# Set Theory

CPSC 2070

**Tables and set definitions can be found on the last page**

## Reading Tables

1. Give the set of nations that had over over 3000 people killed by Ebola by the end of February 2015?

**Answer:**

{Sierra Leone, Liberia}

2. What periods saw at least 500 new Ebola deaths? Write as a set

**Answer:**

{ Aug, Oct, Dec 2014; Feb 2015}

The table only gives total deaths. To find new Ebola deaths, you must subtract (e.g. Liberia Aug 14 = 694, Liberia Jun 14 = 34.

So *new* Ebola deaths in Liberia between June and August were 694 - 34 = 660 )

3. What was the overall death toll of the Ebola outbreak as of the end of 2014?

**Answer:**

As of the end of December 2014, the cumulative death toll was

$$3423 + 2758 + 1708 = 7889$$

## Union

1. What elements are in $B \cup C$?

**Answer:**

B = { a, b, monkey, dog, cat }

C = { a, b, 1, 2 }

B ∪ C = { a, b, monkey, dog, cat, 1, 2 }

Recall that ∪ takes any element that is in *either* B or C. Again, elements which are in both (i.e. *a, b*) only need to be written once, and the order of the elements inside the braces does not matter.

2. What elements are in $B \cup C \cup A$?

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

C = { a, b, 1, 2 }

B ∪ C = { a, b, monkey, dog, cat, 1, 2 }

B ∪ C ∪ A = { a, b, monkey, dog, cat, 1, 2 }

Recall that ∪ takes any element that is in *either* B or C. Again, elements which are in both (i.e. *a, b*) only need to be written once, and the order of the elements inside the braces does not matter.

Note that because A is a subset of (B ∪ C), taking the ∪ does not do anything.

## Intersection

1. What elements are in $A \cap B$?

> **Answer:**
>
> A = { 1, 2, b }
>
> B = { a, b, monkey, dog, cat }
>
> A ∩ B = { b }
>
> ∩ takes only elements that are common to both A and B

2. What elements are in $(C \cap B) \cup A$?

> **Answer:**
>
> A = { 1, 2, b }
>
> B = { a, b, monkey, dog, cat }
>
> C = { a, b, 1, 2 }
>
> $(C \cap B)$ = { a, b }
>
> $(C \cap B) \cup A$ = { a, b, 1, 2 }
>
> As with programming and math generally, use set operations use parentheses to show what order/what operations to do first. Do the intersection first $(C \cap B)$, then take the union of the result with A
>
> The final answer could also be written as 'C', since the set of correct answers is exactly the elements in the set C.

## Complement

1. What elements are in $B'$?

**Answer:**

B = { a, b, monkey, dog, cat }

$\Omega$ = {a, b, c, 1, 2, 3, dog, cat, monkey}

Those elements which are in $\Omega$, but are not in B (definition of B') are: {c, 1, 2, 3}.

Recall that for sets, order does not matter. Therefore it does not matter that the elements are in a different order in B and $\Omega$.

The braces/curly brackets are necessary to show this is a set.

Remember that $B'$, $\overline{B}$, and $B^C$ are three different ways of writing the same thing.

2. What elements are in $\overline{A} \cap B$?

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

$\overline{A}$ = { a, c, 3, dog, cat, monkey }

$\overline{A} \cap B$ = { a, dog, cat, monkey }

It is recommended that multi-step processes like this one be done in stages and that you show all work. For example, on a test, if $\overline{A}$ had been incorrect, partial credit can be given if the $\cap$ was done correctly for the answer you got. If you do not show your work, no partial credit is possible.

Again, keep in mind that complement of A is everything in $\Omega$ which is not in A. If no $\Omega$ is explicitly given, write down an assumption that makes sense and use it (see next example).

3. What elements are in $\overline{\{\text{numbers bigger than } 5\}}$?

**Answer:**

In this case, the universe of consideration ($\Omega$) is not given. A safe assumption would be "numbers smaller than or equal to 5" would be the complement.

**Above and Beyond**

In special contexts, you may be able to make an argument for something else as $\Omega$. If the context is counting people at an event, you could argue that the complement is { 0, 1, 2, 3, 4, 5 }, since counting people requires whole people, and you can't have negative people at an event.

In any case, <u>explicitly state your assumption</u>.

## Other Set Stuff Equality, Subsets, Difference, Symmetric Difference, Disjoint, Cartesian Product, Set Builder

1. Show that $(A \cap B) \cup (B - A) = B$

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

$A \cap B$ = { b }

B - A = { a, monkey, dog, cat }

$(A \cap B) \cup (B - A)$ = { b, a, monkey, dog, cat}

B - A is the difference between set B and set A. Think of it as subtraction. You take all the elements in set B, and remove any that are also in set A.

The definition of two sets being equal is that they have the same elements. These two sets have the same elements, therefore they are equal.

A way to show this formally is to show that $B \subseteq (A \cap B) \cup (B - A)$ (i.e. that all elements in B are also in $(A \cap B) \cup (B - A)$), <u>and also</u> that $(A \cap B) \cup (B - A) \subseteq B$.

2. Show that $A - B \subseteq A$

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

$A - B$ = { 1, 2 }

The definition of $A - B \subseteq A$ is that every element in $A - B$ is also in A. For this small example, it can be shown by inspection.

1 $\in$ A, and 2 $\in$ A. That is all the elements of $A - B$, therefore all elements of $A - B$ are in A.

Since the definition of $\subseteq$ is that all elements of A - B are in A, by definition $A - B \subseteq A$.

3. List all elements of C $\triangle$ B

**Answer:**

B = { a, b, monkey, dog, cat }

C = { a, b, 1, 2 }

$B - C$ = { monkey, dog, cat }

$C - B$ = { 1, 2 }

$C \triangle B = (B - C) \cup (C - B)$ = { 1, 2, monkey, dog, cat }

$\triangle$ is one symbol for the symmetric difference of two sets. The definition is given above. A symmetric difference of B and C is all elements that are in B OR C, but not both.

It is equivalent to XOR in logic.

4. Are $A \cap B$ and C-{b} disjoint?

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

C = { a, b, 1, 2 }

$A \cap B$ = { b }

C - {b} = { a, 1, 2 }

The definition of two sets being disjoint is that they have nothing in common. Therefore yes, $A \cap B$ and C - {b} are disjoint by definition.

5. List all elements of A $\times$ B

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

Recall that the definition of A $\times$ B is a tuple of the form $(a,b)$, where $a$ comes from A and $b$ comes from B.

This is called the Cartesian product of A and B. I sometimes call it a cross product, but that is a bad habit.

All elements:

(1,a),(1,b),(1,monkey),(1,dog),(1,cat)
(2,a),(2,b),(2,monkey),(2,dog),(2,cat)
(b,a),(b,b),(b,monkey),(b,dog),(b,cat)

## Power Sets

1. Give the power set of A

**Answer:**

Remember that the power set of A, which we will write $2^A$ and some people write it $\mathbf{P}$(A), is the set of all possible subsets of A.

A = { 1, 2, b }

Subsets with

0 elements: $\emptyset$

1 element : { 1 }, { 2 }, { b }

2 elements: { 1, 2 }, { 1, b },{ 2, b }

3 elements: { 1, 2, b }

Listing them in this way is not required. You may also give a regular set definition such as

$2^A$ = { $\emptyset$, { 1 }, { 2 }, { b }, { 1, 2 }, { 1, b },{ 2, b }, { 1, 2, b }}

It is also correct to refer to { 1, 2, b } as A in either definition. For example

$2^A$ = { $\emptyset$, { 1 }, { 2 }, { b }, { 1, 2 }, { 1, b },{ 2, b }, A}

2. Give the power set of C

**Answer:**

C = { 1, 2, a, b }

Subsets with

0 elements: $\emptyset$

1 element : { 1 }, { 2 }, { a }, { b }

2 elements: { 1, 2 }, { 1, a }, { 1, b }, {2, a } { 2, b }, { a, b }

3 elements: { 1, 2, a }, { 1, 2, b }, { 1, a, b }, { 2, a, b }

4 elements: { 1, 2, a, b }

Listing them in this way is not required. You may also give a regular set definition such as:

$2^C$ = { ∅, { 1 }, { 2 }, { a }, { b },{ 1, 2 }, { 1, a }, { 1, b }, {2, a } { 2, b }, { a, b },{ 1, 2, a }, { 1, 2, b }, { 2, a, b },{ 1, 2, a, b } }

It is also correct to refer to { 1, 2, a, b } as C in either definition. In both cases, the $2^C$ definition makes clear that the power set is a set made of sets. That is, the second element in $2^C$ is not the number '1', it is the set containing the number 1: { 1 }.

There could be some confusion between $2^C$ being the power set of C and being the complement of 2. There is no set '2', and sets are usually named with capital *letters*, so $2^C$ should always be the power set of C.

3. Give the power set of $(A \cap B) \cup (B \cap C)$

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

C = { a, b, 1, 2 }

$(A \cap B)$ = { b }

$(B \cap C)$ = { a, b }

$(A \cap B) \cup (A \cap C)$ = { a, b }

$2^{(A \cap B) \cup (A \cap C)} = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

Writing the power set out as follows would also be acceptable:

Power Set of $(A \cap B) \cup (A \cap C)$

0 elements: ∅

1 element : { a }, { b }

2 elements: { a, b }

Once you have the elements of the set $(A \cap B) \cup (A \cap C)$, forget how you got them and just focus on making the power set of a set with elements { a, b }.

## More Complicated Examples

1. What elements are in $(A \cap \{1,2,3\}) \cup (\{b, c, dog\} \cap C)$?

**Answer:**

A = { 1, 2, b }

C = { a, b, 1, 2 }

$(A \cap \{1,2,3\})$ = { 1, 2 }

$(\{b, c, dog\} \cap C)$ = { b }

$(A \cap \{1,2,3\}) \cup (\{b, c, dog\} \cap C) = \{1,2\} \cup \{b\}$

$\{1,2\} \cup \{b\} = \{1,2,b\}$

Simply answering 'A' would also be correct, since the set of correct answers is exactly the elements in the set A.

This is another example of a situation in which showing all work is vital. Making a single mistake in the first "$(A \cap \{1,2,3\})$" step would have made all future work incorrect. If each step is shown, we can follow that logic through and the student would have gotten most of the credit for the problem.

For Canvas Quizzes, showing all work helps you pinpoint your mistake more quickly, and so helps you fix your mistake as quickly as possible.

2. What elements are in $(\overline{A} \cap \{3, c, cat\}) \cap (\{b, monkey\} \cap C^C)$?

**Answer:**

A = { 1, 2, b }

C = { a, b, 1, 2 }

$\overline{A}$ = { 3, a, c, dog, cat, monkey }

$\overline{A} \cap \{3, c, cat\}$ = { 3, c, cat }

$C^C$ = { c, 3, dog, cat, monkey}

{ b, monkey } $\cap C^C$ = { monkey}

$\overline{A} \cap \{3, c, cat\} \cap (\{b, monkey\} \cap C^C) = \{3, c, cat\} \cap \{monkey\}$

$\{3, c, cat\} \cap \{monkey\} = \emptyset$

Notice how each half is built up from its parts. For complicated problems like this one, it is very important to stay organized.

Also notice that I am being tricky and using both $C^C$ and $\overline{A}$ in the same problem: two different ways of writing complement.

3. What elements are in $\overline{A \cap \{1,2,3\}} \cup \{b, c, dog\}$?

**Answer:**

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

C = { a, b, 1, 2 }

$A \cap \{1,2,3\}$ = { 1, 2 }

$\overline{(A \cap \{1,2,3\})}$ = { 3, a, b, c, dog, cat, monkey }

$\overline{(A \cap \{1,2,3\})} \cup \{b, c, dog\} = \{3, a, b, c, dog, cat, monkey\}$

This is another case in which { b, c, dog } is a subset of $\overline{(A \cap \{1,2,3\})}$, and so their union is the same as $\overline{(A \cap \{1,2,3\})}$.

4. What elements are in $(A \cap (B \cup C)) \cap ((B - A) \cup (C - B) \cup (C - A))$?

**Answer:**

It is **VERY** important that you pay attention to parentheses in these large problems.

A = { 1, 2, b }

B = { a, b, monkey, dog, cat }

C = { a, b, 1, 2 }

$B \cup C$ = { 1, 2, a, b, monkey, dog, cat }

$A \cap (B \cup C)$ = { 1, 2, b }

B-A = { a, monkey, dog, cat }

C-B = { 1, 2 }

C-A = { a }

$(B - A) \cup (C - B) \cup (C - A)$ = { a, monkey, dog, cat, 1, 2 }

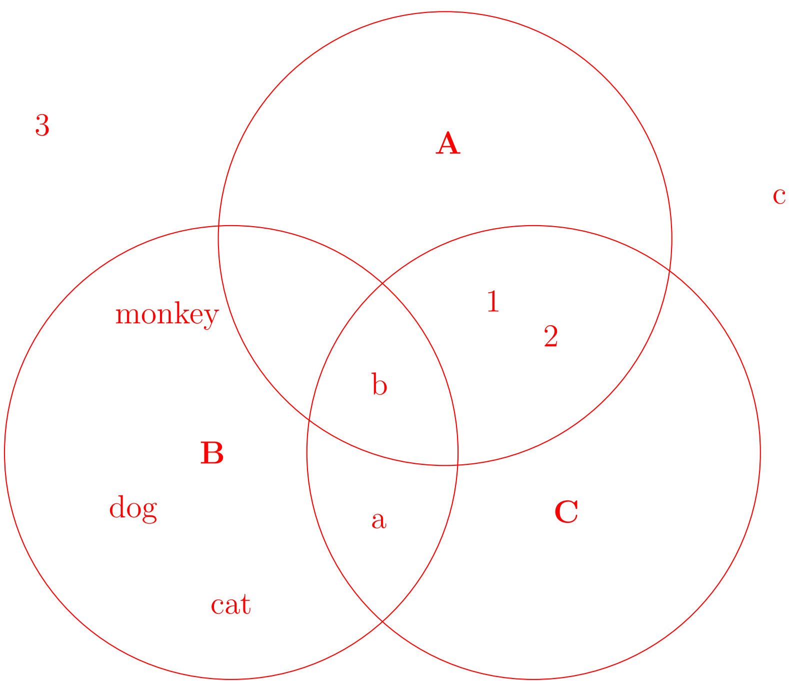
$(A \cap (B \cup C)) \cap ((B - A) \cup (C - B) \cup (C - A))$ = { 1, 2, b} $\cap$ { 1, 2, a, monkey, dog, cat }

$(A \cap (B \cup C)) \cap ((B - A) \cup (C - B) \cup (C - A))$ = { 1, 2 }

## Venn Diagram

1. Draw A, B, and C on a Venn Diagram

**Answer:**



| | **Apr 14** | **Jun 14** | **Aug 14** | **Oct 14** | **Dec 14** | **Feb 15** |
|---|---|---|---|---|---|---|
| **Liberia** | 11 | 34 | 694 | 2413 | 3423 | 4037 |
| **Sierra Leone** | 0 | 34 | 422 | 1510 | 2758 | 3461 |
| **Guinea** | 146 | 270 | 430 | 1018 | 1708 | 2091 |

Table 1: Total Ebola Deaths By Country

| | | |
|---|---|---|
| $\Omega$ | = | **{a, b, c, 1, 2, 3, dog, cat, monkey}** |
| **A** | = | **{ 1, 2, b }** |
| **B** | = | **{a, b, monkey, dog, cat }** |
| **C** | = | **{ a, b, 1, 2 }** |

Table 2: Definitions of Sets A, B, C