What do we talk about when we talk about Majorana neutrinos?

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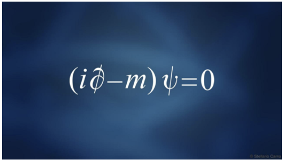
What do we talk about when we talk about Dirac?



- Since neutrinos are fermions of spin 1/2 they can be described by Dirac equation.
- P.A.M. Dirac had proposed his famous equation in 1928, two years before the neutrino was proposed by Pauli.
- In 1929 he published his famous paper predicting antimatter (the positron), who would be discovered shortly after by Andersen. Yet, in 1930, antimatter was a concept as fantastic and hard to believe in as the neutrino itself.

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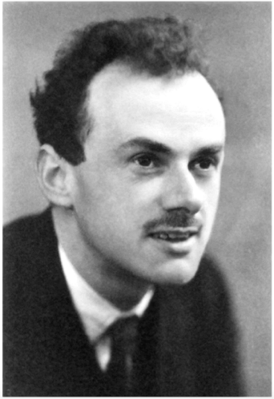

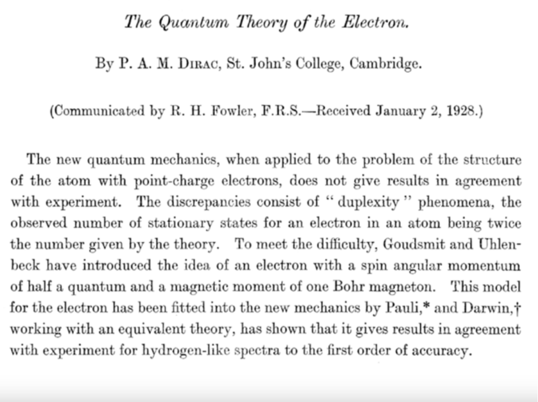

The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charge electrons, does not give results in agreement with experiment. The discrepancies consist of " duplexity " phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton. This model for the electron has been fitted into the new mechanics by Pauli,* and Darwin,† working with an equivalent theory, has shown that it gives results in agreement with experiment for hydrogen-like spectra to the first order of accuracy.

The Dirac equation describes spin-1/2 particles such electrons and neutrinos. It emerges from Dirac's attempt to avoid the negative solutions in the equation of Klein-Gordon which is obtained when one quantizes the relativistic relation:

$$E^2 = p^2 + m^2 \text{ (with c} = 1)$$

through the quantum-mechanical recipe:

$$E \rightarrow i\frac{\partial}{\partial_t},\ \vec{P} \rightarrow -i\vec{\nabla}$$

then we obtain the Klein Gordon equation

$$\boxed{(i\frac{\partial}{\partial_t})^2\psi = [(-i\bar{\nabla})^2 + m^2]\psi}$$

The wavefunction ψ is now a relativistic scalar and the space and time derivatives are both second order. However, the initial values of ψ and $\partial\psi$ can be chosen freely, and as a result the probability density is no longer positive definite. This leaves open the possibility of negative probabilities.

Linearizing $E = \sqrt{p^2 + m^2}$

Dirac approach was to attempt linearizing the relativistic energy-momentum equation

$$E = \sqrt{p^2 + m^2} = \vec{\alpha} \cdot \vec{P} + \beta \cdot m = \alpha_x p_x + \alpha_y p_y + \alpha_z p_z + \beta m$$

Squaring both sides:

$$\begin{aligned} E^2 = p^2 + m^2 =& (\alpha_x p_x + \alpha_y p_y + \alpha_z p_z + \beta m)(\alpha_x p_x + \alpha_y p_y + \alpha_z p_z + \beta m) \\ =& p_x^2 + p_y^2 + p_z^2 + m^2 \end{aligned}$$

The above equation can only be solved if the α_i, β are matrices of at least rank 4, which satisfy:

$$\boxed{\begin{array}{l} \{\alpha_i, \alpha_j\} = 0 \ (i \neq j) \\ \{\alpha_i, \beta\} = 0 \end{array}} \tag{1}$$

where:

$$\{A, B\} = AB + BA$$

Constructing α and β using Pauli matrices

They α, β can be constructed in terms of the Pauli matrices:

$$\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

where I is the 2×2 identity matrix, and the Pauli matrices are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Pauli matrices exhibit clearly the properties of being hermitian and traceless, $\sigma_i^\dagger = \sigma_i$, $Tr\sigma_i = 0$, and $\sigma_i^2 = I$. They satisfy the commutation relations:

$$\boxed{\begin{array}{l} \{\sigma_i, \sigma_j\} = 2\delta_{ij} \ (i, j, k = 1, 2, 3) \\ [\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \end{array}} \tag{2}$$

using the linearized equation

$$E - \vec{\alpha} \cdot \vec{\mathbf{P}} - \beta \cdot m = 0$$

and substituting operators

$$E \to i\frac{\partial}{\partial_t},\ \vec{\mathbf{P}} \to -i\vec{\nabla}$$

One obtains the Dirac equation

$$i\frac{\partial}{\partial_t}\psi = [\vec{\alpha}(-i\vec{\nabla}) + \beta m]\psi$$

Multiply now β from the left and define the gamma matrices:

$$\gamma^0 = \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}\ , \gamma^i = \beta\alpha_i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

To obtain

$$\boxed{(i\gamma^\mu\partial_\mu - m)\psi = 0}$$

The γ matrices are not unique. Any set that satisfy the anticommutation relations (Clifford algebra) can be used:

$$\boxed{\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}}$$

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Plane wave solutions of the Dirac equation

For a particle at rest, $\vec{\mathbf{p}} = 0$ and the Dirac equation becomes:

$$i\gamma^0 \partial_0 \psi = m\psi, \quad or \quad i \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \dot{\psi}_1 \\ \dot{\psi}_2 \\ \dot{\psi}_3 \\ \dot{\psi}_4 \end{pmatrix} = m \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}$$

There are four independent solutions: two describe particle-antiparticle and two describe spin up-down

$$\psi^{(1)} = w^{(1)} e^{-im \cdot t}, \;\; \psi^{(2)} = w^{(2)} e^{-im \cdot t}, \;\; \psi^{(3)} = w^{(3)} e^{+im \cdot t}, \;\; \psi^{(4)} = w^{(4)} e^{im \cdot t}$$

$$w^{(1)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \;\; w^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \;\; w^{(3)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \;\; w^{(4)} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

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Negative energy solutions

Since we define the eigenvalue of the operator $\frac{\partial}{\partial t}$ to be the energy, we find that $\psi^{(1)}$ and $\psi^{(2)}$ are positive energy solutions and $\psi^{(3)}$ and $\psi^{(4)}$ are negative energy solutions.

Surprise!

Dirac's equation predicts negative energy solutions.

Dirac struggle with negative energy solutions: protons

A Theory of Electrons and Protons.

By P. A. M. Dirac, St. John's College, Cambridge.

(Communicated by R. H. Fowler, F.R.S.—Received December 6, 1929.)

§ 2. *Solution of the Negative Energy Difficulty.*

The most stable states for an electron (*i.e.*, the states of lowest energy) are those with negative energy and very high velocity. All the electrons in the world will tend to fall into these states with emission of radiation. The Pauli exclusion principle, however, will come into play and prevent more than one electron going into any one state. Let us assume there are so many electrons in the world that all the most stable states are occupied, or, more accurately, that *all the states of negative energy are occupied except perhaps a few of small velocity.* Any electrons with positive energy will now have very little chance of jumping into negative-energy states and will therefore behave like electrons are observed to behave in the laboratory. We shall have an infinite number of electrons in negative-energy states, and indeed an infinite number per unit volume all over the world, but if their distribution is exactly uniform we should expect them to be completely unobservable. *Only the small departures from exact uniformity, brought about by some of the negative-energy states being unoccupied, can we hope to observe.*

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The opinion of Heisemberg

“The saddest chapter of modern physics is and remains the Dirac theory ... I regard [it] ... as learned trash which no one can take seriously.”

Werner Heisenberg
(1901-1976)
1932 Nobel Laureate

Identifying holes in the negative sea with positrons

Quantised Singularities in the Electromagnetic Field.

By P. A. M. Dirac, F.R.S., St. John's College, Cambridge.

(Received May 29, 1931.)

...It thus appears that we must abandon the identification of the holes with protons and must find some other interpretation for them. Following Oppenheimer,§ we can assume that in the world as we know it, *all*, and not merely nearly all, of the negative-energy states for electrons are occupied. A hole, if there were one, would be a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron. We may call such a particle an anti-electron. We should not expect to find any of them in nature, on account of their rapid rate of recombination with electrons, but if they could be produced experimentally in high vacuum they would be quite stable and amenable to observation. An encounter between two hard γ-rays (of energy at least half a million volts) could lead to the creation simultaneously of an electron and anti-electron, the probability of occurrence of this process being of the same order of magnitude as that of the collision of the two γ-rays on the assumption that they are spheres of the same size as classical

The discovery of the positron (Andersen, 1932)

MARCH 15, 1933 PHYSICAL REVIEW VOLUME 43

The Positive Electron

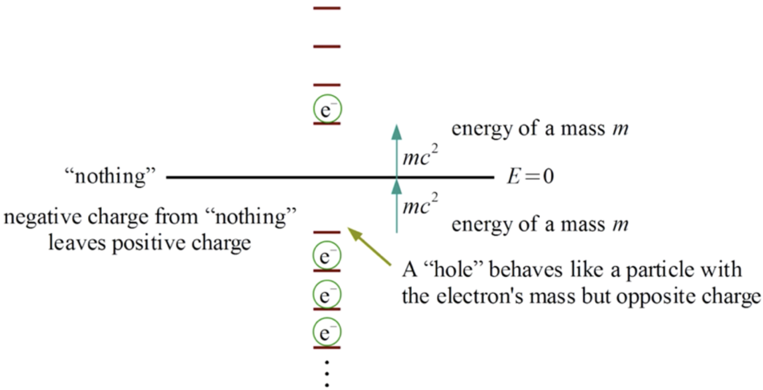
CARL D. ANDERSON, *California Institute of Technology, Pasadena, California*

(Received February 28, 1933)

Out of a group of 1300 photographs of cosmic-ray tracks in a vertical Wilson chamber 15 tracks were of positive particles which could not have a mass as great as that of the proton. From an examination of the energy-loss and ionization produced it is concluded that the charge is less than twice, and is probably exactly equal to, that of the proton. If these particles carry unit positive charge the curvatures and ionizations produced require the mass to be less than twenty times the electron mass. These particles will be called positrons. Because they occur in groups associated with other tracks it is concluded that they must be secondary particles ejected from atomic nuclei.

Editor

The negative sea parable



In spite of the heroic efforts of Dirac and others, RQM is not enough to describe elementary particles. The Dirac equation describes a single electron. Positrons need to be introduced by a sort of magical incantation (the negative sea state). In short, RQM cannot predict creation and annihilation of particles.

Quantum Electrodynamics (QED) was eventually developed as the first Quantum Field Theory (QFT), capable of describing such creation and annihilation of particles. In QED, it is not only the photons that are quanta of a field but also the charged particles, like the electrons and positrons. The fields are operators that create and annihilate their quanta. Thus $\psi(x)$ is no longer interpreted as a waveform that describes probability but as an operator that creates a particle in point x and destroys an antiparticle in x ($\bar{\psi}(x)$ will create an antiparticle in point x and destroy antiparticle in x).

Dirac revisited

Recall the Dirac equation:

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

which we can rewrite (using $i\partial_0 \to E, i\partial_i \to -\vec{\mathbf{p}}$) as:

$$(E - \vec{\mathbf{p}} \cdot \vec{\gamma} - m)\psi = 0$$

Also recall that ψ is a 4-dimensional spinor:

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_2 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

where ψ_L and ψ_R are two-components column matrices.

Dirac equation using Weyl representation of the γ matrices

$$\gamma^0 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \ \vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$$

Then the Dirac equation becomes:

$$\left[E\begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} - \begin{pmatrix} 0 & \vec{\mathbf{p}}\cdot\vec{\sigma} \\ -\vec{\mathbf{p}}\cdot\vec{\sigma} & 0 \end{pmatrix} - \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}\right]\begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = 0$$

which solves into two equations:

$$\boxed{\begin{aligned} (E - \vec{\mathbf{p}}\cdot\vec{\sigma})\psi_R &= m\psi_L \\ (E + \vec{\mathbf{p}}\cdot\vec{\sigma})\psi_L &= m\psi_R. \end{aligned}}$$

Massless particles

In the limit of $m \to 0$ these two equations decouple:

$$\boxed{\begin{aligned}(E + \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_L = 0 \\ (E - \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_R = 0\end{aligned}}$$

Which implies that a four component eigenstate $\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$ splits into two two-component pieces ψ_L and ψ_R, that don't get mixed up by the equations of motion.

Surprise!

Dirac's equation predicts two sorts of massless fermions in Nature. **left-handed fermions** described by ψ_L, and **right-handed fermions** described by ψ_R. In the chiral (Weyl) representation of the equation, they decouple perfectly.

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Chirality operator

Define the **chirality operator** as:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -I & 0 \\ 0 & I \end{pmatrix}$$

(the matrix representation holds only in the chiral representation)

Then a wave function $\begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$ is an eigenstate of γ^5 with eigenvalue 1, while a wave function $\begin{pmatrix} 0 \\ \psi_R \end{pmatrix}$ is an eigenstate of γ^5 with eigenvalue -1.

The Dirac's equation predicts then two sorts of wave functions in the Universe. Left-handed functions with chirality -1 and right handed functions with chirality +1. We can extract the left and right part of a wave function using projection operators:

$$P_L = \frac{1-\gamma^5}{2}, \quad P_R = \frac{1+\gamma^5}{2}$$

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Helicity

Since ψ_L and ψ_R are eigenstates of the massless Dirac equation, then a free and massless left-handed particle can never change into a right-handed particle.
On the other hand, for massless particles, $E = |\delta p|$, and thus, substituting in the massless Dirac equation:

$$\boxed{\begin{aligned} \frac{\vec{\sigma}\cdot\vec{\mathbf{p}}}{\vec{\mathbf{p}}}\psi_R &= \psi_R \\ \frac{\vec{\sigma}\cdot\vec{\mathbf{p}}}{\vec{\mathbf{p}}}\psi_L &= -\psi_L \end{aligned}}$$

This means that the massless states ψ_L and ψ_R are also eigenstates of the helicity operator

$$h = \frac{\vec{\sigma}\cdot\vec{\mathbf{p}}}{\vec{\mathbf{p}}}$$

Or, in other words, for massless particles, helicity and chirality mean the same. Notice that right-handed particles have positive helicity and left-handed particles have negative helicity.

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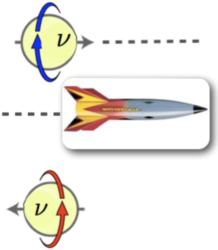

For massive particles, helicity depends on the reference frame and thus is not Lorentz invariant. One can always jump into a reference system faster than that of the particle and see its helicity flip.
But massless particles travel at the speed of light and cannot be overtaken. The helicity becomes a constant of motion.

Massive Dirac particles involve both left- and right- handed wavefunctions coupled by the particle mass. We can think of massive Dirac particles oscillating back and forth in time between left and right handed at a rate determined by their mass. This is immediately evident considering massive Dirac particles at rest:

$$\boxed{\begin{aligned} i\gamma_0\psi_R &= m\psi_L \\ i\gamma_0\psi_L &= m\psi_R \end{aligned}}$$

For the negative solutions of the Dirac equation we can write $E = -|p_0|$, and then, for massless particles:

$$\boxed{\begin{aligned}(-|p_0| + \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_L = 0 \\ (-|p_0| - \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_R = 0\end{aligned}}$$

which implies:

$$\boxed{\begin{aligned}\frac{\vec{\sigma} \cdot \vec{\mathbf{p}}}{\vec{\mathbf{p}}}\psi_R = -\psi_R \\ \frac{\vec{\sigma} \cdot \vec{\mathbf{p}}}{\vec{\mathbf{p}}}\psi_L = \psi_L\end{aligned}}$$

That is, right-handed antiparticles have negative helicity and left-handed antiparticles have positive helicity, exactly the opposed to the case of particles.

The four component solutions to the Dirac equation have left-handed parts in the two upper slots and right handed parts in the two lower slots and can be written as:

$$\psi(x) = \begin{pmatrix} \psi_L(x) \\ \psi_R(x) \end{pmatrix}.$$

The four component wave functions are known as Dirac spinors. The two component ψ_L and ψ_R are known as Weyl spinors.

A symmetric electron-positron theory (1937)

TEORIA SIMMETRICA DELL'ELETTRONE
E DEL POSITRONE

Nota di ETTORE MAJORANA

Sunto. - *Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di* DIRAC *ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di «antiparticelle» corrispondenti ai «vuoti» di energia negativa.*

We show that it is possible to achieve complete formal symmetrization in the electron and positron quantum theory by means of a new quantization process. The meaning of Dirac equations is somewhat modified and **it is no more necessary to speak of negative-energy states; nor to assume, for any other type of particles, especially neutral ones, the existence of antiparticles, corresponding to the "holes" of negative energy.**

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An unpleasant symmetry

- "The interpretation of the so called "negative energy states" proposed by Dirac leads, as it is well known, to a substantially symmetric description of electron and positrons... but it looks to us important, in view of possible extensions, that the notion itself of negative energy state be abandoned."
- "Indeed, we shall see that to be perfectly possible to build, in the most natural way, a theory of the neutral elementary particles without negative states."
- "The new approach allows not only to give a symmetric form to the electron-positron theory, but also to build a substantially novel theory for the particles deprived of electric charge."
- "it is probably not yet possible to ask to the experience to decide between this new theory and the simple extension of the Dirac equations to the neutral particles."

VOW!

- At the time the neutrino had not yet been discovered.
- Even if in those times the only known "charge" was the electric charge, Majorana implicitly assumed particles deprived of all the possible charges.

Majorana questioned whether it was necessary for spin-1/2 particles to have equations that involved complex numbers. What was required were gamma matrices that still satisfy the Clifford algebra but were purely imaginary. Majorana found such constructions in terms of the Pauli matrices:

$$\gamma^0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix},\ \gamma^1 = \begin{pmatrix} i\sigma_1 & 0 \\ 0 & i\sigma_1 \end{pmatrix},\ \gamma^2 = \begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix},\ \gamma^4 = \begin{pmatrix} i\sigma_3 & 0 \\ 0 & i\sigma_3 \end{pmatrix}$$

Recall that σ_1, σ_3 are real, while σ_2 is complex. Thus, all the Majorana gamma matrices are complex. But then, the Majorana spinor ψ_M must be real and is therefore invariant to charge conjugation. The derived field theory can be constructed from one type of operator. Therefore, the particles created are fermions that are their own antiparticles.

Majorana insight

Majorana proposed an alternative to the two-coupled, two-component Dirac equation, namely two independent, relativistic, two-component equations:

$$\boxed{\begin{array}{c}(E + \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_L - m\epsilon\psi_L^* = 0 \\ (E - \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_R - m\epsilon\psi_R^* = 0\end{array}}$$

where

$$\epsilon = i\sigma_2 = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$

If we compare the Dirac and Majorana equations:

$$\begin{aligned}(E + \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_L - m\psi_R &= 0 \\ (E + \vec{\mathbf{p}} \cdot \vec{\sigma})\psi_L - m\epsilon\psi_L^* &= 0\end{aligned}$$

It's obvious that both equations are identical for $m = 0$. In the Standard Model neutrinos are assumed to be massless and thus both theories are identical. Instead, if neutrinos are massive they are not. The remarkable thing about Majorana equation is that it is constructed only with the $\psi_L i$ (or the ψ_R for the second equation), thus eliminating two degrees of freedom.

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In the chiral representation the recipe for obtaining the charge conjugate ψ_C of a Dirac spinor ψ is:

$$\psi_C = C^{-1}\psi C = C_0\psi^*$$

with $C_0 = -i\gamma^2$.

Thus if we start with a Dirac spinor $\psi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$, we obtain:

$$\psi_C = -i\gamma^2 \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}^* = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} \psi_L^* \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i\sigma_2\psi_L^* \end{pmatrix}$$

and for the right handed spinor:

$$\begin{pmatrix} 0 \\ \psi_R \end{pmatrix} \rightarrow \begin{pmatrix} -i\sigma_2\psi_R^* \\ 0 \end{pmatrix}$$

Majorana spinor

In the chiral basis we build a Majorana spinor by stacking together a left-handed Weyl spinor and its charged conjugated as follows:

$$\phi = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ i\sigma_2 \psi_L^* \end{pmatrix} = \begin{pmatrix} \psi_L \\ i\sigma_2 \psi_L^* \end{pmatrix}$$

On the other hand, given a spinor ϕ representing a particle, the charge-conjugate spinor representing an antiparticle is:

$$\boxed{\phi^C = C\gamma^0 \phi^*}$$

where ϕ^* is the complex conjugate of ϕ and the charge conjugation matrix C can be written in the Weyl representation as:

$$C = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}, \quad C\gamma^0 = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}$$

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A particle is its own antiparticle if we can reverse all its charges without effect. Obviously electrons cannot be their own antiparticles, but Dirac neutrinos are also not their own antiparticles, as explicit in the construction of the bi-spinor that separates particles (ψ_L) and antiparticles (ψ_R).

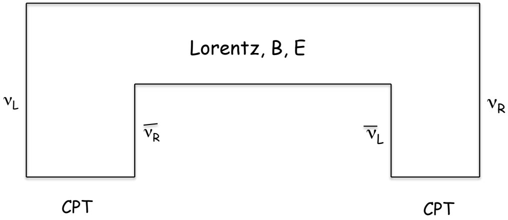
For a Majorana spinor $\phi = \begin{pmatrix} \psi \\ i\sigma_2\psi^* \end{pmatrix}$:

$$\boxed{\begin{aligned} \phi^C &= C\gamma^0\phi^* = \begin{pmatrix} 0 & -i\sigma_2 \\ i\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} \psi^* \\ -i\sigma_2\psi \end{pmatrix} \\ &= \begin{pmatrix} -i^2\sigma_2^2\psi \\ i\sigma_2\psi^* \end{pmatrix} = \begin{pmatrix} \psi \\ i\sigma_2\psi^* \end{pmatrix} = \phi \end{aligned}} \tag{3}$$

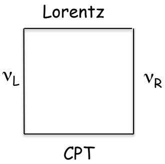
This means that a Majorana particle is identical to its antiparticle.

Majorana and Dirac neutrinos

Four distinct states of a massive Dirac neutrino and the transformation among them.
ν_L can be converted into the opposite helicity state by a Lorentz transformation,
or by the torque exerted be an external B or E field.



Dirac neutrino ν_D



There are only two distinct states of a Majorana neutrino ν_M. Under the Lorentz transformation ν_L is transformed into the same state ν_R as by the Lorentz transformation. The dipole magnetic and electric moments must vanish.

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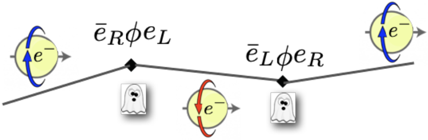
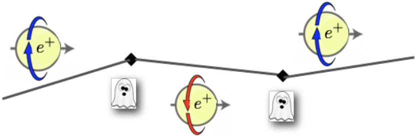
Electron mass



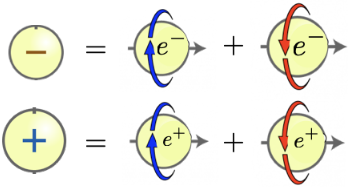
left and right handed states bump against the Higgs field



$$\mathcal{L}_D = \bar{e}_L m_e e_R + h.c.$$

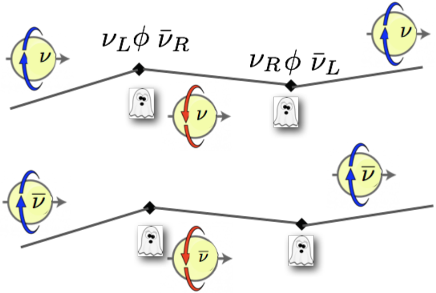
$$\lambda \bar{e}_R \phi e_L \to \lambda v \bar{e}_R e_L$$

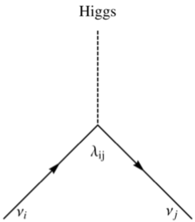
$$m_e = \lambda_e v$$



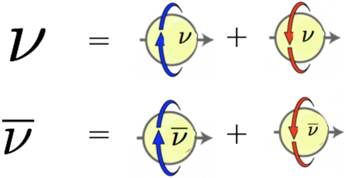
$$e^- = e_L^- + e_R^-$$

$$e^+ = e_L^+ + e_R^+$$





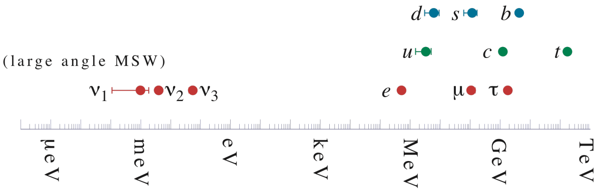
$$-\mathcal{L}_{\rm Dirac} = \bar{\nu}_L m_\nu \nu_R + h.c.$$
$$m_\nu = \lambda_\nu v$$



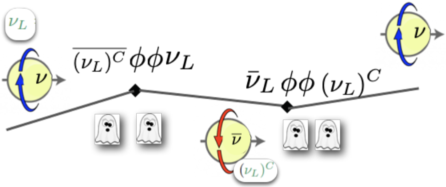
$$\boxed{\nu = \nu_L + \nu_R}$$

$$\boxed{\nu^C = (\nu_L)^C + (\nu_R)^C}$$

Why are the neutrinos so light?

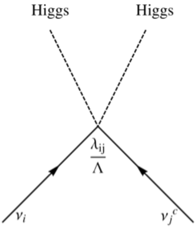


Neutrino mass (Majorana recipe)



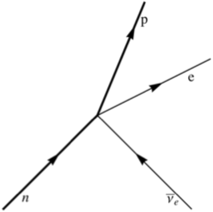
$$\boxed{\nu_L = (\nu_R)^C \qquad (\nu_L)^C = \nu_R}$$

$$-\mathcal{L}_{\text{Majorana}} = \bar{\nu}_L m_\nu \nu_L^c + h.c.$$



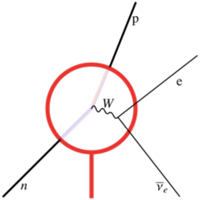
$$m_\nu \sim \lambda \frac{v^2}{\Lambda}$$

Effective theory
(Fermi constant)

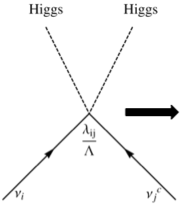


$$G_F \sim \frac{1}{M_W^2}$$

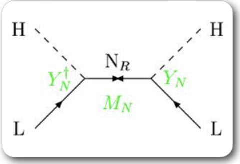
Standard Model



Effective theory
(Λ)



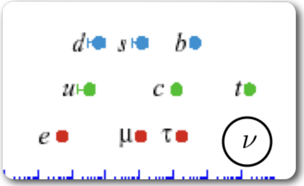
Extension of
Standard Model



$$m_\nu = \frac{\alpha v^2}{\Lambda} \equiv Y_N^T \frac{v^2}{M_N} Y_N$$

See-saw models

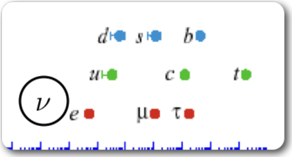
$\mathrm{M_N} = \mathrm{GUT}$



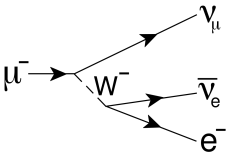
Yukawa

$$m_\nu = \frac{\alpha v^2}{\Lambda} \equiv Y_N^T \frac{v^2}{M_N} Y_N$$

$\mathrm{M_N} = \mathrm{TeV}$



Yukawa



Lepton number counts the total number of leptons and anti-leptons involved in a weak interaction which must be zero. For example, in the decay of μ above, the disappearance of the μ^- (a lepton) is compensated with the appearance of a ν_μ (also a lepton), while the appearance of e^- (a lepton) is compensated with the appearance of a $\bar{\nu_e}$ (an anti-lepton). On the contrary, Majorana neutrinos have no lepton number. As such they may induce processes violating lepton number conservation.

In most processes involving neutrinos the energy of the process is very large compared with the (tiny) neutrino mass. In practice we approach the limit $m = 0$ and Dirac and Majorana become impossible to distinguish.

CP violation and Majorana neutrinos

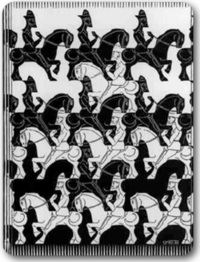
$$N \rightarrow e^- + H^+ \quad \text{and} \quad N \rightarrow e^+ + H^-$$

Standard-Model Higgs

•If there is CP violation in the lepton sector, the heavy Majorana neutrino N can violate CP too and decay with different rates to electrons and positrons. This results in an unequal number of leptons and antileptons in the early universe

•Leptonic asymmetry is later transferred to baryons, resulting in...

The Universe



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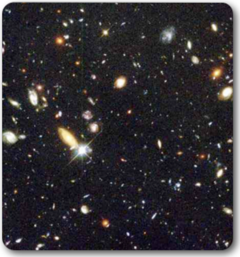
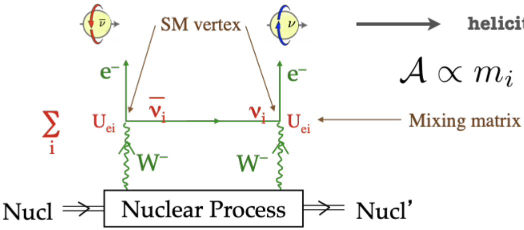


Majorana mass

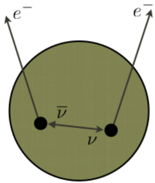


helicity flip $\propto \dfrac{m_i}{E}$

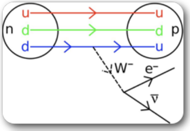
$\mathcal{A} \propto m_i$ **for each** ν_i

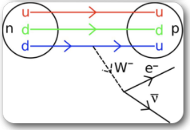
$$m_{\beta\beta} = ||U_{e1}|^2 m_1 + e^{i\alpha_1}|U_{e2}|^2 m_2 + e^{i\alpha_2}|U_{e3}|^2 m_3|$$

The Uei terms are measured by neutrino oscillation experiments. Nothing is known about the two Majorana phases.



$\beta\beta0\nu$





$$(T^{0\nu}_{1/2})^{-1} = \boxed{G^{0\nu}(Q,Z)}\,\boxed{|M^{0\nu}|^2}\, m^2_{\beta\beta}$$

phase-space

nuclear matrix

Majorana neutrino

Two protons decay simultaneously in a heavy isotope
Nuclear physics results in proportionality constants between period and the inverse of the Majorana mass squared

DIPC Donostia International Physics Center
ikerbasque Basque Foundation for Science

Good luck!