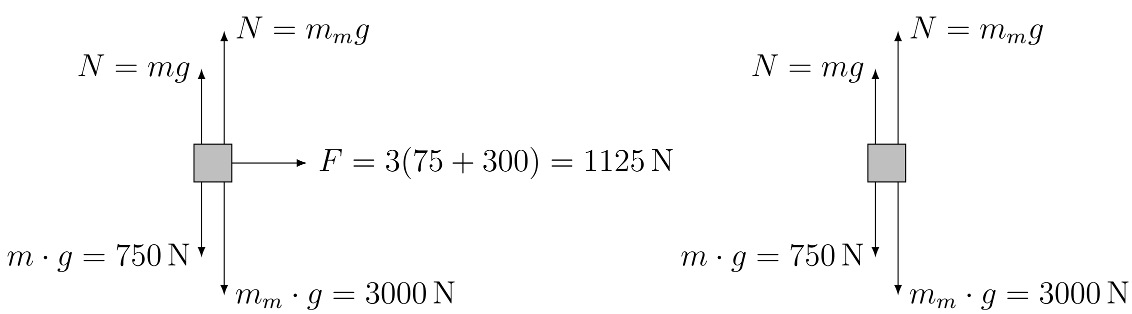
1 MPHY0003

Assumptions:

- $g = 10\,\mathrm{ms}^{-2}$
- speed limit $= 14\,\mathrm{ms}^{-1}$ ($\approx 50\,\mathrm{km/h}$)
- Motorcycle mass, $m_m = 300\,\mathrm{kg}$
- Tree trunk mass, $m_t = 100\,\mathrm{kg}$

i



(a) Accelerating (b) Constant speed

ii

$$\begin{aligned} V = V_0 + at = 14\,\mathrm{ms}^{-1} + 3\,\mathrm{ms}^{-2} \cdot 4.5\,\mathrm{s} \\ V = 27.5\,\mathrm{m/s} \end{aligned} \tag{1}$$

iii

a)

$$
\begin{aligned}
d &= \frac{V^2 - {V_0}^2}{2a} = \frac{27.5^2 - 14^2}{2 \cdot 3} = 93.375\,\mathrm{m} \\
W &= (m + m_m) \cdot a \cdot d = 375\,\mathrm{kg} \cdot 3\,\mathrm{ms}^{-2} \cdot 93.375\,\mathrm{m} \\
W &= 105\,046.9\,\mathrm{J}
\end{aligned} \tag{2}
$$

b)

There are no unbalanced forces in the direction of the motion, therefore $W = 0$.

iv

Given that it is an inelastic collision, both John and the tree trunk will have the same speed V_f after collision:

$$
\begin{aligned}
(m + m_m)V = (m + m_m + m_t)V_f \implies V_f &= \frac{(m + m_m)V}{(m + m_m + m_t)} \\
V_f = \frac{375 \cdot 27.5}{475} &= 21.7\,\mathrm{ms}^{-1}
\end{aligned} \tag{3}
$$

Given an impulse J,

$$
\begin{aligned}
J = Ft = \Delta p \implies F &= \frac{\Delta p}{t} \\
F = \frac{375 \cdot (27.5 - 21.7)}{0.1} &= 21\,750\,\mathrm{N}
\end{aligned} \tag{4}
$$

2 MPHY0005

i

- Heat gain: Metabolism (see item iii (1)) and radiation

$$
\begin{aligned}
&Q_{radiation} = k_r A_{eff} \epsilon \left(T_{amb.} - T_{skin}\right) \\
&Q_{gain} = 12\,\mathrm{min}\left(6101\,\mathrm{J/min} + 60\,\mathrm{min/s} \cdot 8 \cdot 0.97 \cdot [0.8 \cdot 1.8] \cdot [37 - 35]\right) \\
&Q_{gain} = 105\,394\,\mathrm{J}
\end{aligned}
\tag{5}
$$

- Heat loss: diffusion of water through skin ($\approx 400\,\mathrm{mL/day}$) and convection of sweat

$$
\begin{aligned}
&Q_{lost} = Q_{diffusion} + Q_{sweat} \\
&Q_{lost} = 12\,\mathrm{min} \cdot 2260\,\mathrm{kJ\,kg^{-1}} \cdot \left(0.4\,\mathrm{kg/day} + 0.2\,\mathrm{kg\,h^{-1}}\right) \\
&Q_{lost} = 7533\,\mathrm{J} + 90\,400\,\mathrm{J} = 97\,933\,\mathrm{J}
\end{aligned}
\tag{6}
$$

ii

$$
\begin{aligned}
\Delta T &= \frac{Q}{mc} \\
\Delta T &= \frac{(105394 - 97933)\mathrm{J}}{75\,\mathrm{kg} \cdot 3500\,\mathrm{J\,kg^{-1}\,K^{-1}}} \\
\Delta T &= 0.028423 = 0.028\,^{\circ}\mathrm{C}
\end{aligned}
\tag{7}
$$

Provided that no injury disabled John's ability to sweat, there is no reason to expect a significant change in temperature over such a short period of time, and while wearing clothes that provide little insulation. Indeed, had the number been considerably larger, the value for assumption (3) would have been revised.

iii

(1) Despite an expected metabolic rate increase due to the accident (and the consequent release of hormones), it is really hard to quantify it. However, if we assume John is 25 and

sedentary and couple this metabolic increase with other stresses of a normal day, we can expect an average metabolic rate of around 2100 kcal/day = 6101 J/min.

(2) Assume conduction with floor is negligible because of small area for conduction and temperature difference between skin and floor.

(3) The temperature is high enough that we can assume that despite being cooled by the breeze while riding the motorcycle, John would start to sweat as he lied on the floor. Presumably the portion of his body against the floor would lead to sweat building up (instead of vapour), therefore it is estimated that $0.2\,\mathrm{kg\,h^{-1}}$ is turning to vapour.

3 MPHY0002/MPHY0004

Part A

i

Convert range of $1\,\mu\mathrm{V}$ to $100\,\mu\mathrm{V} \implies 0\,\mathrm{V}$ to $5\,\mathrm{V}$

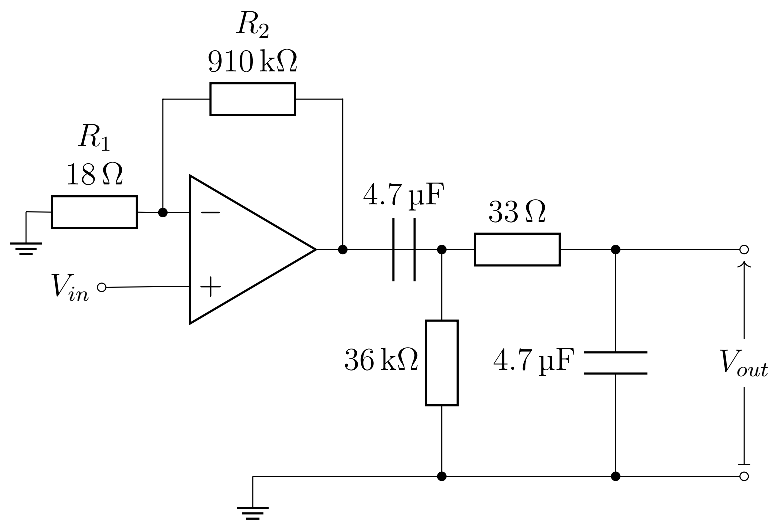
$$\text{Amplification} = \frac{5-0}{10^{-4}-10^{-6}} = 50505 \tag{8}$$

Leading to a range of $50505 \cdot (1\,\mu\mathrm{V}$ to $100\,\mu\mathrm{V}) = 0.050\,505\,\mathrm{V}$ to $5.050\,505\,\mathrm{V}$. Thus requiring a DC offset of $0.050\,505\,\mathrm{V}$.

We can use a band-pass filter or a combination of high-pass and low-pass in series in order to cut off any frequencies outside of the 1 Hz to 1000 Hz range

ii

The amplification is calculated by $1 + \frac{R_2}{R_1} = 50556$, which is appropriately close to the amplification we require. Specifically opted to not use a differential amplifier with an offset, because with the components we use, the required 50 mV offset is very unlikely to be lost within the tolerances of our circuit.



In order to calculate both cut off frequencies for our filters, we use the following expression

$$f_c = \frac{1}{2\pi RC} \tag{9}$$

For the selected resistors and capacitors for both filters, we have the cut-off frequencies of 0.9 Hz and 1026 Hz, respectively.

There are no particular requirements for the op amp itself, though we used LM358 extensively.

Part B

Consider that for $\alpha, \beta, \gamma, \delta$ being above the threshold = 1, and below = 0. Finally, for at least two values = 0 our function $f = 1$ indicates brain damage.

α	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
β	0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
γ	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
δ	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
f	1	1	1	1	1	1	1	0	1	1	1	0	1	0	0	0

Table 1: Truth table where f is our required function, yielding 1 when at least two indicators are below the threshold.

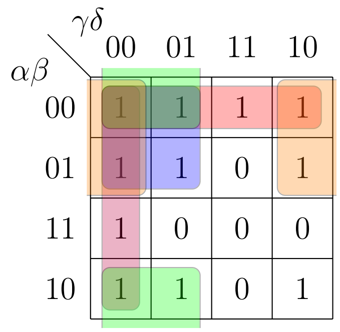


Figure 2: Karnaugh map. Each colour is a different grouping which leads us to the function f below.

As an example, by looking at the green group, we observe that a change in δ and a change in α do not influence the results, which leads us to the $\overline{\gamma\beta}$ term:

$$f = \overline{\alpha\beta} + \overline{\gamma\delta} + \overline{\gamma\alpha} + \overline{\gamma\beta} + \overline{\delta\alpha} + \alpha\bar{\beta}\gamma\bar{\delta} \tag{10}$$

From which we can draw the digital circuit

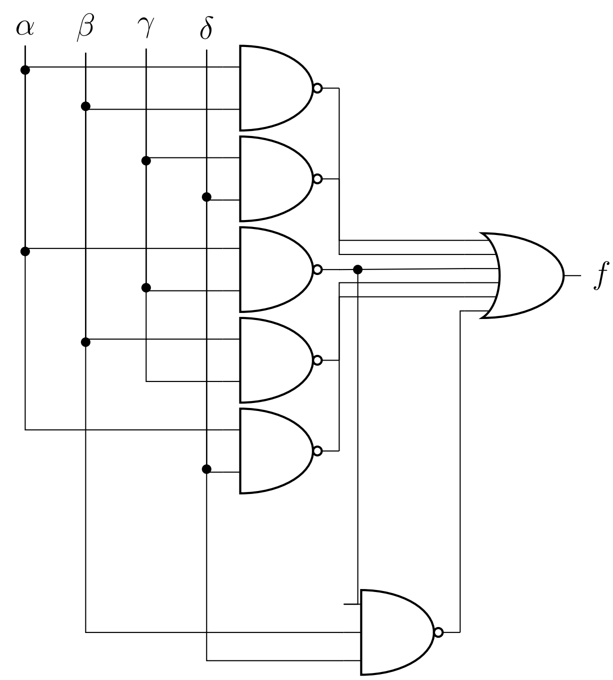


Figure 3: Digital circuit following the description of f.

4 MPHY0001

Part A

i

Picking two adjacent points, one which goes through the haemorrhage and has the number of photons N_h, and a second which does not, and has photons N_n. Furthermore, it is assumed that $1\,\text{cm}^3$ of blood is displacing the brain matter inside the skull, as seen in the cross-sectional drawing below. Note the exact position is irrelevant as can be inferred by the linear attenuation equation seen below.

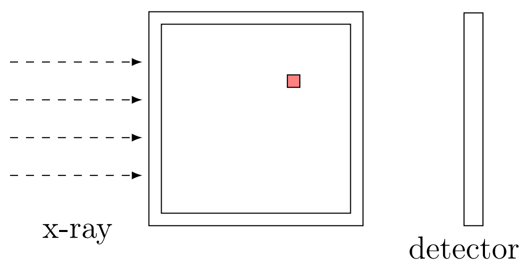


Figure 4: Cross-section of x-ray

$$N = N_0 e^{-\mu x}$$

$$N_h = N_0 \exp(-0.6 - 0.21 \cdot 10 - 0.20 - 0.21 \cdot 4 - 0.6) = 0.013037 \cdot N_0$$

$$N_n = N_0 \exp(-0.6 - 0.21 \cdot 15 - 0.6) = 0.012907 \cdot N_0$$

Given that the xray has an intensity I and energy E, then subject contrast (C_s) is:

$$\begin{aligned} C_s &= \ln\left(\frac{I_h}{I_n}\right) = \ln\left(\frac{E_h}{E_n}\right) = \ln(0.013037) - \ln(0.012907) \\ C_s &= 0.01 \end{aligned} \tag{11}$$

Radiographic contrast(C_r) is the product of gamma(0.5) and $C_s \implies 0.01 \cdot 0.5 = 0.005$.

ii

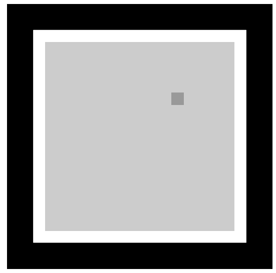


Figure 5: Sketch of cranial CT scan with a haemorrhage.

Part B

i

The ratio of T_2 is larger than the ratio of T_1, suggesting that a T_2 weighted image would provide larger contrast between brain and blood. In order to produce this type of image, we would want $TR >> T_1$ and $TE \approx T_2$.

ii

Contrast of zero means that the intensities would be the same, therefore:

$$92 \cdot [1 - \exp(-500/1500)] \cdot \exp[(-\mathrm{TE}/60)] = 100 \cdot [1 - \exp(-500/2000)] \cdot \exp[(-\mathrm{TE}/240)]$$

Which we solve graphically, as seen in Figure 6 and check the results using Wolframalpha: $\mathrm{TE} = 13.173\,\mathrm{ms}$.

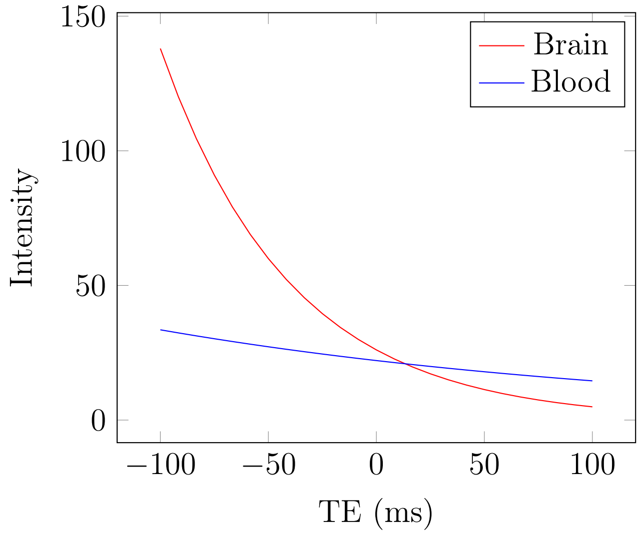


Figure 6: Relationship between Intensity and TE

iii

Using the definition of contrast ratio (C) as the ratio of intensities (high/low):

$$I_{brain} = 92 \cdot [1 - \exp(-3000/1500)] \cdot \exp(-180/60) = 3.9605$$

$$I_{blood} = 100 \cdot [1 - \exp(-3000/2000)] \cdot \exp(-180/240) = 36.697$$

$$\implies C = \frac{I_{blood}}{I_{brain}} = 9.2656$$

Alternatively we can find the Weber Contrast (C_w):

$$C_w = \frac{\text{high - low}}{\text{high}} = \frac{36.697 - 3.9605}{36.697} = 0.89207$$

Part C

i

I_0 is the intensity emitted by the probe, I_{ib} propagates through the brain:

$$\begin{aligned} \alpha &= 10\log_{10}(I_0/I) \\ 13.5 &= 10\log(0.6\cdot I_0/I_{ib}) \\ I_{ib} &= \frac{0.6I_0}{10^{1.35}} = 0.026801\cdot I_0 \end{aligned} \tag{12}$$

I_{ih} is incident on the haemorrhage, I_{rh} is reflected by the haemorrhage:

$$\begin{aligned} & 0.4 = \log_{10}(I_{ib}/I_{ih}) \\ \implies & I_{ih} = \frac{I_{ib}}{10^{0.4}} = 0.010670\cdot I_0 \\ & \frac{I_{rh}}{I_{ih}} = \frac{(Z_2-Z_1)^2}{(Z_2+Z_1)^2} = \frac{(1.6-1.66)^2}{(1.6+1.66)^2} \\ \implies & I_{rh} = 3.3874\times10^{-4}\cdot I_{ih} = 3.6143\times10^{-6}\cdot I_0 \end{aligned} \tag{13}$$

This is once again attenuated by brain matter, the cranium and then loses 60% intensity (I_r):

$$\begin{aligned} I_r &= 0.6\frac{3.6143\times10^{-6}}{10^{0.4}\cdot10^{1.35}}I_0 \\ \frac{I_r}{I_0} &= 3.8563\times10^{-8} = -7.4138\,\mathrm{B} \\ \frac{I_r}{I_0} &= -74.138\,\mathrm{dB} \end{aligned}$$

ii

The results suggest that the intensity of the echo would be too low to form a viable image, in large part due to the attenuation of the signal through the skull (about 22 times). However, the cranium of a newborn infant brain is not yet fully fused yet, and the probe is positioned against these gaps in the cranium – reducing this severe attenuation we experience with John's ultrasound.

5 MPHY0005

(a)

5/5 vision is defined as resolving each optotype with thickness subtended by 1 minute ($1/60° = 0.0166°$) and size subtended by 5 minutes ($5/60° = 0.0833°$):

$$
\begin{aligned}
&h = 2d \cdot \tan\left(\frac{\theta}{2}\right) \\
&\text{Size} = 2 \cdot 5 \cdot \tan(0.0417) = 7.3\,\text{mm} \\
&\text{Thickness} = 2 \cdot 5 \cdot \tan(0.0083) = 1.5\,\text{mm}
\end{aligned} \tag{14}
$$

Acuity is distance of test / distance at which the optotype was resolved. The sizes and thickness are calculated similarly to the above.

Row	Acuity	Size (mm)	Thickness (mm)	Letters
1	5/50	72.7	14.5	E
2	5/25	36.4	7.3	FP
3	5/17.5	25.5	5.1	TOZ
4	5/12.5	18.2	3.6	LPED
5	5/10	14.5	2.9	PECFD
6	5/7.5	10.9	2.2	EDFCZP
7	5/6	8.7	1.7	FELOPZD
8	5/5	7.3	1.5	DEFPOTEC
9	5/4	5.8	1.2	LEFODPCT

Table 2: Snellen chart based on a 5 meter distance utilising standard letters for each row.

(b)

Considering that the farthest John can see is 0.4 m, he is myopic. This is corrected with diverging lenses, which we would expect to be of negative meniscus (concave + convex surface) or biconcave shape.

We calculate the focal length, f, by using the thin lens equation and assuming a distance of 2 cm between the eyes and the glasses:

$$\begin{aligned}
\frac{1}{f} &= \frac{1}{d_o} + \frac{1}{d_i - 0.02} \\
\frac{1}{f} &= \cancelto{0}{\frac{1}{d_o}} - \frac{1}{0.4 - 0.02} \\
f &= -0.38\,\mathrm{m}
\end{aligned} \tag{15}$$

From which we may also deduce the lens power: $\frac{1}{f} = -2.6316\,\mathrm{m}^{-1}$

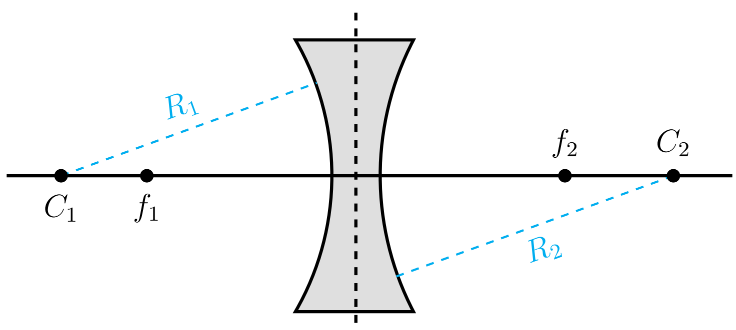


Figure 7: Representation of lens as biconcave for simplicity's sake

Using the lens maker's equation and the following assumptions:

- The refractive index of glass/air is 1.5;
- Thin lenses, $d \approx 0$;
- Lens is biconcave, therefore the radii of curvatuve, $R_1 = -R_2$

$$
\begin{aligned}
\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2}\right] \\
-2.6316 = (1.5-1)(\frac{2}{R}) \\
R = -38.00\,\text{cm}
\end{aligned} \tag{16}
$$

Finally, reflection only accounts for about 4% loss in light transmission, so there is little concern about the glass lenses themselves:

$$
\text{Reflectivity} = \left(\frac{n_2 - n_1}{n_2 + n_1}\right)^2 = \left(\frac{1.5-1}{1.5+1}\right)^2 = 0.04 = 4\% \tag{17}
$$

6 Reflection

Hi, John.

Applying to study biomedical engineering is a fantastic idea! There is an interesting balance of practical work, introduction to real life applications and purely theoretical. Our first engineering challenge, for example, was heavily focused on introducing imaging techniques and making us reflect not only on the techniques themselves, but how practical their use was in a poorer nation. Ultimately getting as far as whether the best solution was even technical in nature. That reflection is probably what I will find most useful for the rest of my life, but the technical and mathematical sides were supported in other classes – including later on having a full module on introduction to medical imaging.

Our second challenge was considerably more driven by our knowledge in building electronic circuits as taught in two of our modules. Largely the purpose of it seemed to have been to interact with other engineering fields, trying to superficially understand what they are doing in order to create one cohesive project. Ultimately working with people with varying degrees of interest was (and is) an important skill to have, and it certainly seems like that was the learning objective.

We have also had a large focus on the human body in particular – even in our Mechanics and Material class, where we discuss standard physics, there is an ongoing conversation about biological materials, for example. An important skill that is used in this class, as well as Physics of the Human Body and Cardiac Engineering, is to break a biological system into a simplified mathematical/physical model. Although these may not be areas I plan to working in, the principle being taught is universal.

By and large the purpose of our first year was to introduce a few concepts, some of which are of very little interest to my pursuit in prosthetics, but it overwhelmingly also gave me really important tools which are completely subject independent.