Question 1

i

$$\begin{aligned}\xi &= A\sin(2\pi f t - kx)\\ v &= \frac{d\xi}{dt} = 2A\pi f\cos(2\pi f t - kx)\end{aligned}$$

v is maximum when $\cos(2\pi f t - kx) = 1$, and therefore

$$v_{max} = 2A\pi f = 2\pi \cdot 4 \cdot 10^6 \cdot 50 \cdot 10^{-9} = 1.2566\,\mathrm{m\,s^{-1}} \approx 1.3\,\mathrm{m\,s^{-1}} \tag{1}$$

ii

$$\frac{E_K}{\text{volume}} = \frac{1}{2}\rho(v_{max})^2 = \frac{(1.2566\,\mathrm{m\,s^{-1}})^2 \cdot 1000\,\mathrm{kg\,m^{-3}}}{2} = 790\,\mathrm{J\,m^{-3}} \approx 800\,\mathrm{J\,m^{-3}} \tag{2}$$

Question 2

$$\begin{aligned}\frac{I_r}{I_i} &= \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2}\\ \frac{4}{100} &= \frac{(Z_2 - 3\cdot 10^6)^2}{(Z_2 + 3\cdot 10^6)^2}\\ \therefore Z_2 &= 2\times 10^6\,\text{rayls or } Z_2 = 4.5\times 10^6\,\text{rayls}\end{aligned} \tag{3}$$

Question 3

$$\begin{aligned}
\text{Number of Lines per image per second} = (N-K+1)f = (300-7+1)25 = 7350 \\
1 \text{ line per } \frac{1}{7350} = 1.36\times10^{-4}\,\text{s} \\
t = \frac{0.01}{1540} = 1\,\text{cm per } 6.494\times10^{-6}\,\text{s} \\
d_{max} = \frac{1.36\times10^{-4}}{2\cdot6.49\times10^{-6}} = 10.5\,\text{cm}
\end{aligned} \tag{4}$$

Question 4

I_0 is the intensity of the initial pulse that propagates through the first medium. I_i, the incident pulse between the media; and I_r, the reflected pulse.

$$\begin{aligned}
&\frac{I_r}{I_i} = \frac{(Z_2-Z_1)^2}{(Z_2+Z_1)^2} = \frac{(1.8-1.2)^2}{(1.8+1.2)^2} \\
&\therefore I_r = 0.04\cdot I_i
\end{aligned} \tag{5}$$

$$\begin{aligned}
30\,\text{dB} = 10\log(I_0/I_r) \implies I_0 = 1000\cdot I_r \\
\therefore I_0 = 40 I_i
\end{aligned} \tag{6}$$

We find the length of the tissue based on the time it takes for the ray to reach the boundary and be reflected:

$$x = ct = 1540\cdot75\times10^{-6}/2 \implies x = 0.057\,750\,\text{m} = 5.8\,\text{cm} \tag{7}$$

$$\begin{aligned}
I_i = I_0\exp(-\mu x) \implies \frac{I_i}{I_0} = \frac{I_i}{40I_i} = \frac{1}{40} = \exp(-5.8\mu) \\
\ln 40 = 5.8\mu \implies \mu = 0.64\,\text{cm}^{-1}
\end{aligned} \tag{8}$$

Question 5

i

$$
\begin{aligned}
v &= \frac{d}{t}\frac{A}{A} = \frac{V}{t}\frac{1}{A} \\
&= \frac{5\times10^{-6}\,\mathrm{m^3\,s^{-1}}}{\left(2\times10^{-3}\,\mathrm{m}\right)^2\pi} = 0.40\,\mathrm{m\,s^{-1}}
\end{aligned} \tag{9}
$$

ii

$$
f_d = \pm\frac{2vf\cos\theta}{c} \tag{10}
$$

$$
f_{da} = \frac{2\cdot0.4\cdot5\times10^6\cdot3/5}{1540} = +1558\,\mathrm{Hz}
$$

$$
f_{db} = 2\cdot0.40\cdot5\times10^6\cos(\pi/2) = 0
$$

Analogously to f_{da}, $f_{dc} = -1558\,\mathrm{Hz}$

iii From the question we infer that at $t = 0$, $\phi/2 = \tan^{-1}(3/4)$

$$
\begin{aligned}
\frac{d\phi}{dt} = 26 &\implies \phi = 26t + C \\
\therefore \phi = 0.026t + 1.2870 &\quad \text{for } t \text{ in ms}
\end{aligned} \tag{11}
$$

Given that $\theta = \pi/2 - \phi/2$, the equation for f_d is plotted in Figure 1.

iv

$$
Q = v_c^{'}A \implies v_c^{'} = \frac{5\times10^{-6}\,\mathrm{m^3\,s^{-1}}}{\left(1\times10^{-3}\,\mathrm{m}\right)^2\pi} = 1.5915\,\mathrm{m\,s^{-1}} \tag{12}
$$

$$
f_{dc}^{'} = \pm\frac{2v_c^{'}f\cos\theta}{c} = -\frac{1.5915\cdot5\times10^6\cdot3/5}{1540} = -6200\,\mathrm{Hz} \tag{13}
$$

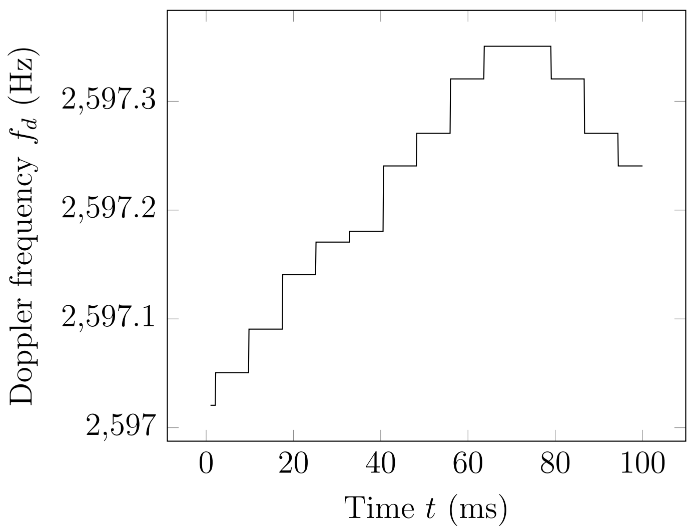

Figure 1: Doppler frequency over a period of 100 ms, starting from A.