LONDON'S GLOBAL UNIVERSITY

Mathematical Modelling & Analysis I

Coursework No 1

Topic Coverage:	
	Topic 1: Building mathematical models **Topic 2: Employ assumptions to simplify systems** **Topic 3: Matrices & Vectors** **Topic 4: Complex Numbers**
Date When Coursework Set	**7 October 2019**
Coursework Submission Deadline	**11 November 2019**
Date of Coursework Return	**12 December 2019**
Mode of Submission	**Submit via Moodle on the "Assessment" tab.**
Expected Time on Task	**6 hours**

About the coursework:

This coursework is made up of four questions, each worth 25 of marks adding up to 100 marks.

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Question 1
Topic 1 – Building Mathematical Models

The attached file shows the student coursework and end-of-year final exam marks for a first year mathematical modelling and analysis class.

a) Using computational software of your choice (e.g. MATLAB or EXCEL), plot a scatterplot of the student coursework marks against the end-of-year final exam marks (putting the exam marks on the *x*-axis). Does the data indicate any relationship between the two sets of data? Explain why. [10%]

b) Fit a straight line through the scatterplot by eye and then work out the slope of the line. Hence compute a relationship between student coursework marks against the end-of-year final exam marks. [20%]

c) Using computational software of your choice (e.g. MATLAB or EXCEL), compute the slope and intersect of an alternative line to the one that you fitted by eye in part (b) above, as well as the uncertainties in the parameters. [10%]

d) Comment on the similarities and differences between the straight line that you fitted by eye, and the one that you obtained using computer software. [10%]

e) Use the equation of the alternative straight line that you computationally determined in part (c) to predict the coursework marks of a student who achieves an end-of-year exam mark of 60%. [15%]

f) Comment on the reliability of the student coursework marks as a predictor of student performance in the end-of-year exams. [10%]

Guidance:
Make sure that you label the x and y-axes of the plots that you created in parts a) and c). Print out these plots, together with their accompanying MATLAB code or Excel worksheets, and submit together with the rest of your coursework.

Question 2
Topic 2 – Employ assumptions to simplify systems

a) An oscillator (a mass on a spring) can be represented by the following equation

$$m\frac{d^2x}{dt^2} = -k \cdot x$$

where m is a mass in kilogram, x is a displacement in meter and k is the spring constant.

(i) What are the dimensions of, x, m and $\boldsymbol{\frac{d^2x}{dt^2}}$ [6%]

(ii) What is the dimension of k? [4%]

b) One solution of the system can be written as

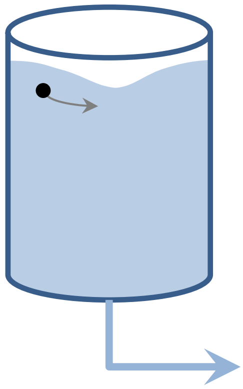
$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}t\right)$$

where A is a constant of 0.009 m and k is 9800 $N.m^{-1}$ and *m* is a mass of 1kg.

Give an order of magnitude approximation of the value of $\boldsymbol{x}$ at time 0.001 s. [15%]

Question 3
Topic 3 – Matrices and Vectors

The position of a particle moving in a liquid flow-field can be described by a time-dependent 3D vector. Let us focus on the liquid flow-field of a cylindrical vessel that is drained (see picture on the right).



A particle's swirling motion due to the vortex generated can be described by the vector $\vec{\boldsymbol{a}}$:

$$\vec{\boldsymbol{a}} = [20\sin(t), 20\cos(t), 0]$$

Moreover, the downwards motion if the particle (due to the drop of the liquid level) can be described by vector $\vec{\boldsymbol{b}}$ as:

$$\vec{\boldsymbol{b}} = [0, 0, 10-(t/4)^2]$$

The trajectory of the particle is then calculated as $\vec{\boldsymbol{p}}=\vec{\boldsymbol{a}}+\vec{\boldsymbol{b}}$. Use position in units of centimetres, and time in units of seconds.

a) Use MATLAB to calculate the values of $\vec{\boldsymbol{p}}$ at 0.4 second intervals from $t = 0$ s up to $t = 40$ s. [20%]

b) Use MATLAB to plot $\vec{\boldsymbol{p}}$ in 3D for $t = 0$ s up to $t = 40$ s. [30%]

c) If the velocity of the particle is given by

$$\vec{\boldsymbol{v}} = \frac{d\vec{\boldsymbol{p}}}{dt} = [20\cos(t), -20\sin(t), -\frac{2t}{16}],$$

calculate the particle's speed at $t = 15$ seconds. [20%]

d) Use MATLAB to plot the particle's speed for $t = 0$ s up to $t = 40$ s. [30%]

Submit the MATLAB M-file code for parts a), b) and d) with your coursework.

Question 4

Topic 4 – Complex Numbers

Ischemic stroke, where an artery blockage deprives the brain from its blood and oxygen supply, is one of the most common types of strokes, afflicting a high number of patients. Regenerative medicine can help to treat such patients by repairing ischemic brain tissue via transplanting stem cells, from allogeneic or autologous sources, directly into the brain. This promotes the rehabilitation of the tissue by promoting neurogenesis, angiogenesis and chemotaxis.

After the administration of the treatment, the rehabilitation of the damaged tissue can be monitored by bioimpedance measurements through impedance spectroscopy techniques. Such measurements involve subjecting the tissue to an alternating current (e.g. at a frequency of 50 kHz), and monitoring the phase angle φ between the observed voltage waveform and the applied current waveform. These observations provide information regarding tissue hydration, cell mass, and cell membrane integrity.

The impedance $\boldsymbol{Z}$ of an electrical circuit is a measure of the opposition that the circuit presents to a current $\boldsymbol{i}(\boldsymbol{t})$ when a voltage $\boldsymbol{v}(\boldsymbol{t})$ is applied. Mathematically $\boldsymbol{Z} = \frac{\boldsymbol{v}(\boldsymbol{t})}{\boldsymbol{i}(\boldsymbol{t})}$.

a) Write down a mathematical expression for De Moivre's theorem, and use the theorem to show that:

$$j^{\alpha} = \cos\left(\frac{\alpha\pi}{2}\right) + jsin\left(\frac{\alpha\pi}{2}\right) \quad \text{Equation (1)}$$

where $j = \sqrt{-1}$. [10%]

b) The impedance of tissue $\boldsymbol{Z}$ at a frequency $\boldsymbol{\omega}$ is described by the Cole equation below:

$$Z = R_{\infty} + \frac{R_0 - R_{\infty}}{1 + (j\omega\tau)^{\alpha}} \quad \text{Equation (2)}$$

where $\boldsymbol{\tau}$ is a time constant and $\boldsymbol{\alpha}$ is a parameter which takes a value between zero and one. Use the equation for the impedance of tissue $\boldsymbol{Z}$ and the equation in part (a) above to show that:

$$Z = R_{\infty} + \frac{(R_0 - R_{\infty})(1 + c\theta^{\alpha} - js\theta^{\alpha})}{1 + 2c\theta^{\alpha} + \theta^{2\alpha}} \quad \text{Equation (3)}$$

where $\theta = \omega\tau$, $c = \cos\left(\alpha\, {}^{\pi}/_{2}\right)$, and $s = \sin\left(\alpha\, {}^{\pi}/_{2}\right)$. [20%]

c) Show that the modulus of the tissue impedance Z is given by:

$$|Z| = \sqrt{\frac{R_{\infty}^2\theta^{2\alpha} + 2cR_0R_{\infty}\theta^{\alpha} + R_0^2}{1 + 2c\theta^{\alpha} + \theta^{2\alpha}}}$$

[20%]

<u>Hint</u>: It may be helpful to first re-express as

$$Z = R_\infty + \Delta \frac{(1 + c\theta^\alpha - js\theta^\alpha)}{q}$$

Where $\Delta = (R_0 - R_\infty)$ and $q = (1 + 2c\theta^\alpha + \theta^{2\alpha})$

d) Using MATLAB or Excel, plot the modulus |Z| as a function of $\log_{10} \omega$ for four different values of $\boldsymbol{\alpha}$ = 0.25 0.5, 0.75, 1, using the parameters $\tau = 1$, $R_\infty = 1$, and $R_0 = 10$. Display the curves on the same graph, using different colours for each curve, over a frequency range from $\omega = 0.001$ to $\omega = 10^3$. *Make sure to clearly show the trends across the whole range of* $\boldsymbol{\omega}$. [40%]

e) Comment on the nature of the graphs that you obtained in part (d). [10%]