# Volatility and Quantitative Research

## U.S. Equity Derivatives Research

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## Index Dispersion Monitor

We highlight the most attractive S&P 500 and NASDAQ-100 dispersion trades based on comparing the current Implied-Realized correlation spread relative to its average levels over the past year for 1-mth, 3-mth, 6-mth and 1-year maturities. We analyze dispersion signals for the index vs. all underlying stocks and for the index vs. top 20 constituents by market cap (see methodology on last page).

| Maturity | Move in Realized Correlation over the Week | Move in Implied Correlation over the Week | Dispersion Signal |
|---|---|---|---|
| 1-Month | 30% -> 23%<br>SPX: Down | 21% -> 20%<br>SPX: Down | SPX: Realized and implied correlation both dropped over the week. The implied less realized correlation spread remains negative, indicating that index implied volatility is a better buy relative to single stock implied volatility. |
| | 29% -> 27%<br>NDX: Down | 20% -> 24%<br>NDX: Up | NDX: Realized correlation dropped and implied correlation rose over the week. The implied less realized correlation remains negative, indicating that index implied volatility is a better buy relative to single stock implied volatility. |
| 3-Month | 24% -> 24%<br>SPX: Flat | 30% -> 28%<br>SPX: Down | SPX: Realized correlation was flat and implied correlation dropped over the week. The implied less realized correlation spread remains positive, indicating that index implied volatility is a better sell relative to single stock implied volatility. |
| | 27% -> 25%<br>NDX: Down | 30% -> 31%<br>NDX: Up | NDX: Realized correlation declined and implied correlation rose over the week. The implied less realized correlation spread remains positive, indicating that index implied volatility is a better sell relative to single stock implied volatility. |
| 6-Month | 26% -> 25%<br>SPX: Down | 34% -> 31%<br>SPX: Down | SPX: Realized and implied correlation both dropped over the week. The implied less realized correlation spread remains positive, indicating that index implied volatility is a better sell relative to single stock implied volatility. |
| | 32% -> 30%<br>NDX: Down | 38% -> 39%<br>NDX: Up | NDX: Realized correlation declined and implied correlation rose slightly over the week. The implied less realized spread remained positive, indicating that index implied volatility is a better sell relative to single stock implied volatility. |
| 1-Year | 26% -> 26%<br>SPX: Flat | 34% -> 32%<br>SPX: Down | SPX: Realized correlation was flat and implied correlation declined over the week. The implied less realized spread remained positive, indicating that index implied volatility is a better sell relative to single stock implied volatility. |

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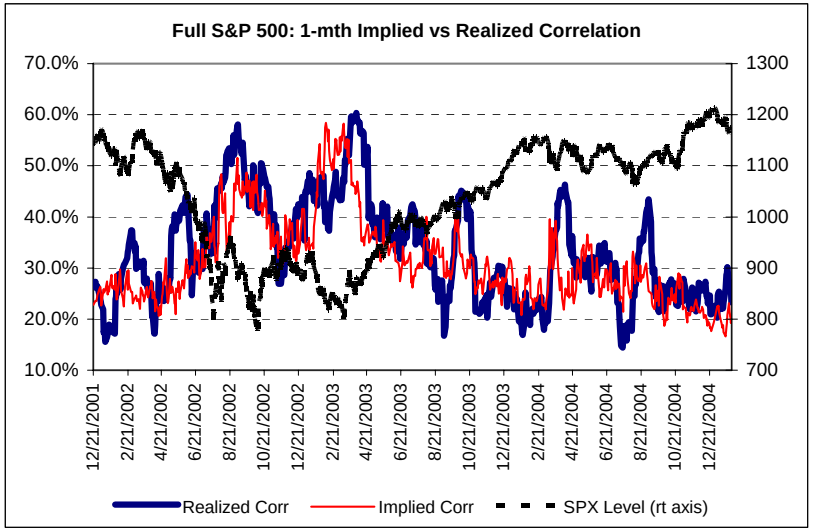
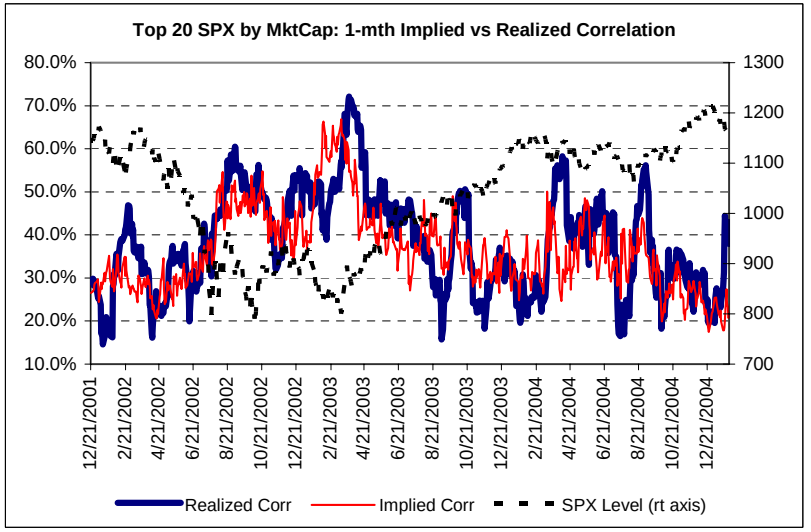
**Equity Derivatives: Volatility & Quantitative Research**

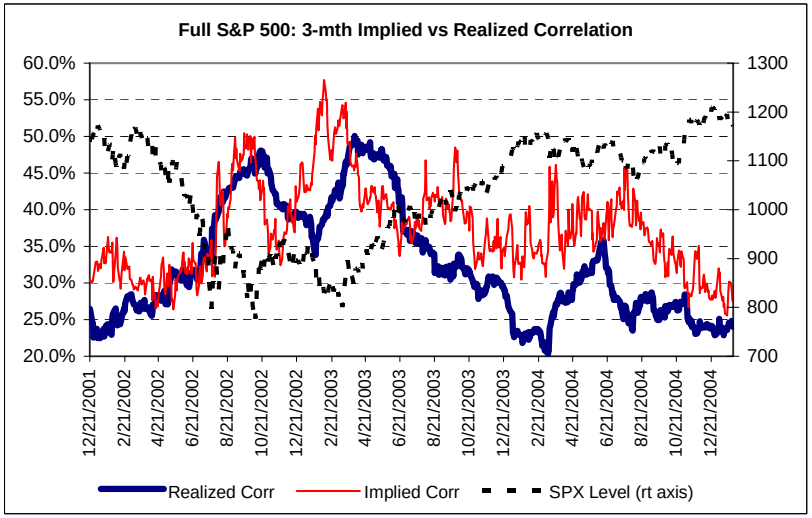
## US Index Dispersion Monitor

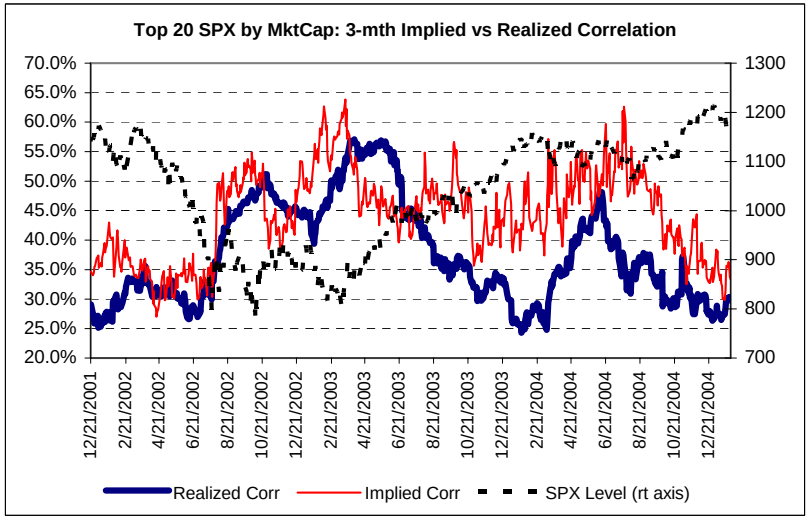
## S&P 500 Index (SPX): 1-mth and 3-mth Correlations

**Date** **1/31/2005**



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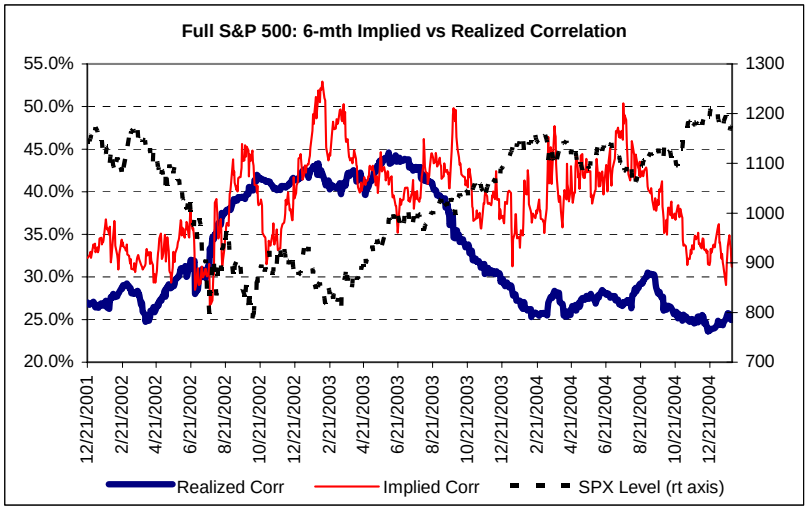
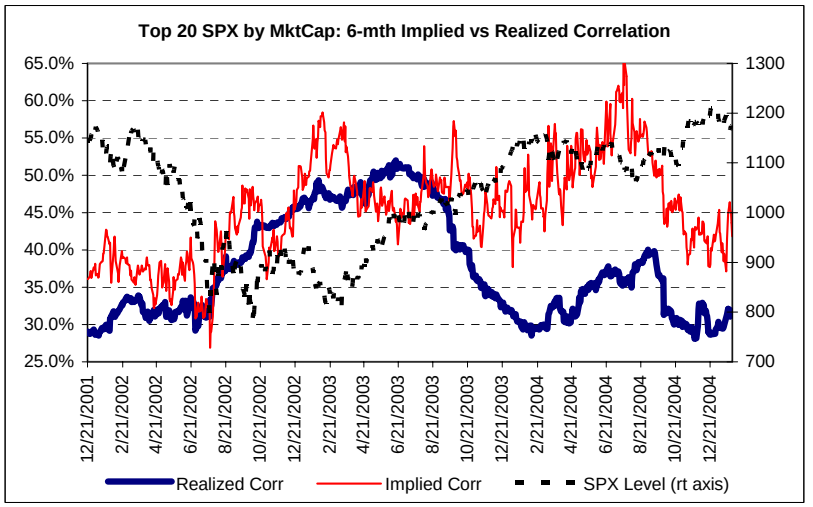
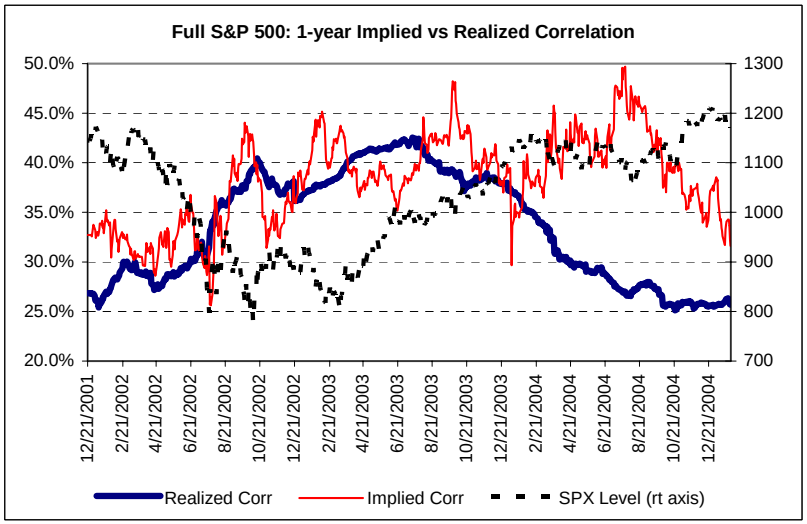
## US Index Dispersion Monitor

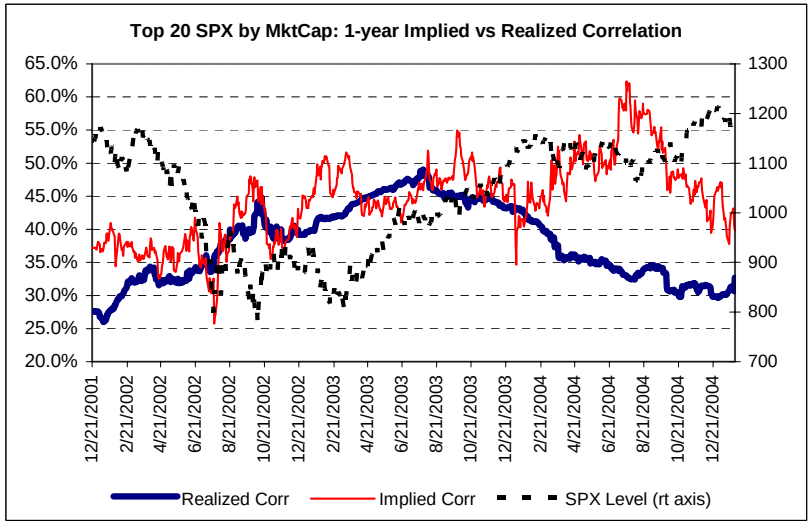
## S&P 500 Index (SPX): 6-mth and 1-year Correlations

**Date** **1/31/2005**



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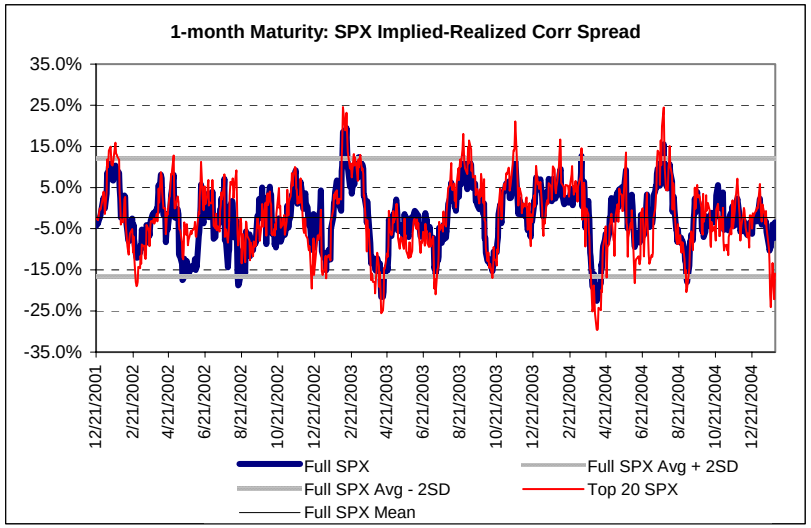
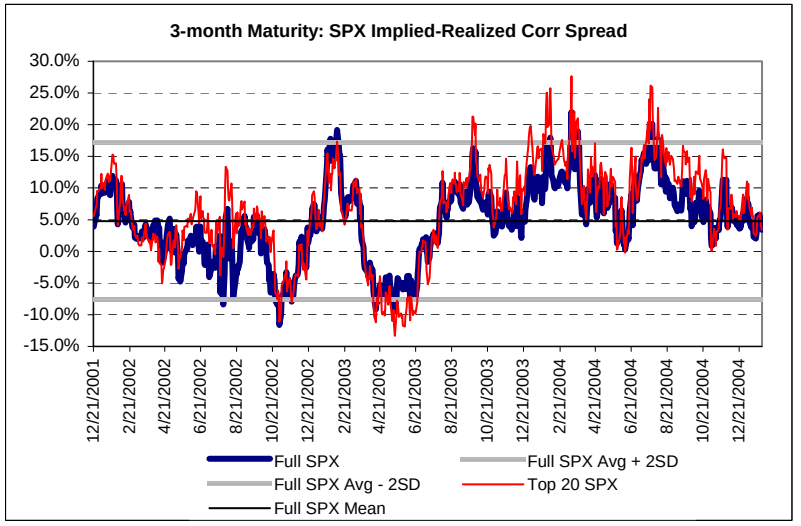
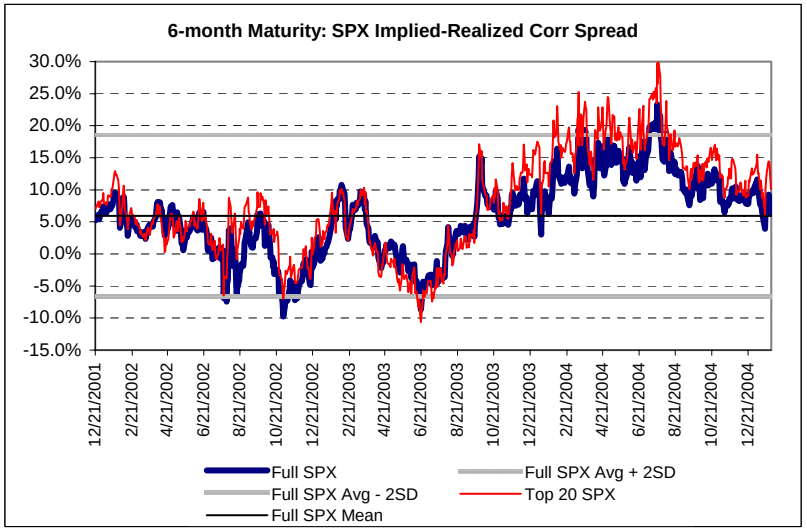
## US Index Dispersion Monitor

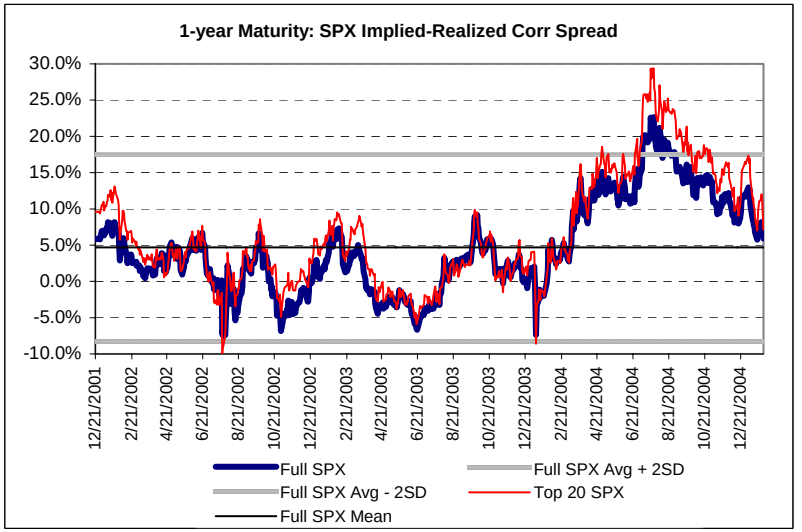
## S&P 500 Index (SPX): Implied - Realized Corr Spreads

**Date** **1/31/2005**



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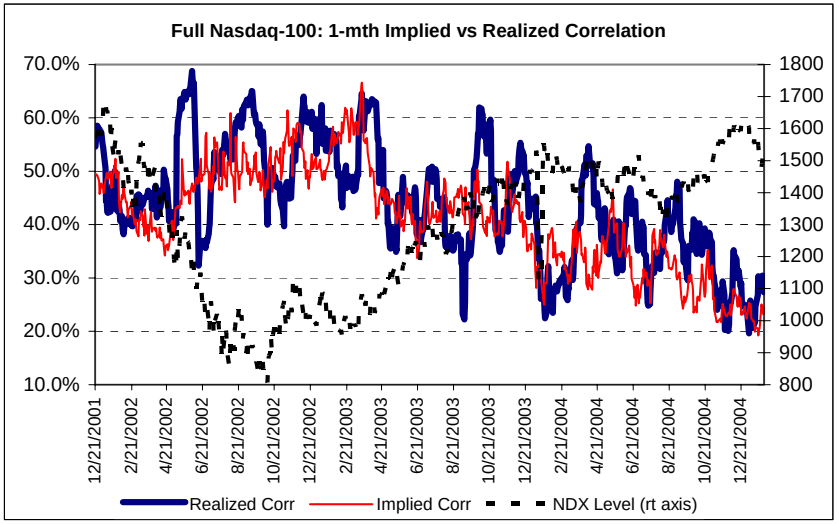
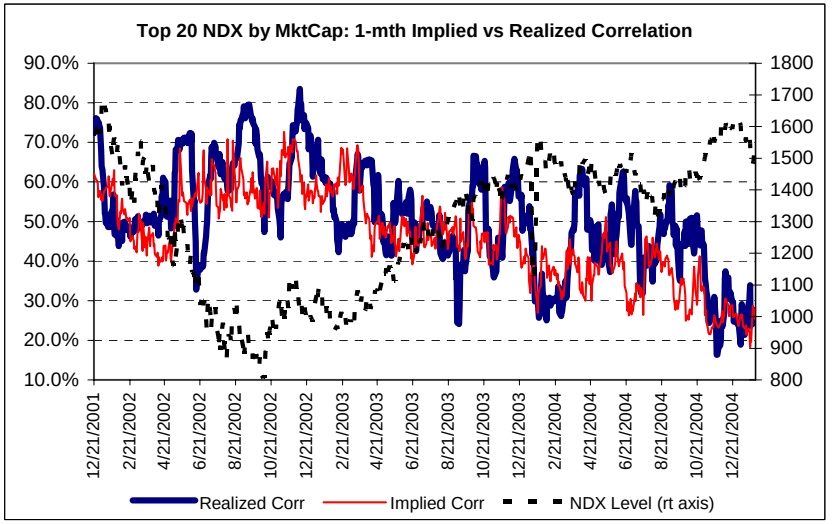
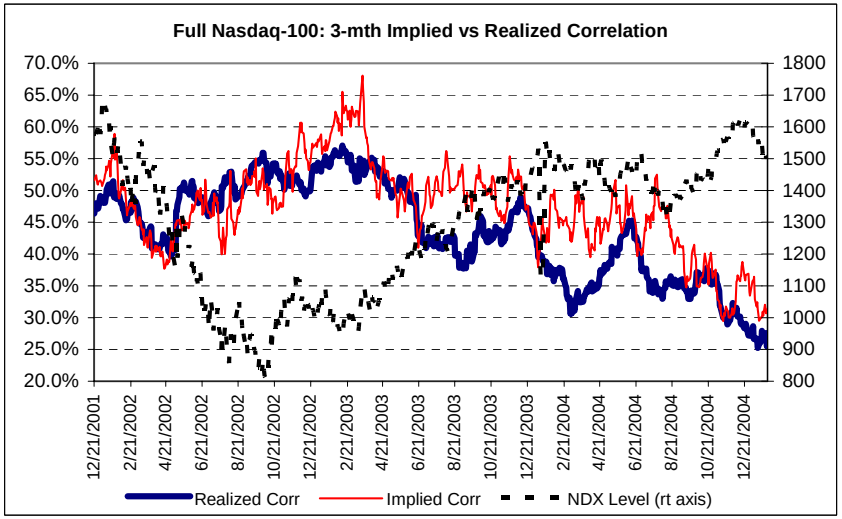
## US Index Dispersion Monitor

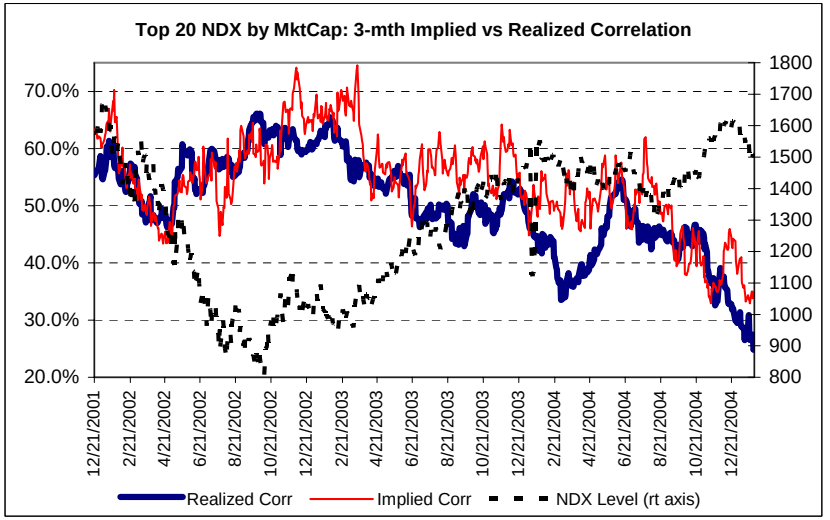
## Nasdaq-100 Index (NDX): 1-mth and 3-mth Correlations

**Date** 1/31/2005



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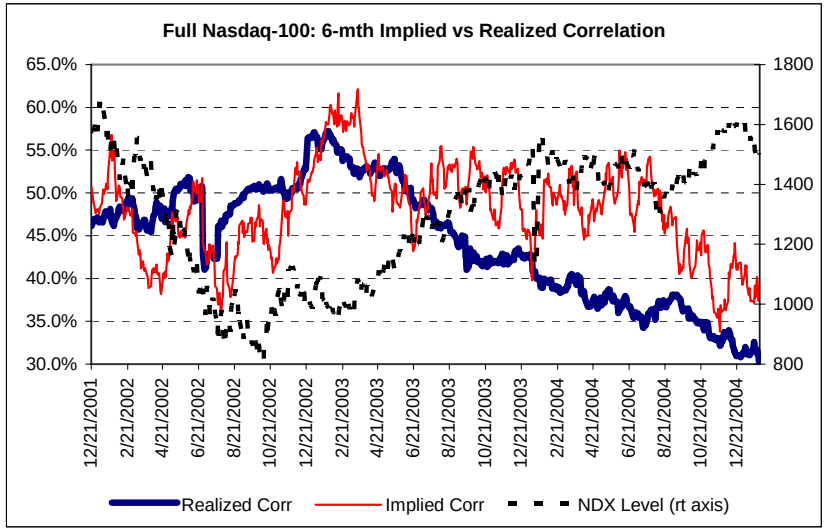
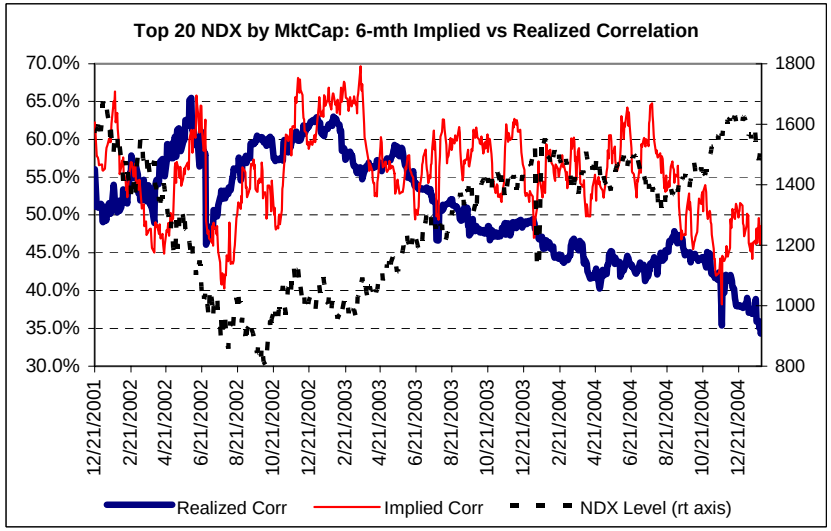
## US Index Dispersion Monitor

## Nasdaq-100 Index (NDX): 6-mth Correlations

**Date** **1/31/2005**



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## US Index Dispersion Monitor

## Nasdaq-100 Index (NDX): Implied - Realized Corr Spreads

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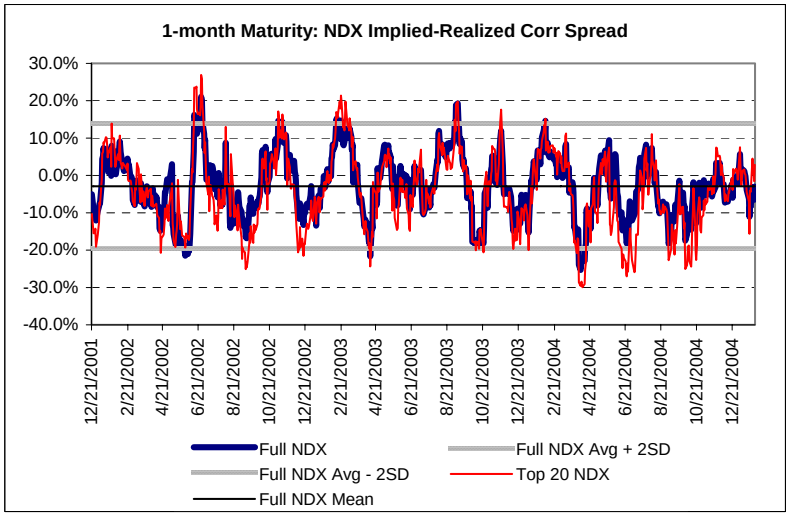


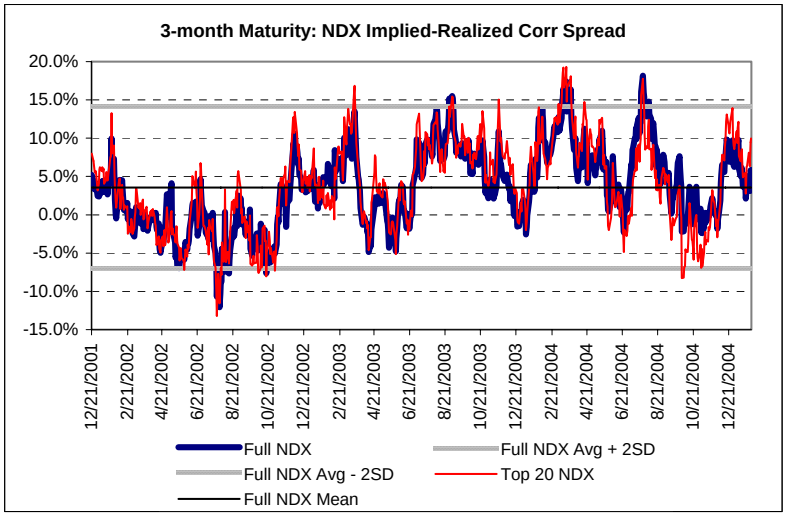


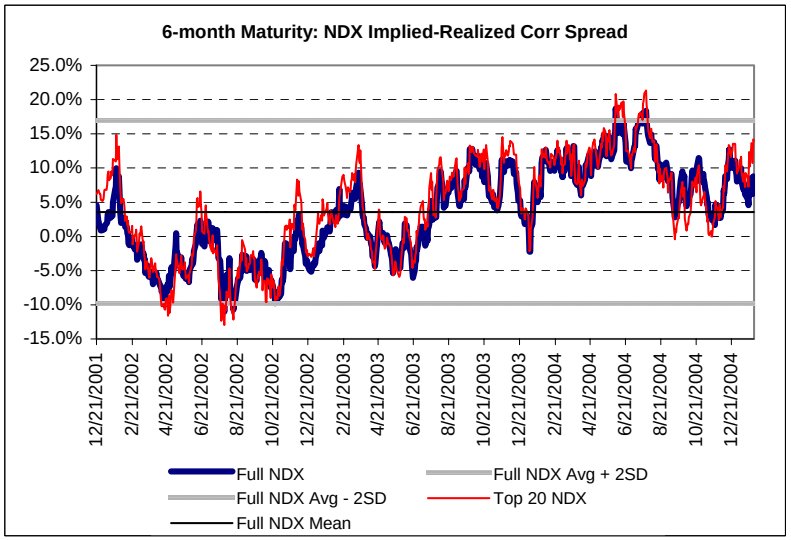

## Introduction to Dispersion Analysis

Comparing index constituent volatility to index volatility can highlight the disparity that often exists between single stock options and index options. We warn, however, that the generally safer and more commonly executed dispersion trade is to buy single stock volatility while selling index volatility, when the conditions are favorable. We analyze the current dispersion signal for all constituent stocks vs. the underlying index as well as the top 20 constituents by market cap (as a proxy for the index) vs. the underlying index, but ensure that the signal MUST exist for the full set of constituents before we recognize a signal for the top 20 constituents by market cap.

## *Dispersion Trading Methodology and Definitions*

**REALIZED (HISTORICAL) CORRELATION:** This is the relationship between the realized (historical) price volatility of the single stock constituents relative to the index. The volatility of an equity index can be computed as the weighted volatility of each individual stock adjusted for the effect of the correlation between stock returns. In the limiting case, where index components all exhibit perfect positive correlation (i.e., they move one-for-one), index volatility is the same as weighted average stock volatility. To the extent that there is less than perfect correlation among stock returns, index volatility will be lower than weighted average single stock volatility. Realized correlation is computed using historical stock returns, and can be thought of as a weighted average of pair-wise correlations between individual components and their respective volatilities, as shown in equation (1) below. We calculate this correlation for across different time horizons to determine if correlation is high or low for the following different maturities: 1 month, 3 month, 6 month and 1 year.

We calculate the realized correlation using all component stocks as well as a subset of the top 20 component stocks by market capitalization (as a proxy portfolio for each index), and use the following formulas to calculated realized correlation We start with the known formula for variance of the portfolio:

$$\sigma^2_p = \Sigma^N_{i=1} \omega_i^2 \sigma_i^2 + [ ( \Sigma^N_{i=1} \Sigma^N_{j=1} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij} )_{\text{for all } i \neq j} ]$$

where: $\sigma^2_p = ( \Sigma^N_{i=1} \Sigma^N_{j=1} \rho_{ij} \omega_i \omega_j \sigma_i \sigma_j )_{\text{for all } i \text{ and } j}$ (the index or portfolio variance, an observed variable)

If we replace $\rho_{ij}$ with $\rho_{\text{Realized}}$ for $i \neq j$, then we can then write the following formula representing the average realized correlation across the stocks in the index or portfolio.

$$\rho_{\text{Realized}} = ( \sigma^2_p - \Sigma^N_{i=1} \omega_i^2 \sigma_i^2 ) / [ ( \Sigma^N_{i=1} \Sigma^N_{j=1} \omega_i \omega_j \sigma_i \sigma_j )_{\text{for all } i \neq j} ]$$

$$= [ ( \Sigma^N_{i=1} \Sigma^N_{j=1} \rho_{ij} \omega_i \omega_j \sigma_i \sigma_j )_{\text{for all } i \text{ and } j} - ( \Sigma^N_{i=1} \omega_i^2 \sigma_i^2 )_{\text{for all } i = j} ] / [ ( \Sigma^N_{i=1} \Sigma^N_{j=1} \omega_i \omega_j \sigma_i \sigma_j )_{\text{for all } i \neq j} ]$$

$$= [ ( \Sigma^N_{i=1} \Sigma^N_{j=1} \rho_{ij} \omega_i \omega_j \sigma_i \sigma_j )_{\text{for all } i \neq j} ] / [ ( \Sigma^N_{i=1} \Sigma^N_{j=1} \omega_i \omega_j \sigma_i \sigma_j )_{\text{for all } i \neq j} ]$$

(Equation 1)

where:

$\rho_{ij}$ = the realized (or historical) correlation between the $i^{th}$ and $j^{th}$ assets

$\omega_i$ = average of the past weight (by MC) for the $i^{th}$ asset in the index (or portfolio) over the respective time horizon

$\omega_j$ = average of the past weight (by MC) for the $j^{th}$ asset in the index (or portfolio) over the respective time horizon.

$\sigma_i$ = the realized volatility for the $i^{ih}$ asset in the index (or portfolio)

$\sigma_j$ = the realized volatility for the $j^{th}$ asset in the index (or portfolio)

$\sigma_p$ = the realized volatility for the index (or portfolio)

**IMPLIED CORRELATION:** Just as an option's implied volatility reconciles the option's market price with known option pricing inputs, implied correlation can be used to reconcile the implied volatility of an index option with that of options on the index or portfolio constituents. The implied average correlation for a portfolio can be computed only for indices or portfolios for which traded index options and constituent stock options exist. While average realized correlation is a backward looking measure that examines historical price movements, implied average correlation is a forward looking measure, since it uses option market prices for stock and index option contracts that expire some time in the future. Similar to realized correlation, we calculate this correlation for across different time horizons to determine if correlation is high or low for the following different maturities: 1 month, 3 month, 6 month and 1 year.

If we replace $\rho_{Realized}$ with a forward looking indicator called $\rho_{Implied}$, we can then create the a formula to represent a forward looking correlation. Our formulation of implied correlation is shown in equation (2) below.

$$\rho_{Implied} = \left(\sigma^2_{pImp} - \Sigma^N_{i=1}\omega_i^2\sigma_{iImp}^2\right) / \left[\left(\Sigma^N_{i=1}\Sigma^N_{j=1}\omega_i\omega_j\sigma_{iImp}\sigma_{jImp}\right)_{for\ all\ i\neq j}\right]$$

$$= \left(\sigma^2_{pImp} - \Sigma^N_{i=1}\omega_i^2\sigma_{iImp}^2\right) / \left[\left(\Sigma^N_{i=1}\omega_i\sigma_{iImp}\right)^2 - \left(\Sigma^N_{i=1}\omega_i^2\sigma_{iImp}^2\right)\right]$$

(Equation 2)

where:

$\sigma_{pImp}$ = option implied volatility on the underlying index

$\omega_i$ = average of the past weight (by MC) for the $i^{th}$ asset in the index (or portfolio) over the respective time horizon

$\sigma_{iImp}$ = the option implied volatility of the $i^{th}$ asset in the index (or portfolio)

$\omega_j$ = average of the past weight (by MC) for the $j^{th}$ asset in the index (or portfolio) over the respective time horizon

$\sigma_{jImp}$ = the option implied volatility of the $j^{ih}$ asset in the index (or portfolio)

Again, since we are using a subset of the top 20 stocks by market capitalization as a proxy portfolio for the each index, we need to use the above the formula to calculated implied correlation for our portfolio of 20 stocks as well as for all stocks of the index in order to show that the signal exists for all the stocks of the index as well as for our proxy 20 stock portfolio.

**DISPERSION TRADING:** Dispersion trading aims to exploit the discrepancy between the pricing of index options and the portfolio of individual stock options. Implied average correlation is important in that it provides information on the relative pricing of the index options and the options on the stocks that comprise the index. For an index that has both exchanged traded options as

well as options on its constituent stocks, one can calculate both realized and implied correlation (as shown in equations 1 and 2). When these correlations diverge, investors may have an opportunity to profit from a dispersion trading strategy. For example, if $\rho_{Implied} > \rho_{Realized}$, index options may be rich compared to stock options and the appropriate dispersion trade would be to sell index options and buy constituent stock options. Conversely, if $\rho_{Implied} < \rho_{Realized}$, the appropriate dispersion trade would be to buy index options and sell constituent stock options. Typically, the more common dispersion trade is to sell the index options and buy the stock options on a delta hedged basis, which generally carries less risk than the opposite trade.

An alternative way to examine dispersion trading opportunities is to monitor the spread between the implied volatility of the index and the weighted average implied volatility of individual stocks, but this will not account for the effects of stock correlation. We define the implied volatility spread in equation (3) below. In general, when S is relatively high, the dispersion strategy may be to sell the index options and buy the individual stock options. Conversely, when S is relatively low, the dispersion strategy may be to buy the index options and to sell the individual stock options. The relative spread in equation (3) can be used to determine potential trade profitability, which would be attractive if the spread is large enough. Typically, this simple calculation is usually a negative number. Therefore, if it is less negative compared to its average, then one could sell index implied volatility and buy single stock implied volatility. However, the proper analysis would be to calculate the implied versus realized correlation but this simple calculation could be used as an initial analysis.

$$S = ( \sigma_{pImp} - \Sigma^{N}_{i=1} \omega_i \sigma_{iImp}) \qquad \text{(Equation 3)}$$

where:

$\sigma_{pImp}$ = option implied volatility on the underlying index

$\omega_i$ = the weight (by MC) for the $i^{th}$ asset in the index (or portfolio)

$\sigma_{iImp}$ = the option implied volatility of the $i^{th}$ asset in the index (or portfolio)

**IMPLIED CORRELATION INCREASES DURING PERIODS OF HIGH REALIZED VOLATILITY:** The degree of correlation among equities varies over time. Increased market volatility, which has been evident over the last year, tends to increase single stock correlation, making this an appropriate time to examine the profit potential of dispersion trading strategies.

## Analyst Certification

The respective research analysts hereby certify: 1) that the views expressed here accurately reflect their personal views about any or all of the subject securities or issuers discussed and (2) no part of our compensation was, is or will be directly or indirectly related to the specific recommendations or views expressed here.

### Important Disclosures:

The analysts responsible for preparing this report have received compensation based upon various factors including the Firm's total revenues, a portion of which is generated by investment banking activities.