Problem Set # 03 ICS 311, Spring 2017

Due: Friday, January 27, 2017 10pm

You may discuss the problems with your classmates, however you must write up the solutions on your own and list the names of every person with whom you discussed each problem.

1 Peer Credit Assignment (1 point extra credit for replying)

Please list the names of the other members of your peer group for this week and the number of extra credit points you think they deserve for their participation in group work.

- You have a total of 6 points to allocate across all of your peers.
- You can distribute the points equally, give them all to one person, or do something in between.
- You need not allocate all the points available to you.
- ***You cannot allocate any points to yourself!*** Points allocated to yourself will not be recorded.

2 Master Method Practice (15 pts)

Use the Master Method to give tight Θ bounds for the following recurrence relations. Show a, b, and $f(n)$. Then explain why it fits one of the cases, if any. If it fits a case, write and *simplify* the final Θ result.

(a) (5 pts) $T(n) = 5T(n/3) + n$

(b) (5 pts) $T(n) = 3T(n/9) + n$

(c) (5 pts) $T(n) = 3T(n/9) + \sqrt{n}$

3 Substitution Method (30 pts)

Use substitution *as directed below* to solve

$$T(n) = 5T(n/3) + n$$

It is strongly recommended that you read page 85-86 ”Subtleties” before trying this!

(a) (10 pts) First, use the result from the Master Method in 2(a) above as your “guess” and inductive assumption. We will do this without Θ and c: just use the algebraic portion. Take the proof up to where it fails and say where and why it fails. (See steps below.)

(b) (10 pts) Redo the proof, but subtracting dn from the guess to construct a new guess. This time it should succeed.

(c) (10 pts) Remember, d is just some constant, so you should determine what d is equal to. Redo the proof with the specific value of d that you computed.

As a reminder, to do a proof by substitution you:

1. Write the definition $T(n) = 5T(n/3) + n$
2. Replace the $T(n/3)$ with your "guess" instantiated for $n/3$ (you can do that by the inductive hypothesis because $n/3$ is smaller than n).
3. Operating only on the right hand side of the equation, transform that side into the exact form of your "guess".
4. Determine any constraints on the constants involved.
5. Show the base case holds.

4 Recursive Algorithm Analysis (30 pts)

In this problem, you will analyze the running time of the following algorithm:

```
SillyRecursion(n)
if (n < 2)
    return 1
else
    x = 0
    for i = 1 to 4
        x = x + SillyRecursion(n/2)
    for i = 1 to n
        for j = 1 to n
            x = x + 1
    return x
```

Write down the recurrence, which defines the running time of the above algorithm and solve it using any method you like.

5 Understanding Binary Search Trees (5 pts)

Suppose we are searching for some key x in a BST. As we perform the search, we compare x to the keys on the path from the root to the leaf in the BST. Which sequence below **cannot** be the sequence of keys examined during the search? Explain why not. **Do not** assume that the search below is performed on the same BST.

(a) 85, 76, 30, 15, 6, 1

(b) 13, 98, 28, 94, 33, 85, 48, 77, 51, 62, 55

(c) 99, 78, 44, 87, 57, 52

(d) 16, 33, 77, 54, 73, 60

(e) 67, 24, 74, 55, 48

6 Counting Binary Trees (35pts)

How many binary tree shapes of n nodes are there with height $n-1$? Prove your answer. *Hint: use induction. To make a good guess, try drawing all possible trees for heights 0, 1, 2,... and identify the pattern.*