1 Derivatives

Given an expression $f(x) = x^2 + 5$, the derivative, $f'(x)$, from first principle is given by:

$$\begin{aligned}
f'(x) &= \lim_{\delta x \to 0} \frac{f(x + \delta x) - f(x)}{\delta x} \qquad (1)\\
&= \lim_{\delta x \to 0} \frac{(x + \delta x)^2 + 5 - (x^2 + 5)}{\delta x}\\
&= \lim_{\delta x \to 0} \frac{x^2 + 2x\delta x + \delta x^2 + 5 - (x^2 + 5)}{\delta x}\\
&= \lim_{\delta x \to 0} \frac{\cancel{\delta x}(2x + \delta x)}{\cancel{\delta x}}\\
f'(x) &= 2x \qquad (2)
\end{aligned}$$

2 Integrals

Let $f : [a, b] \to R$ be a function defined in the closed interval $[a, b]$ and with partitions

$$P = \{[x_0, x_1], [x_1, x_2], \ldots [x_{n-1}, x_n]\}$$

such that

$$a = x_0 < x_1 < x_2 \ldots x_n = b$$

A Riemann sum S is defined as:

$$S = \sum_{i=1}^{n} f(x_i^*)\Delta x_i \qquad (3)$$

Now if f is integrable within the interval and Δx_i approaches zero, we have an integral:

$$\int_a^b f(x)\,dx = \lim_{\Delta x_i \to 0} S = \lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\Delta x_i \qquad (4)$$

And finally, if $F(x)$ is the integral of $f(x)$, then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = \Big[F(x)\Big]_a^b$$