Aggregate Costs of a Gender Gap in the Access to Business Resources

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Abstract

We quantify the aggregate costs of a discriminatory restriction against women in the access to business resources. To do so, we develop a general equilibrium model with an endogenous size distribution of production units, which are run by either female or male entrepreneurs. In this setting, we introduce a distortion that limits the amount of capital that women can use to run their businesses. We calibrate the model to match data from benchmark economies that exhibit relatively egalitarian labor market results between women and men, except in entrepreneurship. Our counterfactual analyses show that a gender-specific capital constraint causes an output loss between 14% and 28% and a fall in aggregate productivity between 12% and 20%. Furthermore, we show that most of the output loss is accounted for by a fall in total factor productivity. Lastly, we show that the aggregate cost of the distortion is mainly triggered by preventing the most skilled women from running bigger businesses, and not the exit of women from entrepreneurship.

Keywords: gender gap, entrepreneurship, aggregate productivity

JEL Classification: E2, J21, J24, O40

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1 Introduction

The empowerment of women in many domains of life is one of the most important transformations in the economic and social worlds in recent decades.[1] However, a significant gender gap in entrepreneurship still persists. In OECD countries, the proportion of sole-proprietor enterprises owned by women is only between 20% and 40% (OECD, 2012). Furthermore, less than 25% of businesses across the world are owned by women according to the Enterprise Surveys elaborated by the World Bank. Discrimination in the access to business resources might be blocking female participation in entrepreneurial activities, as it is implicitly suggested by the literature that studies entrepreneurship in the presence of financial frictions (Buera et al., 2015).

Indeed, data show that women seem to have more difficulties accessing credit than men. Some studies conclude that female entrepreneurs face worse credit conditions than men, even when controlling for firm and entrepreneur characteristics (Agier and Szafarz, 2013; Aristei and Gallo, 2016; Moro et al., 2017). In the same line, the OECD reports that, in the United States, 60% of female entrepreneurs start with resources lower than US$ 5,000, compared to 42% of male entrepreneurs (OECD, 2012). Furthermore, also in the United States, 39% of women entrepreneurs started their business with credit from a bank compared to 47% of male entrepreneurs; similar evidence is observed for several European countries (OECD, 2012). Data from the OECD also show that women entrepreneurs have less access to business financing than men; for instance, only 36% of Danish and 41% of Swedish women business owners report having access to the resources needed to grow their business, compared to 46% of Danish and 50% of Swedish male business owners (OECD, 2016). Lastly, the World Bank IFC has estimated that, worldwide, a US$ 300 billion gap in financing exists for formal, women-owned small businesses, and more than 70% of women-owned small and medium enterprises have inadequate or no access to financial services. Distortions, such as those described, might prevent an efficient allocation of talent, and thus, impact aggregate outcomes, as suggested by Hsieh et al. (2019).

In this paper, we quantify the aggregate costs of a discriminatory restriction against women in the access to business resources. We model a distortion that limits the amount of

[1] Female labor force participation has notably increased in several regions of the world (World Bank WDI dataset), women are equally or more educated than men worldwide (Parro, 2012), and the gender wage gap has also shrunk during past decades (Blau and Kahn, 2017).

capital that female entrepreneurs can employ to run their businesses. The latter distortion is directly motivated by the evidence described above, which suggests the existence of a ceiling in the business resources for women. We insert this type of distortion into a model with heterogeneous agents and an endogenous size distribution of production units. In the model, there is a single representative household composed by female and male members who are endowed with heterogeneous managerial skills. Agents must decide whether to run a business or be employed as a production worker in someone else's firm. Production is carried out with a span-of-control technology that uses managerial skills, capital, female labor, and male labor. Hence, more skilled managers run bigger businesses.[2] However, women can employ up to a given amount of capital to start their businesses. As production workers, agents are identical, although women and men are imperfect substitutes in production. Therefore, women and men are otherwise alike, except by their imperfect substitutability in the labor market as production workers, and the capital limit that women face to run their businesses.

We calibrate the model to match data of a *benchmark economy* that exhibits no labor market gender gaps, except regarding entrepreneurship. We choose Denmark, Norway, and Sweden to build the latter benchmark. Hence, we refer hereafter to the average data of Denmark, Norway, and Sweden as the data of the benchmark economy. As observed in Table 1, in the benchmark economy, the gender gap in education is practically null, the ratio of female-to-male labor force participation is close to one, the gender wage gap is tiny; however, a significant gender gap in entrepreneurship is still observed. Specifically, Table 1 shows that only about a third of individuals running a business are women in the benchmark economy. We calibrate the technology parameters of the model to match data on the size distribution of establishments and employment shares by large establishments in the benchmark economy. The gender-specific friction is calibrated to match the observed gender gap in entrepreneurship.

[2]Hereafter, we will use the terms "entrepreneurs" and "managers" interchangeably.

Table 1: Gender gaps in the benchmark economy

	Denmark	Norway	Sweden	Average
Gender gap in education	1.5%	0.2%	-1.8%	-0.03%
Female-to-male LFP	0.93	0.94	0.96	0.94
Gender wage gap	0.05	0.05	0.08	0.06
Share of female entrepreneurs (avg.)				0.31
OECD	0.32	0.33	0.29	0.31
ILO	0.30	0.30	0.27	0.29
GEM	0.35*	0.35	0.31	0.34

Sources: Barro-Lee Educational Attainment Dataset, OECD Gender Equality Dataset (OECD), International Labor Organization Statistics (ILO), and Global Entrepreneurship Monitor (GEM). Note: Gender gap in education is the difference between men-women in the percentage of 30-34 years olds who attained tertiary education in 2010; Female-to-male LFP is the ratio between female and male labor force participation in the population 15-64 years old in 2019; Gender wage gap is defined as the difference between male and female median wages divided by the male median wages in 2019; Share of female entrepreneurs is defined as the proportion of women in total self-employment (with or without employees). The Global Entrepreneurship Monitor reports the share of female entrepreneurs in early-stage entrepreneurship. *Latest data available for 2014.

We use the calibrated model to perform counterfactual exercises that quantify the effect of the distortion on aggregate outcomes. Specifically, we build five counterfactual scenarios which gradually and progressively loosen the capital limit for female entrepreneurs. We compare different outcomes between the counterfactual scenarios and the benchmark economy. Then, our quantitative analysis responds to the following question: How costly is a capital limit on women businesses that generates a gender gap in entrepreneurship that is similar to that observed in the benchmark economy?

We find that the gender-specific capital constraint causes an output loss between 13.7% and 28.2%.[3] Furthermore, the estimated losses in average output per efficiency unit of labor ranges from 14.5% to 30.1%, in average output per production worker ranges from 11.4% to 24.1%, and in average output per manager ranges from 24.5% to 49.3%. We also find that the distortion causes a fall in total factor productivity (TFP) between 12.2% and 19.5%. In addition, we quantify the contribution of capital intensity and TFP to the fall in output per worker. We find that the fall in productivity accounts for 65%-80% of the output per worker loss. We also show that the potential aggregate gains of releasing the distortion studied in this paper would be mostly driven by the more intensive capital utilization of existing female entrepreneurs (intensive margin) and not by the entry of more women into entrepreneurship (extensive margin).

[3]The range of estimates corresponds to the minimum and maximum effect obtained from the five counterfactual scenarios under consideration.

Several articles have studied the relationship between gender inequality and economic performance (Galor and Weil, 1996; Lagerlof, 2003; Greenwood et al., 2005; Doepke and Tertilt, 2009; Esteve-Volart, 2009; Fernandez, 2009; Ngai and Petrongolo, 2017; among others).[4] However, few papers quantify the macroeconomic effects of gender gaps in the labor market (Hsieh et al., 2019; Cavalcanti and Tavares, 2016; Cuberes and Teigner, 2016; Cuberes and Teigner, 2018). Our goal in this paper is to contribute to this literature by analyzing the macroeconomic effects of a specific distortion that directly impacts the gender gap in entrepreneurship; namely, women's limited access to business resources. As discussed previously, evidence suggests that this distortion is indeed relevant, although their aggregate costs have not been quantified thus far.

The rest of the paper is organized as follows. Section 2 develops the theoretical framework. Section 3 describes the calibration strategy. Section 3 presents and discusses the results of this paper. Finally, Section 5 concludes.

2 Model

We build a one-sector aggregative model in which production is carried out by heterogeneous establishments run by two types of agents: women and men. Each type of agent can also work as a production worker in the establishment run by someone else. This benchmark model is exposed to a supply-driven distortion to female entrepreneurship.

2.1 Preferences

There is a single infinitely lived representative household in the economy. The household comprises at time t a continuum of members of size L_t. The size of the household (population) grows at the constant rate x_L.[5] The members of the household are of two types: women (f) and men (m). We denote by θ_i the fraction of the population of type-i, for $i = \{f, m\}$. The household preferences are described by a time-additively separable utility function over

[4] See also Cuberes and Teigner (2014) for a critical review.

[5] We introduce population growth so the model has standard balanced-growth properties, and thus can be better mapped to data.

sequences of per capita consumption, c_t. The representative household gets flow utility from per capita consumption, by an increasing and concave function, $u(c)$. Then, the representative household maximizes:

$$U(\{c_0, ..., c_\infty\}) = \sum_{t=0}^{\infty} \beta^t L_t u(c_t), \quad u(c_t) = \ln c_t, \tag{1}$$

where $\beta \in (0,1)$ is the discount factor for the future.

2.2 Endowments

The household is endowed with a positive (aggregate) stock of capital at date $t = 0$, that is, $K_0 > 0$. In addition, each household member is endowed with z units of managerial skills. We denote by $g(z)$ the density function of skills, with c.d.f. $G(z)$, and support in $Z = [z^l, z^h]$. The distribution of skills is identical for women and men. Household members are also endowed by one unit of time which they inelastically supply in the labor market as either managers or production workers. We describe later the agents' occupational choice and the associated income in detail.

2.3 Production technology

Each establishment–a production unit–produces a homogeneous output. Production is carried out using labor (n), capital (k), and managerial skills (z). Specifically, women's labor, n^f, and men's labor, n^m, are combined in a CES function with an elasticity of substitution given by σ:[6]

$$n_i = \left((n_i^m)^{\frac{\sigma-1}{\sigma}} + (n_i^f)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{2}$$

where n_i^j is the quantity of labor type-j hired by manager i.

This nested CES function n that aggregates women's and men's labor is then combined with capital using a Cobb-Douglas aggregator:

$$q_i = k_i^{\alpha} n_i^{1-\alpha}, \tag{3}$$

[6]Hereafter, we include the time index t when it is strictly necessary for a clear exposition.

where $\alpha \in (0,1)$. Output q is combined with managerial skills using a Cobb-Douglas aggregator:

$$y_i(z) = Az^{1-\zeta} q_i^{\zeta}. \tag{4}$$

where the term A is common to all production units, and accounts for exogenous productivity growth at a constant rate x_A. The parameter $\zeta \in (0,1)$ governs returns to scale in variable factors at the establishment level, that is, it is the span-of-control parameter.

2.4 The problem of a type-z manager

A manager with skills z maximizes profits by taking wages and the rental price for capital services as given. The static maximization problem of a type-i agent with skills z is

$$\max_{k_i, n_i^f, n_i^m} \left[y_i(z) - w^f n_i^f - w^m n_i^m - Rk_i \right], \tag{5}$$

where R is the rental price for capital services and w^j is the wage of labor type-j for $j \in \{f, m\}$.

The first order conditions of the maximization problem are:

$$[k_i] : \zeta\alpha \frac{y_i(z)}{k_i(z)} = R \tag{6}$$

$$[n_i^f] : \zeta(1-\alpha) \left(\frac{y_i(z)}{n_i(z)} \right) \left(\frac{n_i(z)}{n_i^f(z)} \right)^{\frac{1}{\sigma}} = w^f \tag{7}$$

$$[n_i^m] : \zeta(1-\alpha) \left(\frac{y_i(z)}{n_i(z)} \right) \left(\frac{n_i(z)}{n_i^m(z)} \right)^{\frac{1}{\sigma}} = w^m \tag{8}$$

We use equations (6) to (8) to compute the relative demand of inputs for a type-i manager with skills z. Appendix A provides a formal derivation of these demands.

2.5 Occupational choices

Agents must decide in each period whether to operate a single productive unit or work for a wage. We denote by $M = \{w^f, w^m, R\}$ the market prices of labor and capital inputs. Let $\mathcal{I}_i(z; M)$ be an indicator that describes the optimal occupation choice for an agent with abilities

z that face market prices M. Specifically, we set $\mathcal{I}_i(z;M) = 1$ if the agent optimally decides to operate a business and $\mathcal{I}_i(z;M) = 0$ if the agent decides to work for a wage. An individual that decides to become a manager must receive in equilibrium a compensation higher than the wage earned by a production worker. Denote by $\pi_i(z;M)$ the solution to problem (5) for a type-i manager with skills z. Then,

$$\mathcal{I}_i(z;M) = \begin{cases} 1 \text{ if } \pi_i(z;M) > w^i \\ 0 \text{ if } \pi_i(z;M) \le w^i, \end{cases} \tag{9}$$

and thus, labor market earnings of an agent i, can be expressed as:

$$e_i(z;M) = \mathcal{I}_i(z;M)\pi_i(z;M) + (1 - \mathcal{I}_i(z;M))w^i \tag{10}$$

Therefore, the income per capita generated by members type i is

$$v_i(z;M) = \int_{z\in Z} e_i(z;M)g(z)dz, \tag{11}$$

2.6 The household problem

The representative household must choose the sequence of per capita consumption, the sequence of aggregate capital to carry over to the next period, and the occupation of its members, taking all prices as given:

$$\max_{\{c_t,K_t,\mathcal{I}_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t L_t \log c_t \tag{12}$$

$$\text{s.t. } C_t + K_{t+1} = \sum_{i\in\{f,m\}} L_{i,t} v_{i,t}(z;M_t) + R_t K_t + (1-\delta)K_t,$$

$$K_0 > 0, L_0 > 0,$$

where C_t is aggregate consumption at date t, that is, $C_t = L_t c_t$, and δ is the depreciation rate of capital. The first order conditions for per capita consumption and aggregate capita produces the standard Euler equation for capital accumulation:

$$\frac{c_t}{c_{t+1}} = \beta(R_{t+1} + 1 - \delta) \quad \text{for all } t \geq 0. \tag{13}$$

Notice that equation (9) satisfies the first order condition for the occupation choice of agents.

2.7 Market clearing

We derive now the market clearing conditions of the model. The market clearing condition for capital requires that the aggregate capital that the representative household optimally carries over to the next periods equals the aggregate demand for capital by the productive units:

$$K_t(M_t) = L_t \sum_{i \in \{f,m\}} \theta_i \int_{z \in Z} k_{i,t}(z; M_t)\mathcal{I}_{i,t}(z; M_t)g(z)dz. \tag{14}$$

The market clearing condition for labor market services must equal the supply and demand of production labor. The supply of each type of labor service is determined by the occupational decisions of women and men. Demand is determined by the number of agents of each type that decide to run an establishment and their demands for each type of production labor (women and men). Then,

$$L_t\theta_j \int_{z \in Z} (1 - \mathcal{I}_{j,t}(z; M_t))g(z)dz = L_t \sum_{i \in \{f,m\}} \theta_i \int_{z \in Z} n^j_{i,t}(z; M_t)\mathcal{I}_{i,t}(z; M_t)g(z)dz, \tag{15}$$

for $j = \{f, m\}$. Finally, the market clearing condition for the unique good produced in this economy is given by:

$$L_t \sum_{i \in \{f,m\}} \theta_i \int_{z \in Z} y_{i,t}(z; M_t)\mathcal{I}_{i,t}(z; M_t)g(z)dz = C_t(M_t) + K_{t+1}(M_t) - (1 - \delta)K_t(M_t). \tag{16}$$

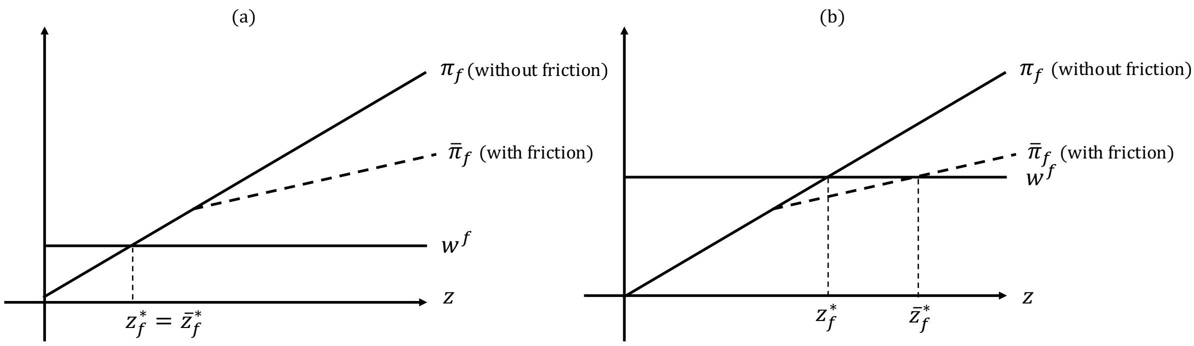
The left-hand side of equation (16) is aggregate income, whereas the right-hand side is aggregate consumption plus aggregate investment.

2.8 Capital constraint for female business owners

In the model economy, women face an exogenous constraint in their access to business resources. The distortion takes the form of a ceiling $\overline{k}$ on the capital amount that women can rent to run their businesses. We now discuss how this distortion impacts female entrepreneurship.

Consider first the partial equilibrium. As a direct consequence of the span-of-control production technology, more skilled women run bigger businesses, and thus, rent larger amounts of capital. Hence, the capital limit $\overline{k}$ is only binding for women with the highest managerial skills. Constrained women face a reduction in their profits as a consequence of the distortion. In contrast, the distortion is innocuous for unconstrained women. Therefore, two cases arise, which are illustrated in panels (a) and (b) of Figure 1.[7] First, suppose low equilibrium wages (panel a). Then, the marginal women entrepreneur is relatively low-skilled, and thus, the capital limit is not binding for her. Hence, the friction is innocuous on the equilibrium number of entrepreneurs. That is, the skill threshold above which women start a business in the frictional economy, $\overline{z}^*_f$, is the same as in the frictionless economy, z^*_f. Suppose now high equilibrium wages (panel b). In this case, the friction $\overline{k}$ constrains the amount of capital available for the marginal women entrepreneurs, who then experience a fall in their profits (dashed line). Therefore, some female entrepreneurs do not have more incentives to run a business, and thus, they become production workers. That is, $z^*_f < \overline{z}^*_f$, and thus, the number of female entrepreneurs falls. In this partial equilibrium analysis, men's choices are not impacted by the distortion.

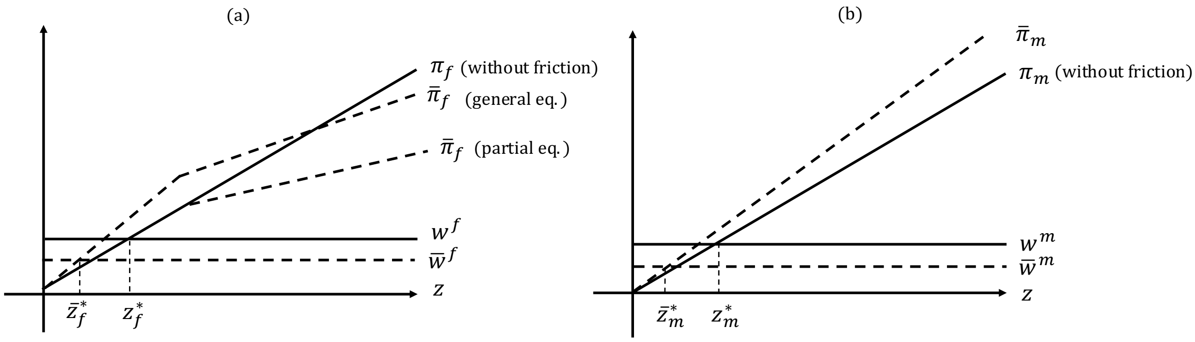
Figure 1: Partial equilibrium analysis



[7]Figures 1 through 3 are drawn only to illustrate the effects and are not intended to show what the actual data looks like.

We now analyze the general equilibrium. Let us start from the scenario depicted in panel (a) of Figure 1; that is, the case in which the capital limit is not binding for the marginal women entrepreneurs. Thus, the distortion does not exert a direct effect on the number of entrepreneurs. This is the partial equilibrium effect analyzed previously. However, existing female entrepreneurs are constrained in their use of capital and must run smaller businesses as a consequence of the distortion. Those firms demand less production labor which pushes wages down.[8] Hence, profits go up and the choice of running a business becomes more attractive for both women and men. Formally, the threshold managerial skill falls; that is $z_f^* > \overline{z}_f^*$ and $z_m^* > \overline{z}_m^*$. Therefore, both female and male entrepreneurship rises. Figure 2 illustrates these first round effects for women (panel a) and men (panel b), where we denote by $\overline{w}$ the frictional wage.

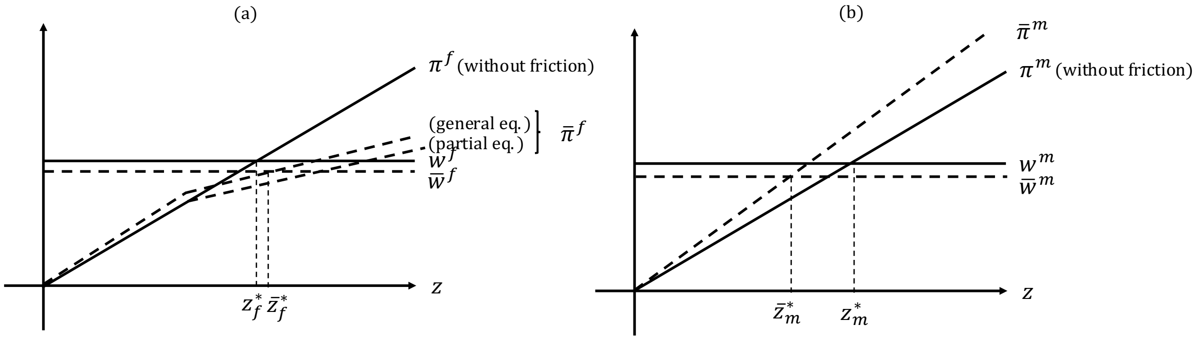
Figure 2: General equilibrium analysis I



Let us start now from the situation depicted in panel (b) of Figure 1. In this case, the direct effect of the capital limit is a fall in the number of female entrepreneurs, the partial equilibrium effect. Furthermore, existing female entrepreneurs are constrained by the distortion, run smaller businesses, and thus, demand less production labor. Hence, supply of female production labor rises and demand falls which unambiguously pushes wages down and profits up. For women, this is a second round effect, and thus, female entrepreneurship falls, although in a lesser magnitude compared to the partial equilibrium depicted in panel (b) of Figure 1. In the case of men, the fall in wages and rise in profits increases the number of entrepreneurs. Therefore, as first order effects, female entrepreneurship falls, whereas the opposite occurs for men. Figure 3 illustrates this case.

[8] The fact that the standard Euler equation for capital accumulation applies in this model implies that the rental rate for capital services is constant across steady states.

Figure 3: General equilibrium analysis II



2.9 Equilibrium

A competitive equilibrium is a set of allocations $\{c_t, K_{t+1}, \{\mathcal{I}_{i,t}(z)\}_{z\epsilon Z}\}_{t=0}^{\infty}$ for the representative household, and a set of prices $\{w_t^f, w_t^m, R_t\}$ such that (i) the household problem (12) is solved by taking all the prices as given, and subject to the constraint $k_{f,t}(z) \leq \overline{k}$ for women (ii) the market for capital services clears for all t (equation 14 holds), (iii) the market for labor services clears for all t (equation 15 holds), and (iv) the market for goods clears for all t (equation 16 holds).

3 Calibration

We calibrate the model to match observations of a benchmark economy, which we build using the average data for Denmark, Norway, and Sweden. As discussed in the introductory section, these economies are close to gender equality in the labor market, except regarding entrepreneurship.[9] We now explain the specific data used to calibrate the model.

We choose a model period of a year. Along the balanced-growth path, consumption per capita and output per capita grow at a common rate x_c. To compute x_c, we consider the average annual growth rate of real GDP per capita in the benchmark economy during the period 1971-2019. Using data from the World Bank WDI dataset, we get $x_c = 0.018$. The U.S. Bureau of Economic Analysis (BEA) provides estimates of depreciation rates for different types of assets.

[9]Recall that we refer to the average data of Denmark, Norway, and Sweden as the data of the benchmark economy.

This technological parameter should not be very different between the U.S. economy and our benchmark economy, so we take BEA data to calibrate this parameter. Specifically, we assume that the stock of capital is comprised of private nonresidential equipment and structures. We consider the simple average of the depreciation rates reported by the BEA for these types of assets to calibrate $\delta = 0.0978$. In addition, in the model, $\zeta\alpha$ equals the share of capital. As we will explain later, we follow Guner et al. (2008) to calibrate ζ. Then, we pick α such that $\zeta\alpha$ equals the capital income share of 30% observed in the U.S. data. The elasticity of substitution between women and men, σ, is calibrated to match the observed gender wage gap in the benchmark economy. Labor force participation is assumed to be egalitarian across gender in order to let the elasticity of substitution σ capture the whole gender wage gap that is observed in the benchmark economy. The discount factor β takes the standard value of 0.95 reported in the literature.

We now explain how we calibrate the span-of-control parameter, ζ, and the distribution of managerial skills. The span-of-control parameter, ζ, and the parameters governing the distribution of managerial abilities $g(z)$ determine the size distribution of establishments, that is, the mean establishment size, the range of employment levels, as well as the share of total labor employed by establishments of different sizes. Following Guner et al. (2008), we choose to calibrate $g(z)$ and ζ so they are consistent with the data on the fraction of establishments at different employment levels, and the share of total employment accounted for by large establishments. We use data from Eurostat Structural Business Statistics to compute the following targets (i) mean establishment size, (ii) the fraction of establishments with 0-9 employees, 10-19 employees, 20-49 employees, 50 or more employees, and (iii) the share of total employment accounted for by large establishments, defined as those with 50 or more employees. We then select ζ and $g(z)$ to match these statistics. We assume that log-managerial ability is distributed according to a (truncated) normal distribution with mean m and variance s^2. We impose that this distribution accounts for the bulk of production units, with a total mass of $1 - \tilde{g}$. To account for the remainder of the distribution of establishments, we select a top value for managerial skill, $\tilde{z}$ and its corresponding fraction, $\tilde{g}$. Hereafter we refer to managers with skills $\tilde{z}$ as "superstars." Thus, the bottom side of the distribution of managerial skill is characterized by a log-normal distribution while the very top is captured by an extreme value for managerial skill.[10] Lastly, the capital limit $\bar{k}$ is calibrated to match the fraction of female entrepreneurs in the benchmark economy.

Hence, there are seven parameters, σ, ζ, m, s^2, $\tilde{g}$, $\tilde{z}$, and $\bar{k}$ that we choose in order to match eight observations: gender wage gap, mean size, the fraction of establishments corresponding to four different levels of employees, the share of employment accounted for by establishments with 50 or more employees, and the fraction of female entrepreneurs. Table 2 summarizes our choices and Table 3 shows the performance of the model in terms of our targets.[11]

Table 2: Calibrated parameters

Parameter	Value	Explanation
Productivity (A)	1	normalization
Labor force participation ($\theta_f = \theta_m$)	0.5	normalization based on data
Discount factor (β)	0.95	extracted from literature
Output per capita growth (x_c)	1.8%	extracted from data
Depreciation rate (δ)	0.0978	extracted from data
Share of capital ($\zeta\alpha$)	0.3	extracted from data
CES elasticity (σ)	1.1357	to match data
Span-of-control (ζ)	0.6219	to match data
Mean log-entrepreneurship skills (μ)	-0.3989	to match data
Variance of log-entrepreneurship skills (s^2)	4.9929	to match data
Highest entrepreneurship ability ($\tilde{z}$)	2944	to match data
Mass of highest entrepreneurship ability ($\tilde{g}$)	0.0022	to match data
Capital limit ($\bar{k}$)	5.2860	to match data

Table 3: Performance of the model

Statistic	Data	Model
Gender wage gap	0.06	0.06
Fraction of female entrepreneurs	0.31	0.33
Mean firm size	5.90	5.94
Fraction of establishments at:		
0-9 employees	0.92	0.91
10-19 employees	0.043	0.049
20-49 employees	0.025	0.028
50+ employees	0.014	0.016
Share of employment at:		
50+ employees	0.54	0.56

[10]These few but highly talented managers allow us to generate the right amount of factor demand by large establishments.

[11]In all simulations we approximate $g(z)$ by a discrete distribution with 5,000 grid points. Appendix B explains how we solve the model.

4 Results

In this section we estimate the aggregate costs of the distortion described in Section 2.8. To do so, we build five counterfactual scenarios which gradually and progressively loosen the capital limit for female entrepreneurs. The first scenario reduces by 10% the gap between $\overline{k}$ and the maximum amount of capital used by non-superstar female entrepreneurs in a frictionless economy (S1).[12] In the second and third scenarios, the latter gap is shortened by 20% (S2) and 50% (S3), respectively. In a fourth counterfactual scenario, all women are unconstrained, except the superstar managers (S4). Lastly, we consider a counterfactual economy where no woman is subject to a capital limit to run her business (S5). Then, we compare different outcomes in the baseline scenario to those in each of the five counterfactual economies. We consider three outcomes: aggregate output, output per worker, and total factor productivity (TFP).

Aggregate output, Y, is the total production carried out by both female and male entrepreneurs:[13]

$$Y = \theta_f \int_{z_f^*} y_f(z) g(z) dz + \theta_m \int_{z_m^*} y_m(z) g(z) dz, \tag{17}$$

where z_i^* is the managerial skill of the marginal entrepreneur among type-i agents.

In this first analysis, we also compute the ratio of the average output produced per female manager in the counterfactual scenario to the output produced in the baseline scenario, R_f. This outcome is used to discuss the results for aggregate output losses, and we define it as follows:

$$R_f = \frac{\frac{\int_{z_f^*} y_f(z) g(z) dz}{\int_{z_f^*} g(z) dz}}{\frac{\int_{\overline{z}_f^*} y_f(z) g(z) dz}{\int_{\overline{z}_f^*} g(z) dz}}, \tag{18}$$

where $\overline{z}_f^*$ and z_f^* are the managerial skill of the marginal female entrepreneur in the baseline and counterfactual scenarios, respectively.

Table 4 presents the results. We observe that the distortion causes a fall in aggregate output between 13.7% and 28.2%, depending on the counterfactual scenario considered. Coun-

[12]The demand of capital by superstars is several times bigger than the amount demanded by the most skilled women in the rest of the distribution. Thus, considering a reduction of 10% in the gap between $\overline{k}$ and the maximum amount of capital used by all women (that is, including superstars) would produce an abrupt rise in the capital limit, which prevents the gradualness that the counterfactual exercise seeks to achieve.

[13]Hereafter, we normalize $L = 1$.

terfactual S1 implies that the distortion causes an output loss of 13.7%. In the case of the second and third counterfactuals (S2 and S3), we observe a fall in aggregate output by 16.3% and 19.5%, respectively, which is somewhat higher compared to the first scenario. Next, when comparing the baseline economy with one in which all women are unconstrained, except superstar managers, that is, scenario S4, the distortion causes an output loss of 21.7%. Lastly, output loss reaches 28.2% when the baseline economy is compared with a frictionless economy (S5).

Table 4: Aggregate output losses

	S1	S2	S3	S4	S5
Aggregate output, Y (%)	13.7	16.3	19.5	21.7	28.2
Ratio output per female manager, R_f (level)	4.1	4.8	5.9	6.8	9.9

Therefore, our counterfactual analysis suggests significant potential output gains derived from a small loosening in the capital limit for women entrepreneurs. However, potential gains are marginally smaller when the capital limit is further loosened. An exception to the latter occurs when the economy moves from a scenario where only the superstars are constrained to one where no women face a capital limit. The intuition behind the latter is as follows. The distortion forces women to run smaller businesses than what would be the optimal size, causing the marginal product of capital to be well above the rental price of it. Indeed, we observe in the second row of Table 4 that the ratio of the average output produced in establishments run by women in the counterfactual scenario to the output produced in the baseline scenario ranges from 4.1 (S1) to 9.9 (S5). Therefore, a small loosening of the distortion allows women to materialize the high marginal product of capital, but these benefits begin to evaporate when capital use becomes more and more intensive. However, superstar managers are highly skilled compared to the mass of managers, and thus, capital-skill complementarity in production counters somewhat the effect of capital intensity on the marginal product of capital. Hence, the marginal productivity of capital is still significant in businesses run by superstar female managers.

Next, we evaluate three measures of output per worker. Concretely, we build the average output per efficiency unit of labor, q_1, as

$$q_1 = \theta_f \frac{\int_{z_f^*} y_f(z)g(z)dz}{1 - \int_{z_f^*} g(z)dz + \int_{z_f^*} zg(z)dz} + \theta_m \frac{\int_{z_m^*} y_m(z)g(z)dz}{1 - \int_{z_m^*} g(z)dz + \int_{z_m^*} zg(z)dz}, \tag{19}$$

the average output per production worker, q_2, as

$$q_2 = \theta_f \frac{\int_{z_f^*} y_f(z)g(z)dz}{1 - \int_{z_f^*} g(z)dz} + \theta_m \frac{\int_{z_m^*} y_m(z)g(z)dz}{1 - \int_{z_m^*} g(z)dz}, \tag{20}$$

and the average output per manager, q_3, as

$$q_3 = \theta_f \frac{\int_{z_f^*} y_f(z)g(z)dz}{\int_{z_f^*} g(z)dz} + \theta_m \frac{\int_{z_m^*} y_m(z)g(z)dz}{\int_{z_m^*} g(z)dz}. \tag{21}$$

Table 5 presents the results using the same counterfactual scenarios as in Table 4. We observe that average output per efficiency unit of labor falls by 14.5% when the distortion considered in S1 moves to the level $\overline{k}$. As concluded for aggregate output, this loss gradually increases when considering counterfactuals that progressively loosen the capital limit for women, reaching 30.1% when the baseline economy is compared with a frictionless one. In the case of average output per production worker the losses range from 11.4% to 24.1%, whereas for average output per manager the losses range from 24.5% to 49.3%.

Table 5: Average output per worker losses

(%)	S1	S2	S3	S4	S5
Output per efficiency unit of labor, q_1	14.5	17.4	20.8	23.1	30.1
Output per production worker, q_2	11.4	13.6	16.4	18.3	24.1
Output per manager, q_3	24.5	29.5	35.0	38.9	49.3

Our next outcome measures aggregate productivity losses. We calculate this variable in two alternative ways. The first measure (TFP_1) considers TFP as the residual from an aggregate technology under a capital share of $\alpha\zeta$ and labor share of $1-\alpha\zeta$, with no distinctions between workers and managers in the labor force. Concretely, we calculate

$$TFP_1 = \frac{Y/L}{(K/L)^{\alpha\zeta}}, \tag{22}$$

where

$$L = \theta_f \int_{z_f^*} n_f(z) g(z) dz + \theta_m \int_{z_m^*} n_m(z) g(z) dz, \tag{23}$$

$$K = \theta_f \int_{z_f^*} k_f(z) g(z) dz + \theta_m \int_{z_m^*} k_m(z) g(z) dz. \tag{24}$$

The second measure (TFP_2) considers a separation between workers and managers by their efficiency units and define aggregate labor as $L + \tilde{L}$, where $\tilde{L}$ is

$$\tilde{L} = \theta_f \int_{z_f^*} z g(z) dz + \theta_m \int_{z_m^*} z g(z) dz. \tag{25}$$

In this case, we have

$$TFP_2 = \frac{Y}{K^{\alpha\zeta} \left(L + \tilde{L} \right)^{1-\alpha\zeta}}. \tag{26}$$

We present in Table 6 the results for TFP. We observe that aggregate TFP falls, approximately, between 12.2% and 19.5% as a consequence of the distortion. In this case, we observe moderate gains, in terms of TFP, from moving across the five counterfactual scenarios considered in Table 6.

Table 6: TFP losses

(%)	S1	S2	S3	S4	S5
TFP_1	12.3	13.9	15.8	17.2	19.4
TFP_2	12.2	13.8	15.8	17.2	19.5

Overall, this subsection shows that the capital limit $\overline{k}$ for women causes a fall in aggregate output between 13.7% and 28.2%, in average output per efficiency unit of labor between 14.5% and 30.1%, and in TFP between 12.2% and 19.5%. How do these effects compare to those found in the literature for other distortions? In this regard, the literature on the aggregate costs of financial frictions is indeed extensive, so we document here only a few studies that will provide an order of magnitude of the effects found in this paper.

Erosa (2001) analyzes financial frictions that arise from costly intermediation. The author estimates gains in output per capita of 40% from eliminating this type of distortion. Buera et al. (2011) analyze aggregate cost from financial frictions that take the form of imperfect enforceability contracts. The authors find that the aggregate TFP of the country with the least financial development would be almost 40% below the U.S. level. Greenwood et al. (2013) examine what they call the capital deepening and reallocation effects from financial intermediation. The analysis performed by the authors suggests that a country like Uganda could increase its output by 116% percent if it could adopt the world's best practices in the financial sector which, however, amounts to only 29% of the gap between Uganda's potential and actual output. Midrigan and Yi Xu (2014) analyze financial constraints arising from limits on the amount of debt and equity that agents can issue. They find that financial frictions may reduce the level of aggregate TFP by up to 40%. Hence, this brief discussion suggests that our results are somewhat smaller than those triggered by other frictions in the capital market, although still significant.

4.1 Decompositions

In this section, we analyze the aggregate effects of the distortion triggered by the capital limit $\overline{k}$ along two dimensions. First, we decompose the contribution of capital intensity and TFP to the fall in aggregate output per worker. Second, we assess whether the distortion is mainly channelized through the occupational choice or the production choice of women.

4.1.1 Accounting for losses

We quantify now the contribution of capital intensity and TFP to the fall in output per worker. To do so, we perform two decompositions that are consistent with the two measures of TFP defined by equations (22) and (26). Implicit to the definition TFP_s, for $s = 1, 2$, we have the following aggregate production function:

$$\frac{Y_s}{L_s} = \left(\frac{K}{L_s}\right)^{\alpha\zeta} TFP_s, \tag{27}$$

where $L_1 = L$ and $L_2 = L + \tilde{L}$. Hence, we can directly compute:

$$X_{Y_s/L_s} = \alpha\zeta X_{K/L_s} + X_{TFP_s}, \tag{28}$$

where X_a represents the difference of the log of variable a between the baseline and the counterfactual scenario. Then, we have that the relative contribution of capital intensity and aggregate productivity is $\alpha\zeta X_{K/L_s}/X_{Y_s/L_s} \times 100$ and $X_{TFP_s}/X_{Y_s/L_s} \times 100$, respectively.

Table 7 shows the decomposition implied by equation (28). We observe that aggregate productivity contributes by about 65%-80% to the fall in output per worker, whereas the fall in capital intensity contributes to the rest.

Table 7: Contribution to output per worker losses

		Output Y_1/L_1			
(%)	S1	S2	S3	S4	S5
Capital intensity	18.9	21.5	23.7	24.6	34.6
TFP	81.1	78.5	76.3	75.4	65.4
		Output Y_2/L_2			
(%)	S1	S2	S3	S4	S5
Capital intensity	18.8	21.4	23.6	24.5	34.6
TFP	81.2	78.6	76.4	75.5	65.4

4.1.2 Intensive versus extensive margin

As discussed in Subsection 2.8, the capital limit of $\overline{k}$ for female entrepreneurs brings two consequences, one at an intensive margin level, and the other at an extensive margin level. First, the distortion may produce an exit of women from entrepreneurship (extensive margin effect). Second, women who stay producing despite the capital limit must run smaller businesses (intensive margin effect). We now compare the output loss generated by the intensive and extensive margin effects. We start by defining the extensive margin effect (EM) in the following way:

$$EM = \theta_f \int_{z_f^*}^{\overline{z}_f^*} y_f^*(z)g(z)dz, \tag{29}$$

where $\overline{z}_f^*$ is the skill level of marginal female entrepreneurs in the baseline economy, z_f^* is the analogous threshold in the counterfactual economy, and $y^*(z)$ denotes the output produced by women in the counterfactual economy.[14] Then, EM shows the output produced by women who are entrepreneurs in the counterfactual economy but who leave entrepreneurship in the baseline economy as a consequence of the distortion. That is, EM is the aggregate output loss triggered by the exit of women with skills $z \in [z_f^*, \overline{z}_f^*]$ from entrepreneurship.

[14] Note that women with skills $z \in [z_f^*, \overline{z}_f^*]$ are not entrepreneurs in the baseline economy, and thus, they do not produce any output.

Next, we define the intensive margin effect, IM as

$$IM = \theta_f \int_{\overline{z}_f^*} (y^*(z) - \overline{y}_f(z))g(z)dz, \tag{30}$$

where $\overline{y}(z)$ is the output produced by women in the baseline economy. Therefore, IM measures the change in the output produced by women who run a business even in the presence of a capital limit. Table 8 presents the ratio IM/EM.

Table 8: Extensive versus intensive margin

	S1	S2	S3	S4	S5
IM/EM	6.5	8.8	12.3	15.4	30.8

We observe that the bulk of output losses come from the fact that female entrepreneurs who stay producing in spite of the distortion run smaller businesses, that is the intensive margin effect. The contribution of women who move to production labor is indeed small, as observed in Table 8. Those are low skilled women who run small businesses, and thus, their exit from entrepreneurship causes small output losses.

5 Conclusions

We developed a general equilibrium model to quantify the aggregate consequences of a discriminatory restriction for women in their access to business resources. Our counterfactual analysis shows that a gender-specific capital restriction causes an aggregate output loss between 14% and 28% and a total factor productivity loss between 12% and 20%. About two-thirds of the output loss is explained by the fall in aggregate productivity. Furthermore, we show that the potential aggregate gains of loosening the distortion studied in this paper would be mostly driven by the more intensive use of capital among existing female entrepreneurs and not by the entry of more women into entrepreneurship. Overall, our results suggest significant aggregate gains from a more egalitarian access to business resources across gender. Further research should assess the effectiveness of specific policies that aim to reduce different sources of discrimination against women in entrepreneurship.

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Appendix A: Inputs demands

In this appendix, we derive the demand level for each of the inputs of the production technology of our model economy. We first derive the demands from unconstrained agents, and then, the demands from constrained agents.

We use first equations (3), (4), and (6) to get:

$$k(z) = \left(\frac{\zeta\alpha}{R}\right)^{\frac{1}{1-\zeta\alpha}} A^{\frac{1}{1-\zeta\alpha}} z^{\frac{1-\zeta}{1-\zeta\alpha}} n(z)^{\frac{\zeta(1-\alpha)}{1-\zeta\alpha}}. \tag{A1}$$

Then, we plug (A1) in (3) and use (4) to get:

$$\frac{y(z)}{n(z)} = \left(\frac{\zeta\alpha}{R}\right)^{\frac{\zeta\alpha}{1-\zeta\alpha}} A^{\frac{1}{1-\zeta\alpha}} \left(\frac{z}{n(z)}\right)^{\frac{1-\zeta}{1-\zeta\alpha}}. \tag{A2}$$

Let $\Omega_0 = \left(\frac{\zeta\alpha}{R}\right)^{\frac{\zeta\alpha}{1-\zeta\alpha}} A^{\frac{1}{1-\zeta\alpha}}$. Then, we can express equation (A2) as:

$$\frac{y(z)}{n(z)} = \Omega_0 \left(\frac{z}{n(z)}\right)^{\frac{1-\zeta}{1-\zeta\alpha}}. \tag{A3}$$

Next we use equations (7) and (A3) to get:

$$n(z) = \Omega_1^{\frac{1-\zeta\alpha}{1-\zeta}} \left(\frac{n(z)}{n^f(z)}\right)^{\frac{1-\zeta\alpha}{(1-\zeta)\sigma}} z, \tag{A4}$$

where $\Omega_1 = \frac{\zeta(1-\alpha)\Omega_0}{w^f}$.

We can substitute back (A4) in (A3) to get:

$$\frac{y(z)}{n(z)} = \frac{\Omega_0}{\Omega_1} \left(\frac{n^f(z)}{n(z)}\right)^{\frac{1}{\sigma}}. \tag{A5}$$

We derive now expressions for $n^f(z)/n(z)$ and $n^f(z)/n^m(z)$. From (2) we get:

$$\frac{n(z)}{n^f(z)} = \left(\left(\frac{n^m(z)}{n^f(z)}\right)^{\frac{\sigma-1}{\sigma}} + 1\right)^{\frac{\sigma}{\sigma-1}}. \tag{A6}$$

From equations (7) and (8) we get:

$$\frac{n^f(z)}{n^m(z)} = \left(\frac{w^m}{w^f}\right)^{\sigma}. \tag{A7}$$

From equation (8) we compute:

$$n^m(z) = \left(\frac{\zeta(1-\alpha)(y/n)}{w^m}\right)^{\sigma} n(z). \tag{A8}$$

We directly express:

$$n^f(z) = \left(\frac{n^f(z)}{n^m(z)}\right) n^m(z). \tag{A9}$$

From equation (6) we can get:

$$k(z) = \left(\frac{\zeta\alpha}{R}\right)\left(\frac{y(z)}{n(z)}\right) n(z). \tag{A10}$$

Let $M = \{w^f, w^m, R\}$. Substituting back the expressions derived for y/n, $n(z)$, $n(z)/n^f(z)$, and (A7) into equations (A8) to (A10) we can express the inputs demand system as:

$$k(z; M) = \Phi_0(M)z, \tag{A15}$$

$$n^f(z; M) = \Phi_1(M)z, \tag{A16}$$

$$n^m(z; M) = \Phi_2(M)z, \tag{A17}$$

We now consider the case in which $k = \overline{k}$. Then, we plug this condition in (3) and use (4) to get:

$$y = Az^{1-\zeta}\overline{k}^{\zeta\alpha} n^{(1-\alpha)\zeta}, \tag{A18}$$

Next we use equations (7) and (A18) to get:

$$n(z) = \overline{\Omega_0}^{\frac{1}{\omega_0}} z^{\frac{1-\zeta}{\omega_0}} \overline{k}^{\frac{\alpha\zeta}{\omega_0}} (n^f(z))^{\frac{-1}{\sigma\omega_0}} \tag{A19}$$

where $\overline{\Omega}_0 = \frac{\zeta(1-\alpha)A}{w^f}$ and $\omega_0 = 1 - \frac{1}{\sigma} - (1-\alpha)\zeta$.

From equations (7) and (8) we get:

$$\frac{n^m(z)}{n^f(z)} = \left(\frac{w^f}{w^m}\right)^{\sigma}. \tag{A20}$$

From (2) we get:

$$n(z) = \left(\left(\frac{n^m(z)}{n^f(z)}\right)^{\frac{\sigma-1}{\sigma}} + 1\right)^{\frac{\sigma}{\sigma-1}} n^f(z). \tag{A21}$$

Let $\overline{\Omega}_1 = \left(\left(\frac{n^m(z)}{n^f(z)}\right)^{\frac{\sigma-1}{\sigma}} + 1\right)^{\frac{\sigma}{\sigma-1}}$. Then, using (A19) and (A20) in (A21), we get:

$$\overline{n}^f(z) = \overline{\Omega}_0^{\frac{\sigma}{1+\sigma\omega_0}} \overline{\Omega}_1^{\frac{-\sigma\omega_0}{1+\sigma\omega_0}} \overline{k}^{\frac{\alpha\zeta\sigma}{1+\sigma\omega_0}} z^{\frac{(1-\zeta)\sigma}{1+\sigma\omega_0}} \tag{A22}$$

Then, substituting back (A22) in (A20), we can get $\overline{n}^m(z)$. Then, the demand of constrained entrepreneurs is given by the following system:

$$k(z;M) = \overline{k}, \tag{A23}$$

$$n^f(z;M) = \overline{n}^f(z;M), \tag{A24}$$

$$n^m(z;M) = \overline{n}^m(z;M), \tag{A25}$$

Appendix B: Solving the benchmark model

We first discretize the ability space so it contains Z_n evenly spaced points between z^l and z^h. Then, we use a lognormal density distribution to compute the probability vector for each of the points in the spaces Z_n. Denote by p the latter vector. Once we have built the ability spaces and the discrete density function for them, we solve the household problem for fixed wages and rental price of capital. In Appendix A we showed that, for unconstrained entrepreneurs:

$$k(z;M) = \Phi_0(M)z, \tag{B1}$$

$$n^f(z;M) = \Phi_2(M)z, \tag{B2}$$

$$n^m(z;M) = \Phi_1(M)z. \tag{B3}$$

and for constrained entrepreneurs:

$$k(z;M)=\overline{k}, \tag{B4}$$

$$n^f(z;M)=\overline{n}^f(z;M), \tag{B5}$$

$$n^m(z;M)=\overline{n}^m(z;M), \tag{B6}$$

Then, we use the inputs demands to compute the profits, which is the value of being a manager: $\pi_i(z;M)$, for $i=\{f,m\}$. The value of being a production worker is simply w^i. Then, considering the value of being a manager and the value of being a production worker, we build the indicator function that identifies the members of the household who become a manager:

$$\mathcal{I}_i(z;M)=\begin{cases}1 \text{ if } \pi_i(z;M)>w^i] \\ 0 \text{ if } \pi_i(z;M)\leq w^i].\end{cases} \tag{B7}$$

Notice that the vector $\mathcal{I}_i(z)$ also identifies the skill size of the marginal manager:

$$z_i^*(M)=\min_{z\in Z}\{z\in Z:\mathcal{I}_i(z;M)=1\}. \tag{B8}$$

Next, we build the aggregate demands and supply of labor and capital services. Denote by N_i^j the aggregate demand for labor service type j by group i. Analogously, denote by K_i the aggregate demand for capital services by group i. The aggregate demand for labor services type j of agent type i is:

$$N_i^j(M)=\sum_{z=z^l}^{z^h} n_i^j(z;M)\mathcal{I}_i(z;M)g(z)dz, \tag{B9}$$

The aggregate demand for capital services by agent type i is:

$$K_i(M)=\sum_{z=z^l}^{z^h} k_i(z;M)\mathcal{I}_i(z;M)g(z)dz. \tag{B10}$$

Next, we compute the aggregate demand for each of the four inputs:

$$N^f(M) = \sum_{i=\{f,m\}} \theta_i N_i^f(M), \tag{B11}$$

$$N^m(M) = \sum_{i=\{f,m\}} \theta_i N_i^m(M), \tag{B12}$$

$$K(M) = \sum_{i=\{f,m\}} \theta_i K_i(M), \tag{B13}$$

Then, we compute the supply of labor inputs:

$$S_i(M) = \theta_i \sum_{z=z^l}^{zh} (1 - \mathcal{I}_i(z;M))g(z)dz \tag{B14}$$

For the supply of capital we initially set $K = \hat{K}$. Then, given the guess for the capital stock, we compute equilibrium prices. We iterate on prices until all markets clear and find equilibrium prices for a given stock of capital. Then, we evaluate if the resulting rental rate for capital services differs from $((1 + x_c)/\beta) + \delta - 1$ and we iterate on capital until the R is equal to $((1 + x_c)/\beta) + \delta - 1$, using a a bijective algorithm.