Sizing Procedure

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1 Introduction

Pitching Moment

$M_A = M_{acwf} + L_{wf}\left(x_{cg} - x_{ac}\right) - L_h l_h$

$\frac{1}{2}\rho V^2 Sc C_{mA} = \frac{1}{2}\rho V^2 Sc C_{macwf} + \frac{1}{2}\rho V^2 S C_{Lwf}\left(x_{cg} - x_{ac}\right) - \frac{1}{2}\rho V_h^2 S_h C_{Lh} l_h$

$C_{mA} = C_{macwf} + C_{Lwf}\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - C_{Lh}\frac{V_h^2}{V^2}\frac{S_h}{S}\frac{l_h}{c}$

$C_{mA} = C_{macwf} + C_{Lwf}\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - C_{Lh}\eta_h \bar{V}_h$

$C_{mA} = C_{macwf} + a_{wf}\alpha_{wf}\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - a_h\alpha_h\eta_h \bar{V}_h$

$C_{mA} = C_{macwf} + a_{wf}\alpha_{wf}\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - a_h\left(\alpha_{wf} - i_{wf} + i_h - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha_{wf}\right)\eta_h \bar{V}_h$

$C_{mA} = C_{m0} + C_{m\alpha}\alpha_{wf}$

$$\begin{cases} C_{m0} = & C_{macwf} - a_h\eta_h\bar{V}_h\left(i_t - i_{wf} - \varepsilon_0\right) \\ C_{m\alpha} = & a_{wf}\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - a_h\eta_h\bar{V}_h\left(1 - \frac{d\varepsilon}{d\alpha}\right) \end{cases}$$

Longitudinal Stability

Neutral Point ($\bar{x}_{cg}$ so that $C_{m\alpha} = 0$)

$NP = \bar{x}_{ac} + \frac{a_h}{a_{wf}}\eta_h\bar{V}_h\left(1 - \frac{d\varepsilon}{d\alpha}\right)$

Static Margin

$SM = NP - \bar{x}_{cg}$

Wetted Area after CG

$$S_{wet-aft} = S_{wet-aft-fus} + S_{wet-h}$$

$$S_{wet-aft} = 4D_f d + S_{wet-rear-fus} + 2S_h$$

$$S_{wet-aft} = 4D_f d + S_{wet-rear-fus} + 2\frac{\bar{V}_h S c_w}{l_h}$$

$$S_{wet-aft} = 4D_f d + S_{wet-rear-fus} + 2\frac{\bar{V}_h S c_w}{d+L_{frear}-(1-xMAC_h)c_h}$$

$$\frac{\partial S_{wet-aft}}{\partial d} = 4D_f - 2\frac{\bar{V}_h S c}{\left[d+L_{frear}-(1-xMAC_h)MAC_h\right]^2}$$

2 Positioning Procedure

Unknowns:

$x(1) = x_{ac}$

$x(2) = L_{fbody}$

$x(3) = d$

Constraints to satisfy:

$SM = 0.2$

$\frac{\partial S_{wet-aft}}{\partial d} = 0$

$d = L_{fnose} + L_{fbody} - x_{CG}(L_{fbody})$

Nonlinear system of equations:

$$y(1) = \frac{x(1)}{c_w} + \frac{a_h}{a_{wf}}\eta_h \bar{V}_h\left(1 - \frac{d\varepsilon}{d\alpha}\right) - \frac{x_{CG}(x(2))}{c_w} - 0.2 = 0$$

$$y(2) = 4D_f - 2\frac{\bar{V}_h S c}{\left[x(3)+L_{frear}-(1-xMAC_h)c_h\right]^2} = 0$$

$$y(3) = x(3) - \left[L_{fnose} + x(2) - x_{CG}(x(2))\right] = 0$$

Assumptions:

- $V_h = 0.5$
- $a_h = 2\pi$
- $\eta_h = 0.98$

- $A_h = \frac{2}{3} A_w$
- $D_f = 0.12$
- $L_{fnose} = 1.5 D_f$
- $L_{frear} = 2 D_f$
- $c_h = c_w$
- $xMAC_h = xMAC_w$
- $xTE_h = L_f$
- relative position of the parts within the plane

3 Hstab Sizing Procedure

$x_{CG} = f\left(L_{fbody}\right)$

$l_h = L_f - (1 - xMAC_h) c_h - x_{CG}$

$S_h = \frac{\bar{V}_h S c_w}{l_h}$

Required C_{Lh} for longitudinal trim:

$C_{macwf} + C_{Lwf}\left(\bar{x}_{cg} - \bar{x}_{ac}\right) - C_{Lh}\eta_h \bar{V}_h = 0$

$C_{Lh} = \frac{C_{macwf} + C_{Lwf}(\bar{x}_{cg} - \bar{x}_{ac})}{\eta_h \bar{V}_h}$

Required α_h for longitudinal trim (iterate to find α_h so that $C_{Lh} = C'_{Lh}$):

$C_{Lh} \approx a_h \alpha_h$

$\alpha_h = \frac{C_{Lh}}{a_h}$

$C'_{Lh} = LiftingLine\left(\alpha_h\right)$

Required i_h:

$\alpha_h = \alpha_{wf} - i_{wf} + i_h - \varepsilon_0 - \frac{d\varepsilon}{d\alpha}\alpha_{wf}$

$i_h = \alpha_h - \alpha_{wf} + i_{wf} + \varepsilon_0 + \frac{d\varepsilon}{d\alpha}\alpha_{wf}$