Poisson Surface Reconstruction on non-Cartesian Lattices

***Abstract*—In this work, we cast the well-known Poisson surface reconstruction algorithm into a more general setting, we reformulate it in terms of shift invariant spaces (spaces that are spanned by lattice translates of an admissible generating function). The treatment is general, but our specific interest is in reconstructing a surface of a solid model from an oriented point-set within function spaces defined over the body centred cubic lattice. We also propose a general framework for approximating the solution to Poisson?s equation in a hypercube with zero Dirichlet boundary conditions, and a new variational scheme to re-sample the initial oriented point-set onto a regular grid. Once the points have been re-sampled onto a grid, the rest of the pipeline is purely based on digital filtering. We also analyze the error incurred via the digital filtering approximation methodology and propose a Fourier domain error kernel that can be used to design solution filters that fully exploit the approximation capabilities of the target space. Finally, we show that a host of Poisson-like methods fail to take advantage of the full approximation spaces over which they are defined.**

I. Introduction

Reconstructing a solid model from a scattered set of points is a common and well studied problem in computer graphics. Interest in this problem is rooted in the desire to convert real life data-sets – acquired from range and now increasingly from commodity hardware like the Kinect – to 3D polygonal models. The idea also has a applications in model re-meshing and hole filling. [[XXX This needs more meat, it's a pretty weak ending to the intro paragraph.]]

Given an oriented point-set, a common approach is to find an implicit function whose iso-contour corresponds to an approximation of the original surface. In this work we take a "signal processing" approach to this problem – we cast the problem into a general shift-invariant setting. More explicitly, we reconstruct the desired implicit function within a function space spanned by lattice translates of a single generating function. The extension to this class of function spaces allows us to consider anchoring our basis functions at non-Cartesian lattice sites (Section **??**). Our approach is general and applicable to any valid lattice and generator, but our implementation is narrowed to applicable function spaces over the *Body Centred Cubic* (BCC) and *Cartesian* (CC) lattices.

The advantage of moving to the BCC lattice is two fold. First, it's well known that the BCC lattice generates the optimal sampling pattern in $\mathbb{R}^3$. The argument for optimality is simple; the sampling of a function with respect to the BCC lattice is equivalent to a periodization of it's frequency spectrum about the BCC's dual lattice, the *Face Centered Cubic* (FCC) lattice. Optimality follows from the observation that the FCC lattice is the optimal sphere packing lattice in three dimensions, that is, it packs frequency replica as tightly as possible in the Fourier domain, resulting in less *pre-aliasing* from sampling. For isotropically band-limited functions, this tighter packing of frequency content allows for about 30% more information to be captured when compared to samplings generated from a Cartesian lattice. The second advantage to moving to the BCC lattice is that there exist reconstruction filters on the BCC lattice that outperform commonly used filters of equivalent order on the CC lattice in terms of both speed and accuracy [**?**] (in software implementations) – these filters are also more compact than their typical CC counterparts. In the context of our surface reconstruction scheme, this gives rise to regularization matrices that are more sparse on the BCC lattice and tend to provide faster convergence when optimizing the initial point set Section **??**.

Our surface reconstruction methodology follows the general Poisson approach, but with a few twists. First, we construct a smoothed approximation to the gradient field of a model's indicator function. The divergence of the gradient field is estimated and subsequently fed into a Poisson solver that outputs the coefficients needed to approximate the smoothed indicator function in a target shift invariant space. This Poisson solver is tailored to cater to the approximation power provided by each space. However, our approach is novel in two notable ways. Firstly, we employ a variational scheme to obtain a lattice-based approximation of the gradient field. This scheme depends only on the generating kernel of the target space and optimizes a functional that incorporates interpolation and smoothness constraints. Secondly, we utilize the theory behind shift-invariant spaces to seek discretizations of the divergence and Laplacian operators that are tailored to exploit the full approximation capabilities of the target space.

Additionally, we also consider the possibility of approximating the smoothed gradient field representation within function spaces that are spanned by shifted versions of a single generating kernel. In the context of gradient estimation, introducing a shift in the direction of the derivative has shown to improve overall gradient approximation fidelity in terms of gradient orientation and magnitude, as well as displaying the ability to capture higher frequency details that are smoothed out in non-shifted schemes [**?**]. Since the divergence operator is intimately connected to the gradient operator, these results are of interest to us. Using an appropriate discretization of the derivative/divergence operator is an important step in recovering the indicator function of the initial model. [[XXX Need to emphasize the error-kern here]]

To summarize, our contributions are as follows:

- We cast the Poisson surface reconstruction problem into the framework of shift-invariant spaces.
- Specifically, we investigate the reconstruction of surfaces

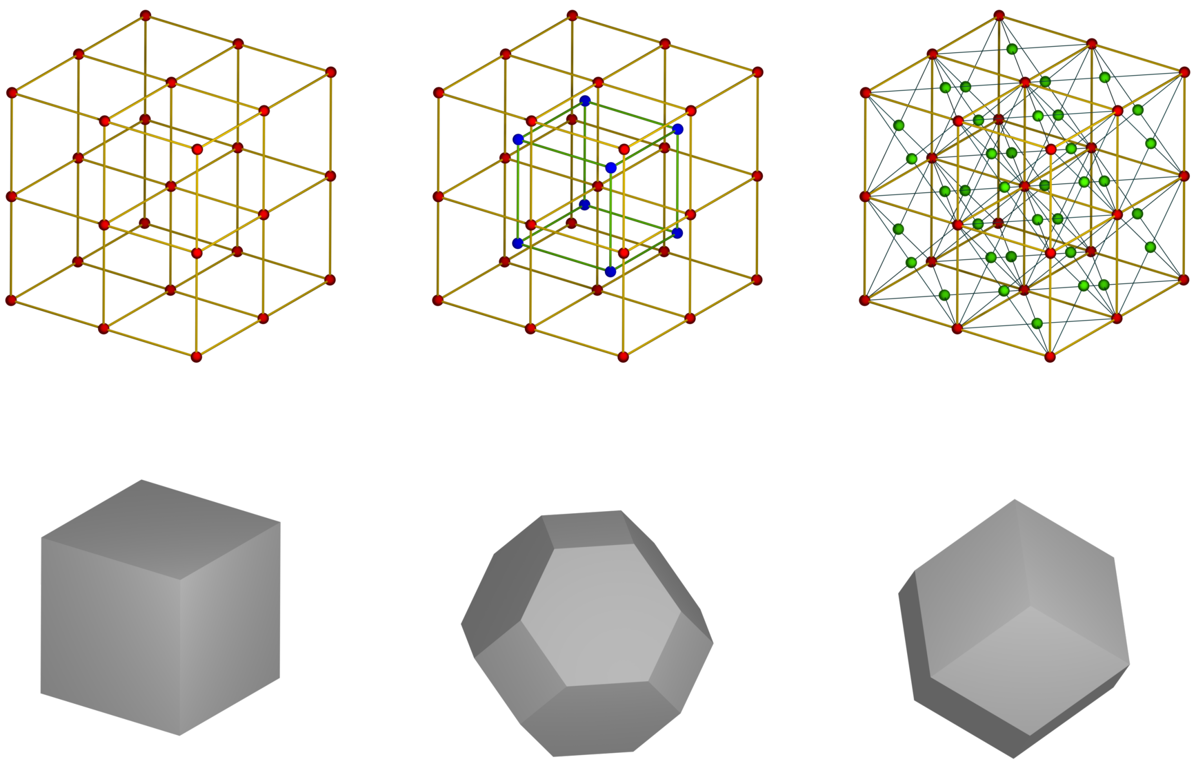

Fig. 1. Sampling lattices (top) with with their corresponding Voronoi regions (bottom). Left to right are the CC, BCC and FCC lattices respectively.

in both a sub-optimal (Cartesian) and an optimal (BCC) box-spline function space.

- We present a variational scheme that not only considers the usual centered generating kernels, but allows for the possibility of recovering each component of the gradient field in a space spanned by a shifted kernel. On the Cartesian lattice, these components are orthogonal whereas on the BCC lattice, the components are non-orthogonal and correspond to the directions associated with the underlying box-spline matrix.
- We present an extension of the error kernel of Blu and Unser to general linear operators, and show how to design filters that respect the approximation order of the approximation space.
- While our focus is on comparing the surface reconstruction algorithm on both the CC and BCC lattices, we also provide qualitative and quantitative comparisons between the presented technique (on both the Cartesian and BCC lattices) and similar methods.

[[XXX OVERALL: What is lacking from this intro is the true purpose of this work (from the surface reconstruction aspect.) We want to show that the BCC beats the CC lattice. NOT that our scheme is better than Kazhdan's. We do show this, but we need to emphasize it above!]]

II. Conclusion

The conclusion goes here.

Appendix A
Proof of the First Zonklar Equation

Appendix one text goes here.

Appendix B

Appendix two text goes here.

Acknowledgment

The authors would like to thank...

References

[1] H. Kopka and P. W. Daly, *A Guide to LaTeX*, 3rd ed. Harlow, England: Addison-Wesley, 1999.

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Michael Shell Biography text here.

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