and entrepreneurship participation, and subsequently improve the economy's aggregate output and productivity.

[12]This finding is insensitive to any model parameters, which suggests that the wage adjustment is due to the analyzed equilibrium condition and the general equilibrium forces.

[13]Nollenberger and Rodríguez-Planas (2013) studied the Spanish case where offering full-time public childcare for three-year-olds' increased female labor participation in the 30-year-old mother cohort. Baker et al. (2008) studied the effects of highly subsidized childcare in Quebec and found that this policy significantly increased the labor supply but at the detriment of mothers' health and parental relationship.

An extra childcare policy might lead to decreasing returns in countries with already strong childcare policies. To illustrate this point, Lundin et al. (2008) found that setting a maximum price in private childcare facilities had small effects in Sweden, a country with high female labor participation and extensive childcare policies. Since a profit gap reduces aggregate output and productivity, economies can focus on bridging this gap to improve macroeconomic variables. Although there is no clear explanation for this gap, incentivizing research in this area can prove beneficial. That said, the report *Closing the Gender Gap* (2012) recommends specialized government assistance for female entrepreneurs, even though the efficiency of this policy remains untested.

The wage gap and the capital gap had a negligible and a small effect on aggregate output and productivity. The wage gap initially boosts aggregate output, but the subsequent increasing wages completely compensate for this effect. The capital gap initially reduces output. As prices begin to decrease, productivity rises in all existing firms, which compensate for the initial negative effect. However, the exit of the most skilled women entrepreneurs can reduce output and productivity, albeit by a small amount. Equalizing credit conditions for male and female entrepreneurs could help mitigate short-run macroeconomic effects. This could be achieved via increasing microfinance regulations or by promoting more female loan officers in commercial banks. First, women entrepreneurs heavily rely on microfinance which often provides worse credit conditions than traditional banks. For example, in Brazil there is no systematic microfinance regulation. Agier and Szafarz (2013) argues that this is an opportunity for external players, such as government institutions, to reduce the gender bias and incentivize women's entrepreneurship. Second, a policy incentivizing banks to hire female loan officers that handle female cases can improve their access to credit (Beck et al., 2005).

There are three ways in which this thesis could be improved. First, a different calibration strategy could improve the discussion of the estimated effects. For instance, a effect decomposition can be achieved if each distortion is calibrated for an average developed country. Second, relaxing the assumption that mothers are the only ones that pay the costs of childcare are relevant in studying paternal and maternal leave policies. Third, introducing an education process that shapes the skill distribution could lead to the wage gap having an impact on macroeconomic variables.

References

Acemoglu, D., D. H. Autor, and D. Lyle (2004). Women, war and wages: The effect of female labor supply on the wage structure at midcentury. *Journal of Political Economy*. 4

Agier, I. and A. Szafarz (2013, feb). Microfinance and Gender: Is There a Glass Ceiling on Loan Size? *World Development 42*, 165–181. 1, 6

Aldrich, H. E. and J. E. Cliff (2003, sep). The pervasive effects of family on entrepreneurship: toward a family embeddedness perspective. *Journal of Business Venturing 18*(5), 573–596. 6

Aristei, D. and M. Gallo (2016, aug). Does gender matter for firms' access to credit? Evidence from international data. *Finance Research Letters 18*, 67–75. 1

Asiedu, E., J. A. Freeman, and A. Nti-Addae (2012, may). Access to Credit by Small Businesses: How Relevant Are Race, Ethnicity, and Gender? *American Economic Review 102*(3), 532–537. 1

Asiedu, E., I. Kalonda-Kanyama, L. Ndikumana, and A. Nti-Addae (2013, may). Access to Credit by Firms in Sub-Saharan Africa: How Relevant is Gender? *American Economic Review 103*(3), 293–297. 1

Aterido, R., T. Beck, and L. Iacovone (2013, jul). Access to Finance in Sub-Saharan Africa: Is There a Gender Gap? *World Development 47*, 102–120. 1

Baker, M., J. Gruber, and K. Milligan (2008, aug). Universal Child Care, Maternal Labor Supply, and Family WellâBeing. *Journal of Political Economy 116*(4), 709–745. 1, 13

Barro, R. J. and J. W. Lee (2013). A new data set of educational attainment in the world, 1950-2010. *Journal of Development Economics*. 4

Beck, T., A. Demirgüç-Kunt, and V. Maksimovic (2005, feb). Financial and legal constraints to growth: Does firm size matter? *Journal of Finance 60*(1), 137–177. 6

Bellucci, A., A. Borisov, and A. Zazzaro (2010, dec). Does gender matter in bank-firm relationships? Evidence from small business lending. *Journal of Banking & Finance 34*(12), 2968–2984. 1

Bird, S. R. and S. G. Sapp (2004, feb). Understanding the Gender Gap in Small Business Success. *Gender & Society 18*(1), 5–28. 1

Blau, F. D. (2015). Gender, Economics of. In *International Encyclopedia of the Social & Behavioral Sciences*, pp. 757–763. Elsevier. 1

Blecker, R. A. and S. Seguino (2002b). Macroeconomic effects of reducing gender wage inequality in an export-oriented, semi-industrialized economy. *Review of Development Economics*. 11

Bönte, W. and M. Piegeler (2013, dec). Gender gap in latent and nascent entrepreneurship: driven by competitiveness. *Small Business Economics 41*(4), 961–987. 2

Caliendo, M., F. M. Fossen, A. Kritikos, and M. Wetter (2015, mar). The Gender Gap in Entrepreneurship: Not just a Matter of Personality. *CESifo Economic Studies 61*(1), 202–238. 2

Cuberes, D., A. M. Munoz-Boudet, and M. Teignier (2019, jan). How Costly Are Labor Gender Gaps? Estimates by Age Group for the Balkans and Turkey. *Eastern European Economics 57*(1), 86–101. 2

Cuberes, D. and M. Teignier (2016, mar). Aggregate Effects of Gender Gaps in the Labor Market: A Quantitative Estimate. *Journal of Human Capital 10*(1), 1–32. 2

Cuberes, D. and M. Teignier (2018). Macroeconomic costs of gender gaps in a model with entrepreneurship and household production. *B.E. Journal of Macroeconomics*. 2, 1

de Mel, S., D. McKenzie, and C. Woodruff (2008, nov). Returns to Capital in Microenterprises: Evidence from a Field Experiment. *Quarterly Journal of Economics 123*(4), 1329–1372. 1

Delecourt, S. and O. Ng (2019). Does Gender Matter for Small Business Performance? Experimental Evidence from India. *Working Paper*. 1

Esteve-Volart, B. (2009). Gender Discrimination and Growth: Theory and Evidence from India. *Working Paper*. 2

Friedman, D. and G. S. Becker (1993, mar). A Treatise on the Family. *Contemporary Sociology 22*(2), 225. 1

Gopinath, G., Å. Kalemli-Özcan, L. Karabarbounis, and C. Villegas-Sanchez (2017). Capital allocation and productivity in south europe. *Quarterly Journal of Economics*. 4

Greenwood, J., A. Seshadri, and M. Yorukoglu (2005, jan). Engines of liberation. *Review of Economic Studies 72*(1), 109–133. 2

Guner, N., G. Ventura, and Y. Xu (2008, oct). Macroeconomic implications of size-dependent policies. *Review of Economic Dynamics 11*(4), 721–744. 4

Guzman, J. and A. O. Kacperczyk (2019, sep). Gender gap in entrepreneurship. *Research Policy 48*(7), 1666–1680. 2

Hardy, M. and G. Kagy (2018). Mind The (Profit) Gap: Why Are Female Enterprise Owners Earning Less Than Men? *AEA Papers and Proceedings 108*, 252–55. 1

Havránek, T. (2015). Measuring intertemporal substitution: The importance of method choices and selective reporting. *Journal of the European Economic Association*. 4

Hsieh, C.-T., E. Hurst, C. I. Jones, and P. J. Klenow (2019). The Allocation of Talent and U.S. Economic Growth. *Econometrica 87*(5), 1439–1474. 1

Jamali, D. (2009, jun). Constraints and opportunities facing women entrepreneurs in developing countries. *Gender in Management: An International Journal 24*(4), 232–251. 2

Lundin, D., E. Mörk, and B. Öckert (2008, aug). How far can reduced childcare prices push female labour supply? *Labour Economics 15*(4), 647–659. 1, 6

Markussen, S. and K. Røed (2017, feb). The gender gap in entrepreneurship: The role of peer effects. *Journal of Economic Behavior & Organization 134*, 356–373. 2

Max, S. and V. Ballereau (2013, mar). Theorizing about gender and entrepreneurship: bridging the gap with social psychology. *International Journal of Gender and Entrepreneurship 5*(1), 97–110. 2

Morchio, I. and C. Moser (2020). The Gender Pay Gap: Micro Sources and Macro Consequences. *Working Paper*. 1

Moro, A., T. P. Wisniewski, and G. M. Mantovani (2017, jul). Does a manager's gender matter when accessing credit? Evidence from European data. *Journal of Banking & Finance 80*, 119–134. 1

Morrison, A., D. Raju, and N. Sinha (2007). Gender Equality, Poverty and Economic Growth. *Development*. 11

Nollenberger, N. and N. Rodríguez-Planas (2013, oct). Full-time universal childcare in a context of low maternal employment: Quasi-experimental evidence from Spain. *Labour Economics 36*, 124–136. 13

OECD (2012, dec). *Closing the Gender Gap*. OECD Publishing. 1, 6

Olivetti, C. and B. Petrongolo (2017, feb). The Economic Consequences of Family Policies: Lessons from a Century of Legislation in High-Income Countries. *Journal of Economic Perspectives 31*(1), 205–230. 1

Parro, F. (2012, jun). International Evidence on the Gender Gap in Education over the Past Six Decades: A Puzzle and an Answer to It. *Journal of Human Capital 6*(2), 150–185. 1

Poschke, M. (2013, may). 'entrepreneurs out of necessity': a snapshot. *Applied Economics Letters 20*(7), 658–663. 2

Ribes-Giner, G., I. Moya-Clemente, R. Cervelló-Royo, and M. Perello-Marin (2018, aug). Domestic economic and social conditions empowering female entrepreneurship. *Journal of Business Research 89*, 182–189. 1

Sauer, R. M. and T. Wilson (2016, jul). The rise of female entrepreneurs: New evidence on gender differences in liquidity constraints. *European Economic Review 86*, 73–86. 1, 2

Shinnar, R. S., O. Giacomin, and F. Janssen (2012, may). Entrepreneurial Perceptions and Intentions: The Role of Gender and Culture. *Entrepreneurship Theory and Practice 36*(3), 465–493. 2

Thébaud, S. (2010, sep). Gender and Entrepreneurship as a Career Choice. *Social Psychology Quarterly 73*(3), 288–304. 2

Toossi, M. (2006). A new look at long-term labor force projections to 2050. *Monthly Labor Review*. 1

Watson, J., R. Newby, and A. Mahuka (2009, mar). Gender and the sme 'finance gap'. *International Journal of Gender and Entrepreneurship 1*(1), 42–56. 2

Appendix A: inputs demands in the absence of capital gap

In this appendix, I use equations (2) through (10) to derive the demand level for each of the inputs of the production technology of our model economy. I use first equations (4), (5), and (7) to get:

$$k(z) = \left(\frac{\zeta\alpha}{R}\right)^{\frac{1}{1-\zeta\alpha}} A^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}} z^{\frac{1-\zeta}{1-\zeta\alpha}} n(z)^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}}. \tag{A1}$$

Then, I plug (A1) in (4) and use (5) to get:

$$\frac{y(z)}{n(z)} = \left(\frac{\zeta\alpha}{R}\right)^{\frac{\zeta\alpha}{1-\zeta\alpha}} A^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}} \left(\frac{z}{n(z)}\right)^{\frac{1-\zeta}{1-\zeta\alpha}}. \tag{A2}$$

I prove later that the ratio $y(z)/n(z)$ does not depend on z. Let $\Omega_0 = \left(\frac{\zeta\alpha}{R}\right)^{\frac{\zeta\alpha}{1-\zeta\alpha}} A^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}}$. Then, I can express equation (A2) as:

$$\frac{y(z)}{n(z)} = \Omega_0 \left(\frac{z}{n}\right)^{\frac{1-\zeta}{1-\zeta\alpha}}. \tag{A3}$$

Next I use equations (9) and (A3) to get:

$$n(z) = \Omega_1^{\frac{1-\zeta\alpha}{1-\zeta}} \left(\frac{n(z)}{n^w(z)}\right)^{\frac{1-\zeta\alpha}{(1-\zeta)\sigma}} \left(\frac{n^w(z)}{n^f(z)}\right)^{\frac{1-\zeta\alpha}{(1-\zeta)\rho}} z, \tag{A4}$$

where $\Omega_1 = \frac{\zeta(1-\alpha)(1-\omega)^{\frac{1}{\sigma}}\mu^{\frac{1}{\rho}}\Omega_0}{wf(1-\tau_w)}$.

I can substitute back (A4) in (A3) to get:

$$\frac{y(z)}{n(z)} = \frac{\Omega_0}{\Omega_1}\left(\frac{n^w(z)}{n(z)}\right)^{\frac{1}{\sigma}}\left(\frac{n^f(z)}{n^w(z)}\right)^{\frac{1}{\rho}}. \tag{A5}$$

I now derive expressions for $n^w(z)/n(z)$ and $n^f(z)/n^w(z)$. From (3) I get:

$$\frac{n(z)}{n^w(z)} = \left(\omega^{\frac{1}{\sigma}}\left(\frac{n^m(z)}{n^w(z)}\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega)^{\frac{1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}. \tag{A6}$$

From equations (8) and (9) I get:

$$\frac{n^m(z)}{n^w(z)} = \left(\frac{\omega}{1-\omega}\right)\left(\frac{1}{\mu}\right)^{\frac{\sigma}{\rho}}\left(\frac{n^f(z)}{n^w(z)}\right)^{\frac{\sigma}{\rho}}\left(\frac{w^f(1-\tau_w)}{w^m}\right)^{\sigma}. \tag{A7}$$

From equation (2) I can derive:

$$\frac{n^w(z)}{n^f(z)} = \left(\mu^{\frac{1}{\rho}} + (1-\mu)^{\frac{1}{\rho}}\left(\frac{n^s(z)}{n^f(z)}\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}. \tag{A8}$$

From equations (9) and (10) I get:

$$\frac{n^s(z)}{n^f(z)} = \left(\frac{w^f}{w^s}\right)^{\rho}\left(\frac{1-\mu}{\mu}\right). \tag{A9}$$

From equation (9) I derive:

$$n^w(z) = \left(\frac{\zeta(1-\alpha)(1-\omega)^{\frac{1}{\sigma}}\mu^{\frac{1}{\rho}}(y/n)}{w^f(1-\tau_w)}\right)^{\sigma}\left(\frac{n^w(z)}{n^f(z)}\right)^{\frac{\sigma}{\rho}} n(z). \tag{A10}$$

From equation (8) I compute:

$$n^m(z) = \left(\frac{\zeta(1-\alpha)\omega^{\frac{1}{\sigma}}(y/n)}{w^m}\right)^{\sigma} n(z). \tag{A11}$$

I directly express:

$$n^f(z) = \left(\frac{n^f(z)}{n^w(z)}\right) n^w(z). \tag{A12}$$

From (A9) I can derive:

$$n^s(z) = \left(\frac{w^f}{w^s}\right)^{\rho} \left(\frac{1-\mu}{\mu}\right) n^f(z). \tag{A13}$$

From equation (7) I can get:

$$k(z) = \left(\frac{\zeta\alpha}{R}\right) \left(\frac{y(z)}{n(z)}\right) n(z). \tag{A14}$$

Then, substituting back the expressions derived for y/n, $n(z)$, $n^f(z)/n^w(z)$, and $n^f(z)$ into equations (A11) to (A14) I can express the inputs demand system as:

$$k(z;M) = \Phi_0(M)z, \tag{A15}$$

$$n^f(z;M) = \Phi_1(M)z, \tag{A16}$$

$$n^s(z;M) = \Phi_2(M)z, \tag{A17}$$

$$n^m(z;M) = \Phi_3(M)z. \tag{A18}$$

Appendix B: inputs demands in the presense of a capital gap

In this appendix, I use equations (2) through (10) and (A6) through (A14) to derive the demand level for each of the inputs of the production technology of our model economy for constrained female entrepreneurs.

Using (4), (5) and (7) I derive the ratio of product and labor demand:

$$\frac{y(z)}{n(z)} = \Theta_1 n^{(1-\alpha)\zeta} z^{1-\zeta}. \tag{B1}$$

Let $\Theta_1 = \bar{k}^{\alpha\zeta} A^{(1-\alpha)\zeta}$.

Then, combining (9) and (B1) I can calculate the labor demand:

$$n(z) = \Theta_2^{\frac{1}{(\alpha-1)\zeta}} \left(\frac{n(z)}{n^w(z)}\right)^{\frac{1}{\sigma(\alpha-1)\zeta}} \left(\frac{n^w(z)}{n^f(z)}\right)^{\frac{1}{\rho(\alpha-1)\zeta}} z^{\frac{1-\zeta}{(\alpha-1)\zeta}}. \tag{B2}$$

where $\Theta_2 = \frac{\zeta(1-\alpha)(1-\omega)^{\frac{1}{\sigma}}\mu^{\frac{1}{\rho}}\Theta_1}{wf(1-\tau_w)}$.

Plugging (B2) back into (B1) I can prove that y/n for constrained entrepreneurs does not depend on z:

$$\frac{y}{n} = \Theta_1\Theta_2^{\frac{1-\alpha}{\alpha-1}} \left(\frac{n(z)}{n^w(z)}\right)^{\frac{1-\alpha}{\sigma(\alpha-1)}} \left(\frac{n^w(z)}{n^f(z)}\right)^{\frac{1-\alpha}{\rho(\alpha-1)}}. \tag{B3}$$

Using the equations on (A6) through (A14), I can derive the input demand system for capital constrained female entrepreneurs as

$$k(z;M) = \bar{k}, \tag{B4}$$

$$n^f(z;M) = \Phi_5(M)z, \tag{B5}$$

$$n^s(z;M) = \Phi_6(M)z, \tag{B6}$$

$$n^m(z;M) = \Phi_7(M)z. \tag{B7}$$

Appendix C: solving the equilibrium of the benchmark economy

I first discretize the ability space of each of the agents of the economy so it contains Z_n^i evenly spaced points between $\underline{z}^i$ and $\overline{z}^i$. Then, I use a lognormal density distribution to compute the probability vector for each of the points in the spaces Z_n^i. Denote by p^i the latter vector and by $p^i(z)$ the element corresponding to agent type-z. Once I have built the ability spaces and the discrete density function for them, I solve the household problem for fixed wages and rental price of capital. In Appendix A I showed that:

$$k(z; M; T) = \Phi_0(M; T)z, \tag{D1}$$

$$n^f(z; M; T) = \Phi_1(M; T)z, \tag{D2}$$

$$n^s(z; M; T) = \Phi_2(M; T)z, \tag{D3}$$

$$n^m(z; M; T) = \Phi_3(M; T)z. \tag{D4}$$

And in Appendix B I showed that capital constrained female entrepreneurs follow the equations:

$$k(z; M; T) = \bar{k}, \tag{D5}$$

$$n^f(z; M; T) = \Phi_5(M; T)z, \tag{D6}$$

$$n^s(z; M; T) = \Phi_6(M; T)z, \tag{D7}$$

$$n^m(z; M; T) = \Phi_7(M; T)z. \tag{D8}$$

Then, I use the inputs demands to compute the profits, which is the value of being a manager: $\pi^i(z; M; T)$. The value of being a production worker is simply w^i. Then, considering the value o being a manager and the value of being a production worker, I build the indicator function that identifies the members of the household who become a male entrepreneur:

$$\mathcal{I}_t^m(z; M; T) = \begin{cases} 1 \text{ if } \pi^m(z; M; T) > w^m] \\ 0 \text{ if } \pi^m(z; M; T) \leq w^m]. \end{cases} \tag{D9}$$

For female entrepreneurs have to account for labor distortions. Then,

$$\mathcal{I}_t^i(z; M; T) = \begin{cases} 1 \text{ if } \pi^i(z; M; T)\,(1 - \tau_\pi) > w^i\,(1 - \tau_w)] \\ 0 \text{ if } \pi^i(z; M; T)\,(1 - \tau_\pi) \leq w^i\,(1 - \tau_w)]. \end{cases} \tag{D10}$$

for all $i \in \{f, s\}$.

Notice that the vector $\mathcal{I}_t^i(z)$ also identifies the skill size of the marginal manager:

$$\tilde{z}^i(z; M; T) = \min_{z \in Z^i}\{z \in Z^i : \mathcal{I}_t^i(z; M; T) = 1\}. \tag{D11}$$

Next, I compute the earnings of mothers as follows:

$$e^f(z; M; T) = \mathcal{I}_t^f(z; M; T)\pi^f(z; M; T)\,(1 - \tau_\pi) + (1 - \mathcal{I}_t^f(z; M; T))w^f\,(1 - \tau_w)\,. \tag{D11}$$

Once I have computed the earnings of mother, I build the labor market participation vector for mothers. First, I parametrize the household production function, $(1 + \tau_h)h(z)$. Then, I build:

$$\mathcal{P}^f(z; M; T) = \begin{cases} 1 \text{ if } e_t^f(z; M; T) > (1 + \tau_h)h(z) \\ 0 \text{ if } e_t^f(z; M; T) \leq (1 + \tau_h)h(z). \end{cases} \tag{D12}$$

Next, I build the aggregate demands and supply of labor and capital services. Denote by $[N^j]^i$ the aggregate demand for labor service type j by group i. Analogously, denote by K^i the aggregate demand for capital services by group i. The aggregate demand for labor services type j of agent type i is:

$$[N^j(M; T)]^i = \sum_{z=\underline{z}^i}^{\overline{z}^i} p^i(z)[n^j(z; M; T)]^i \mathcal{I}^i(z; M; T)\mathcal{P}^i(z; M; T), \tag{C13}$$

The aggregate demand for capital services by agent type i is:

$$K^i(M; T) = \sum_{z=\underline{z}^i}^{\overline{z}^i} p^i(z)[n^j(z; M; T)]^i \mathcal{I}^i(z; M; T)\mathcal{P}^i(z; M; T). \tag{C14}$$

Notice that, by assumption, $\mathcal{P}^s(z; M; T) = \mathcal{P}^m(z; M; T) = 1$.

Next, I compute the aggregate demand for each of the four inputs:

$$N^f(M;T) = \sum_{i=\{f,s,m\}} \theta^i [N^f(M;T)]^i, \tag{C15}$$

$$N^s(M;T) = \sum_{i=\{f,s,m\}} \theta^i [N^s(M;T)]^i, \tag{C16}$$

$$N^m(M;T) = \sum_{i=\{f,s,m\}} \theta^i [N^m(M;T)]^i, \tag{C17}$$

$$K(M;T) = \sum_{i=\{f,s,m\}} \theta^i K^i(M;T). \tag{C18}$$

Then, I compute the inputs supply. The supply of labor inputs are as follows:

$$S^i(M;T) = \sum_{z=\underline{z}^i}^{\overline{z}^i} p^i(z)(1-\mathcal{I}^i(z;M;T))\mathcal{P}^i(z;M;T) \tag{C19}$$

For the supply of capital I initially set $K = \hat{K}$. Then, given the the stock of capital I evaluate the market clearing conditions:

$$O(M,\hat{K}) = \frac{(N^f(M;T) - S^f(M;T)^2}{N^f(M;T) + S^f(M;T)} + \frac{(N^s(M;T) - S^s(M;T))^2}{N^s(M;T) + S^s(M;T)} + \frac{(N^m(M;T) - S^m(M;T))^2}{N^m + S^m} + \frac{(K(M;T) - \hat{K})^2}{K(M;T) + \hat{K}}$$

Then, I iterate on capital until the interest rate is equal to $(1/\beta) + \delta - 1$. To do so, I first make a guess on a minimum and maximum amount of capital such the equilibrium R is higher and lower than $(1/\beta) + \delta - 1$, respectively. After doing so, I use a bijective algorithm to find the level of capital such that $R = (1/\beta) + \delta - 1$.