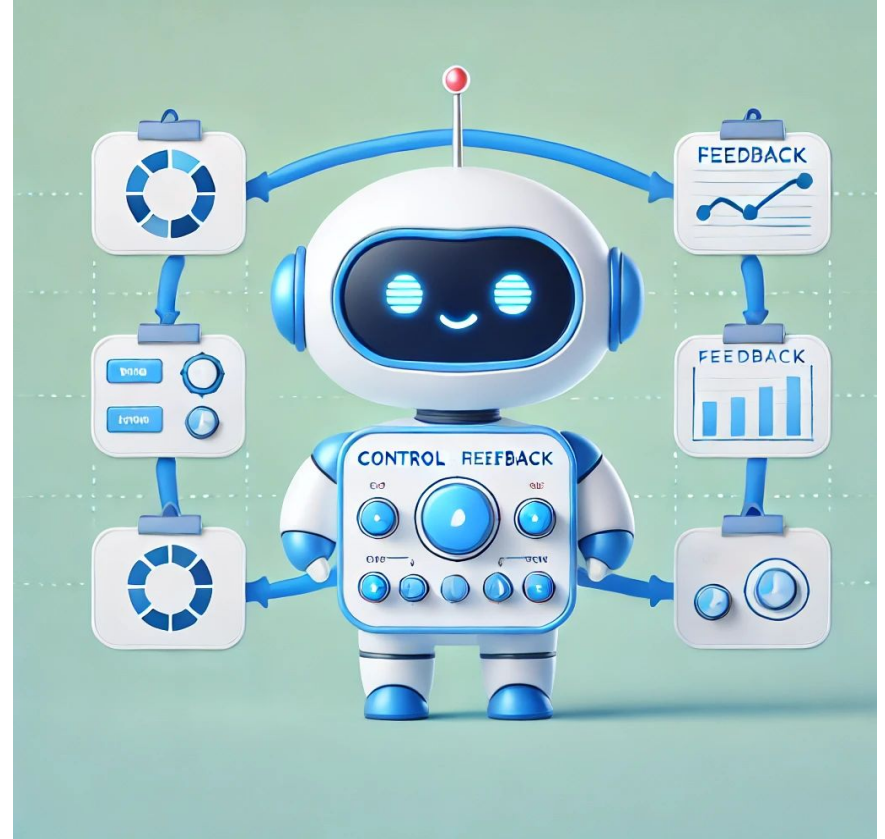


# Feedback Control

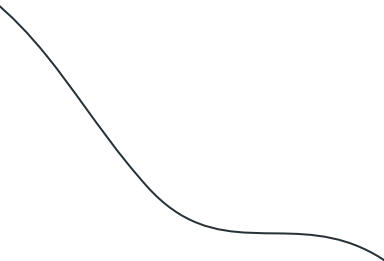
07/16/2025

Prof. Jizhong Xiao



# Outline



1. Open-loop and Closed-loop Control
  2. PID Control Explained
  3. PID Math Demystified
  4. Project Assignment
- 



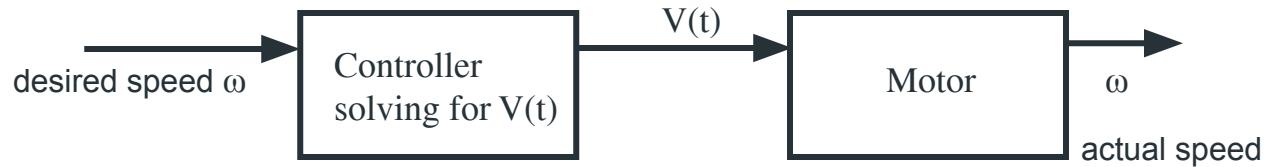
1

# **Open-Loop v.s. Closed Loop Control**

# Open-loop vs. Close-loop Control

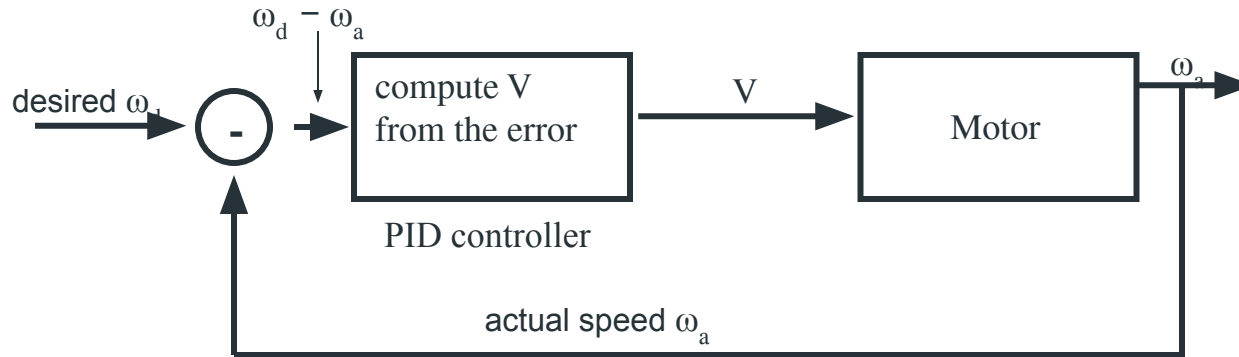
---

Open-loop Control:



If desired speed  $\omega_d \neq$  actual speed  $\omega_a$ , So what?

Closed-loop Control: using feedback

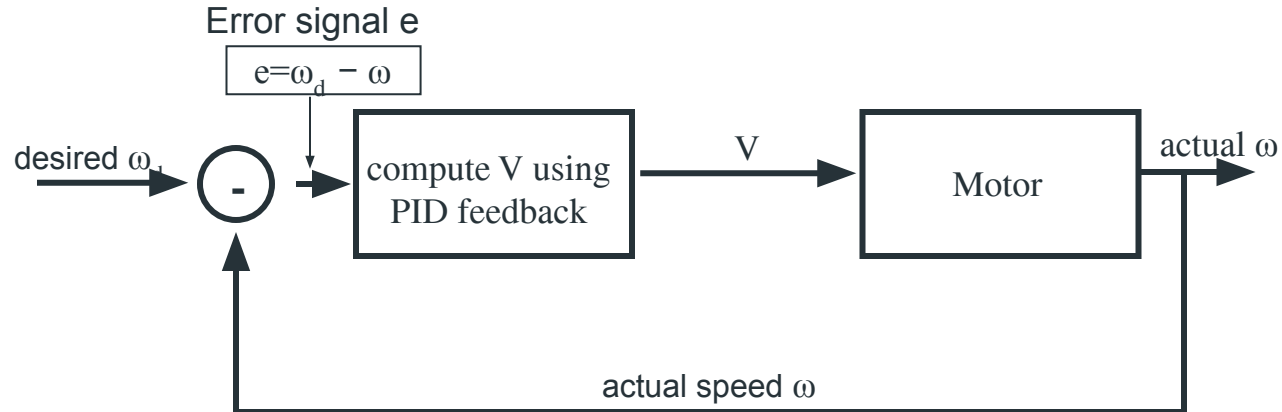


# PID Controller

PID control: Proportional / Integral / Derivative control

$$V = K_p (\omega_d - \omega) + K_i \int (\omega_d - \omega) dt + K_d \frac{de}{dt}$$

$$V = K_p \cdot (e + K_i \int e + K_d \frac{de}{dt})$$



# Implementing PID

---

Use discrete approximations to the I and D terms:

- Proportional term:  $e_i = \omega_{\text{desired}} - \omega_{\text{actual}}$  at time  $i$
- Integral term:  $\sum_{i=0}^{i=\text{now}} e_i$
- Derivative term:  $e_i - 2e_{i-1} + e_{i-2}$



2

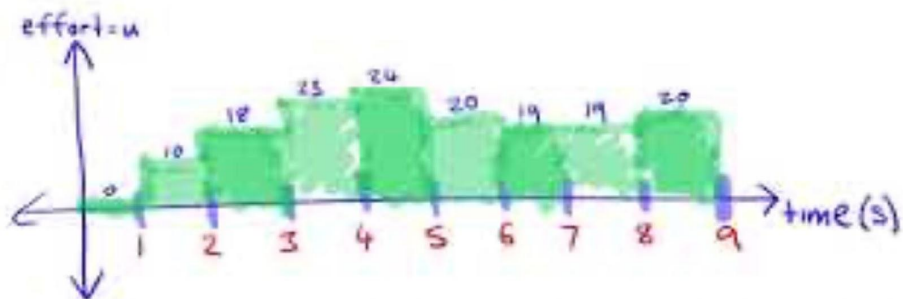
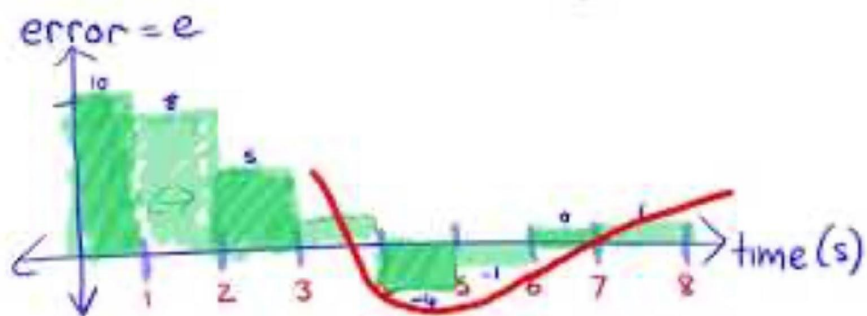
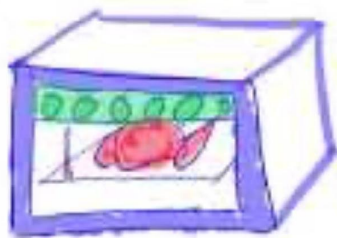
# **PID Control Explained**

## PID Control Explained

PID

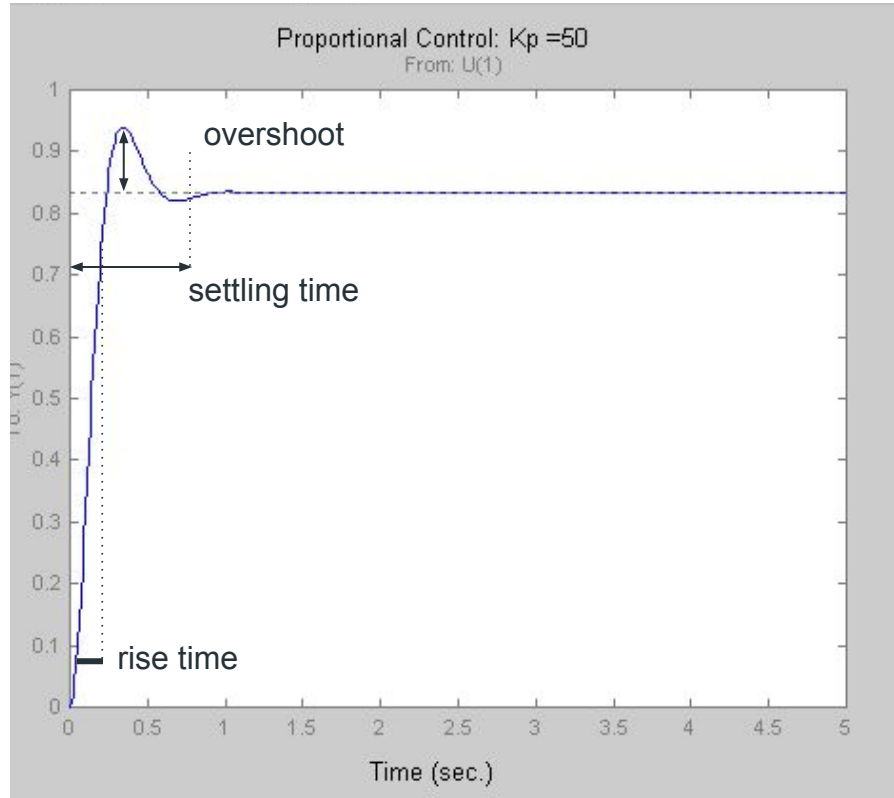
$$u_I = k_I \int e \cdot dt$$

$$e \rightarrow 0$$





# Evaluating the response



steady-state error

**ss error** -- difference from the system's desired value

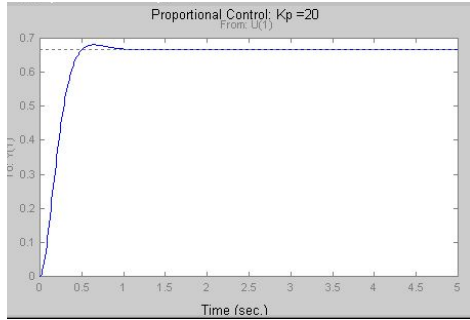
**overshoot** -- % of final value exceeded at first oscillation

**rise time** -- time to span from 10% to 90% of the final value

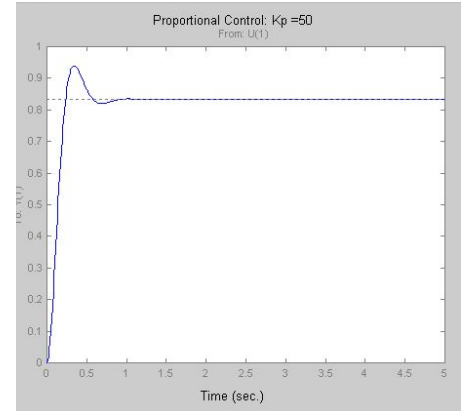
**settling time** -- time to reach within 2% of the final value

How can we eliminate the steady-state error?

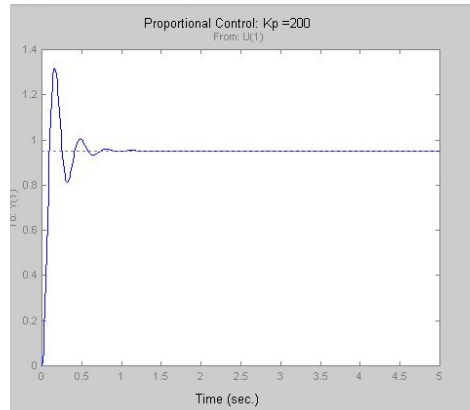
# Control Performance, P-type



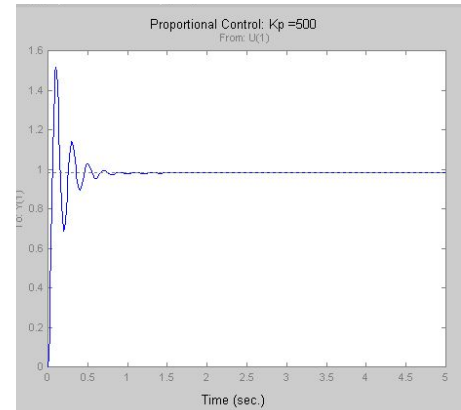
$K_p = 20$



$K_p = 50$

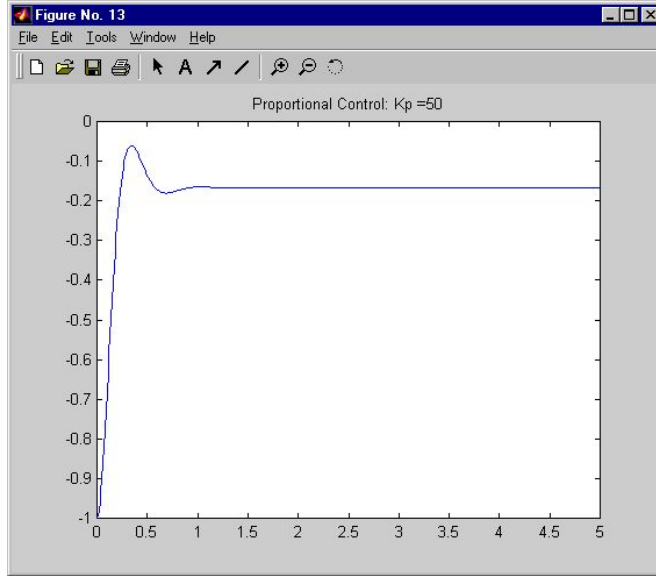


$K_p = 200$

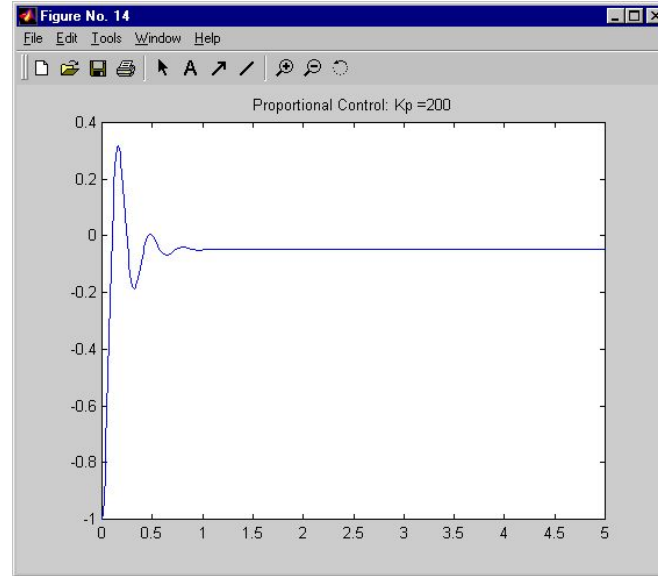


$K_p = 500$

# Steady-state Errors, P-type



$$K_p = 50$$



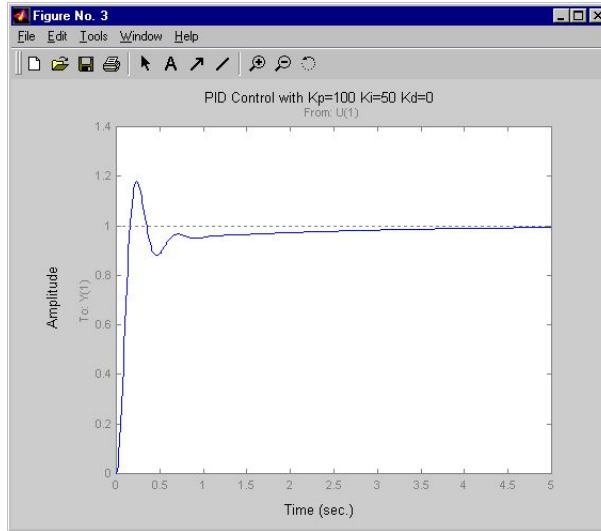
$$K_p = 200$$

# Control Performance, PI - type

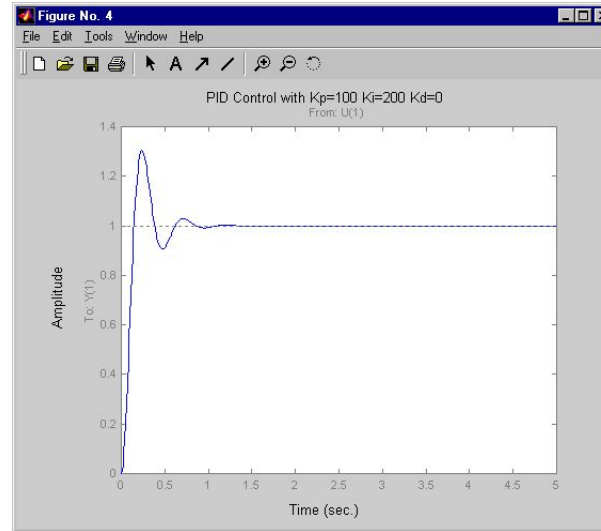
---

---

$$K_p = 100$$

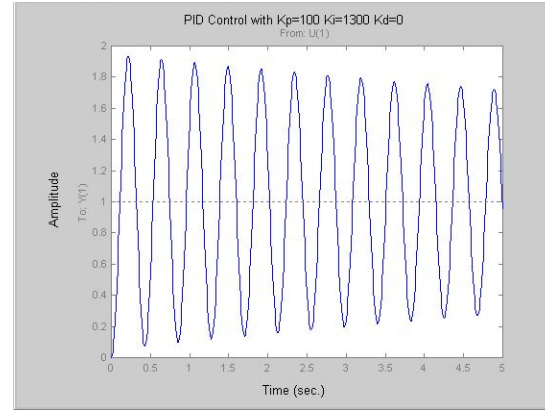
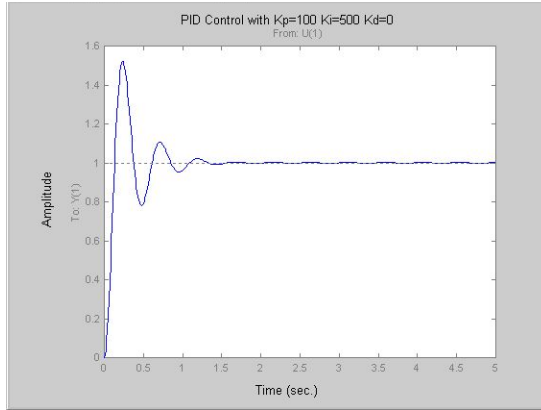


$$K_i = 50$$

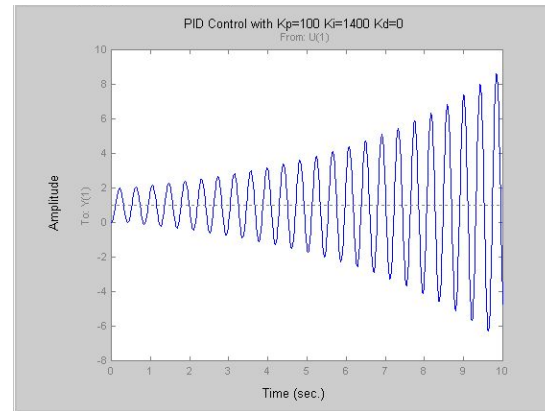
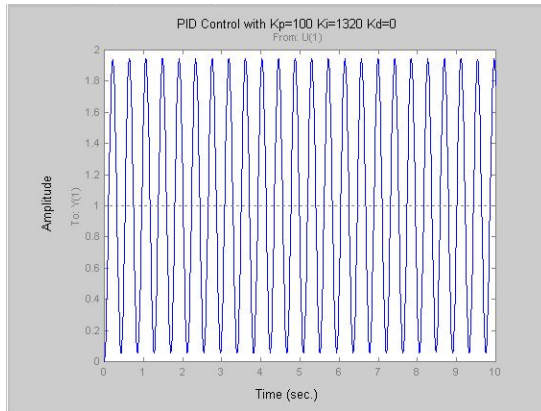


$$K_i = 200$$

# You've been integrated...



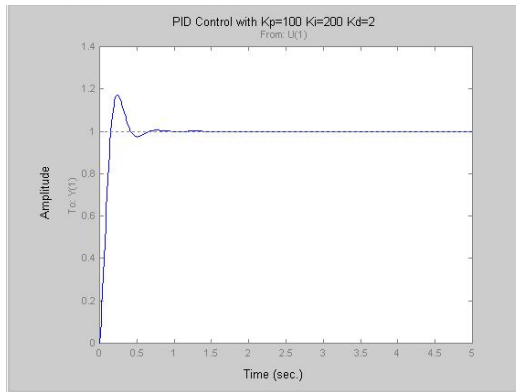
$$K_p = 100$$



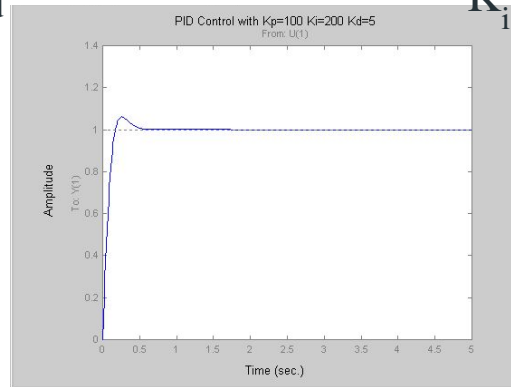
instability &  
oscillation

# Control Performance, PID-type

$K_d = 2$

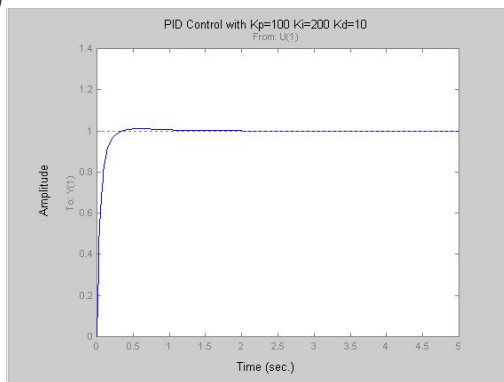


$K_d = 5$

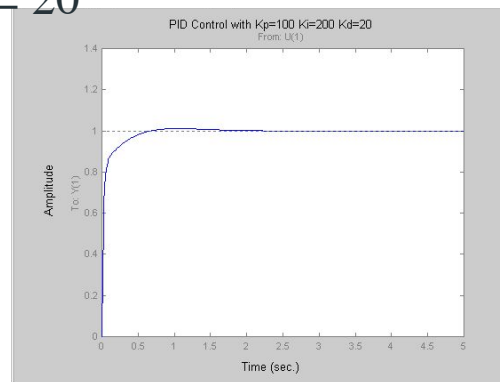


$K_p = 100$   
 $K_i = 200$

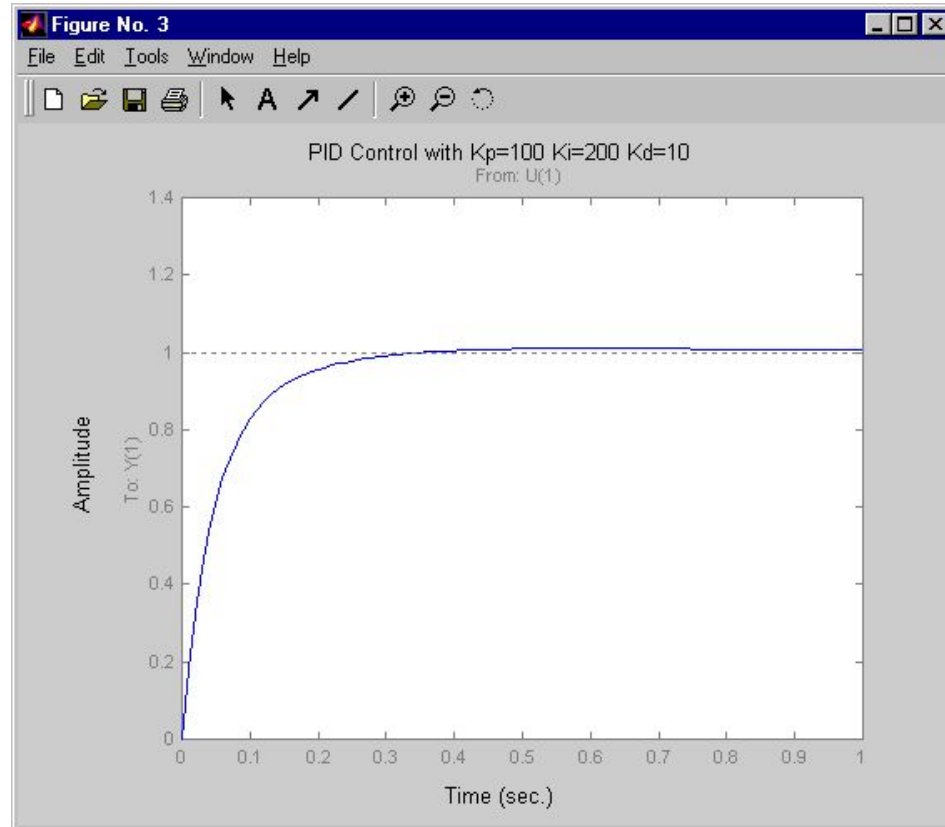
$K_d = 10$



$K_d = 20$



# PID final control



# PID Tuning

How to get the PID parameter values ?

(1) If the system has a known mathematical model (i.e., the transfer function), analytical methods can be used (e.g., root-locus method) to meet the transient and steady-state specs.

(2) When the system dynamics are not precisely known, we must resort to experimental approaches.

Ziegler-Nichols Rules for Tuning PID Controller:

Using only Proportional control, turn up the gain until the system oscillates w/o dying down, i.e., is marginally stable. Assume that  $K$  and  $P$  are the resulting gain and oscillation period, respectively.

Then, use

for P control

$$K_p = 0.5 K$$

for PI control

$$K_p = 0.45 K$$

$$K_i = 1.2 / P$$

for PID control

$$K_p = 0.6 K$$

$$K_i = 2.0 / P$$

$$K_d = P / 8.0$$

Ziegler-Nichols Tuning  
for second or higher  
order systems





# **4** **Project** **Assignment**

# Project Assignment

---

Use ultrasonic sensor as the feedback sensor, implement PID controller to make the 4WD robot approaching the wall 4 meters away as fast as possible, as close as possible.

Hints:

Approaching the wall □ no collision with the wall, “no overshoot”

As fast as possible □ minimize settling time

As close as possible □ minimize steady state error

# **Thank you, enjoy the design!**





**3**

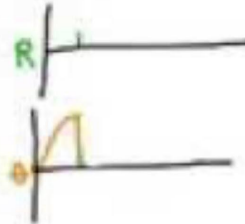
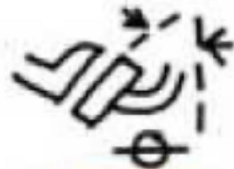
**PID Math  
Demystified**

## PID Math Demystified

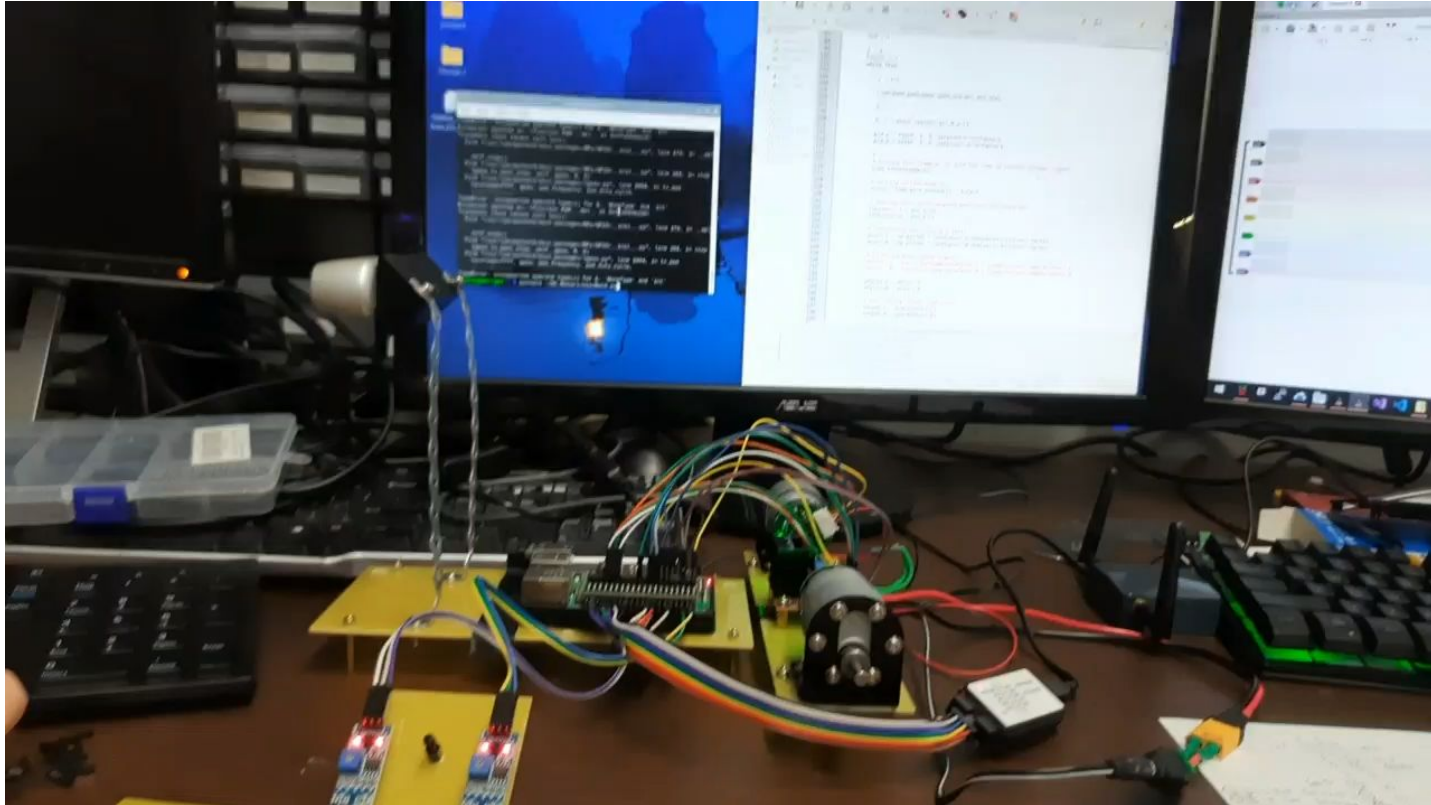
### Proportional + Integral

$$u(t) = K \times e(t) + \sum \frac{K}{\tau} e(t)$$

Error := Setpoint - ProcessValue;  
Reset := Reset + K/tau\_i \* Error;  
Output := K \* Error + Reset;

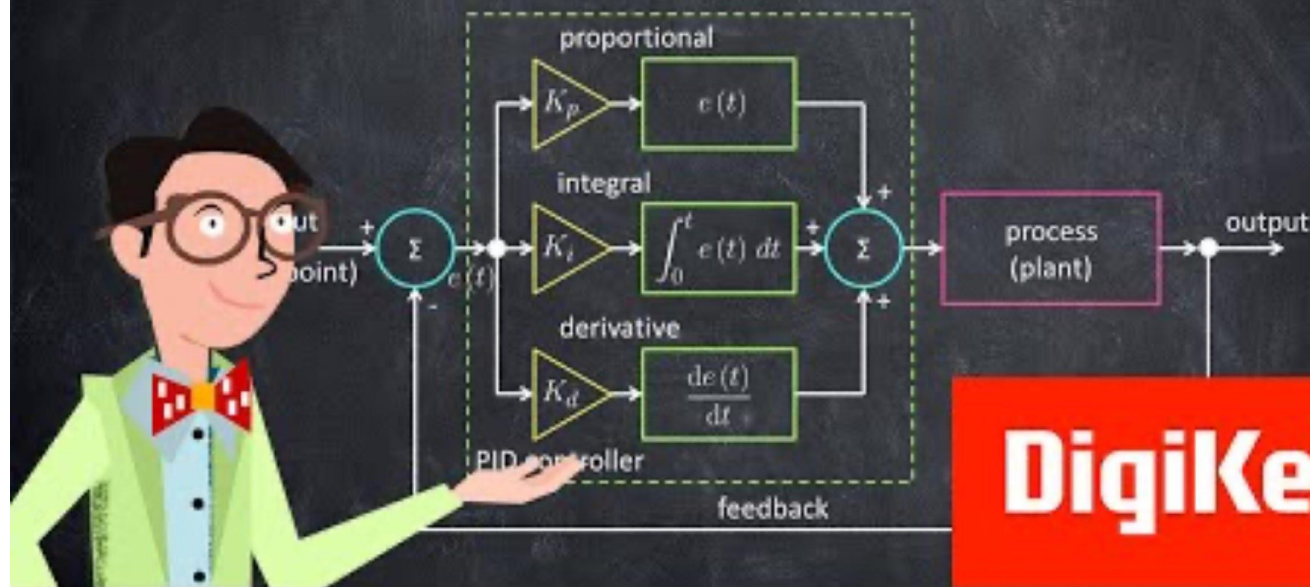


## Motor Control Experiment at CCNY Robotics Lab



# Shawn Hymel

Presents



## DigiKey

# PID Controller



*By Ted Mortenson*

