# Mobile Robot Locomotion

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ACADEMY

# Outline

1. Wheeled Mobile Robots
    - Wheel Types
2. Mobile Robot Locomotion
    - Differential Drive
    - Tricycle
    - Synchronous Drive
    - Omni-directional
    - Car Drive (Ackerman Steering)

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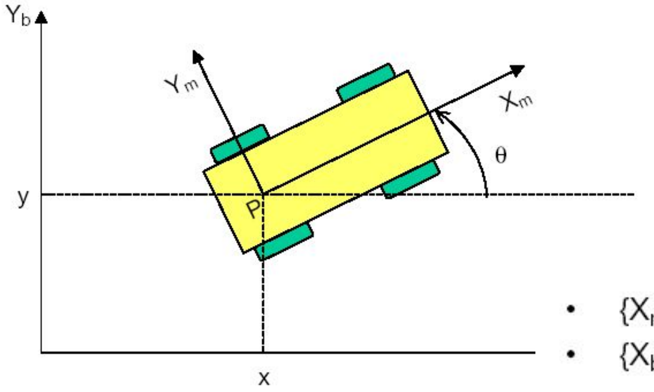
# Wheeled Mobile Robots

# Wheeled Mobile Robots

- **Combination of various physical (hardware) and computational (software) components**

- **A collection of subsystems:**
    - **Locomotion:** how the robot moves through its environment
    - **Sensing:** how the robot measures properties of itself and its environment
    - **Control:** how the robot generate physical actions
    - **Reasoning:** how the robot maps measurements into actions
    - **Communication:** how the robots communicate with each other or with an outside operator

# Notation




Posture: position(x, y) and orientation θ

- $\{X_m, Y_m\}$ – moving frame
- $\{X_b, Y_b\}$ – base frame

$$q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

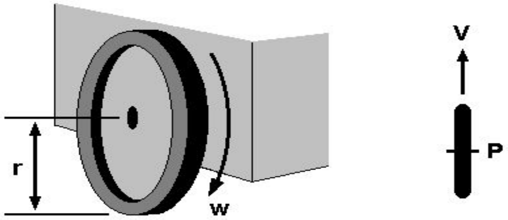
robot posture in base frame

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

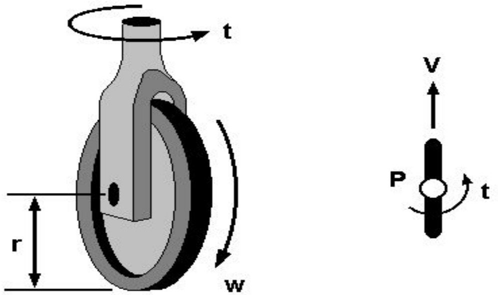
Rotation matrix expressing the orientation of the base frame with respect to the moving frame

# Wheel Types

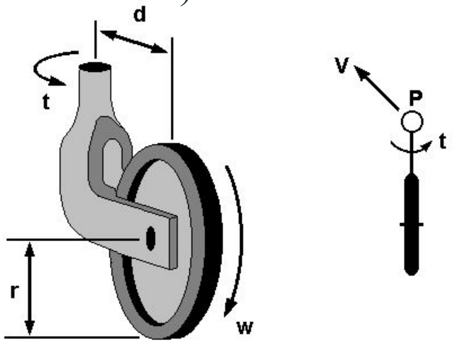
Fixed wheel



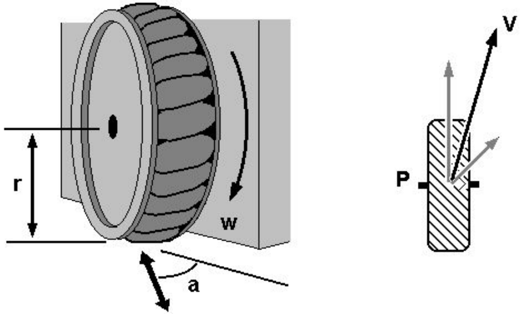

Centered orientable wheel



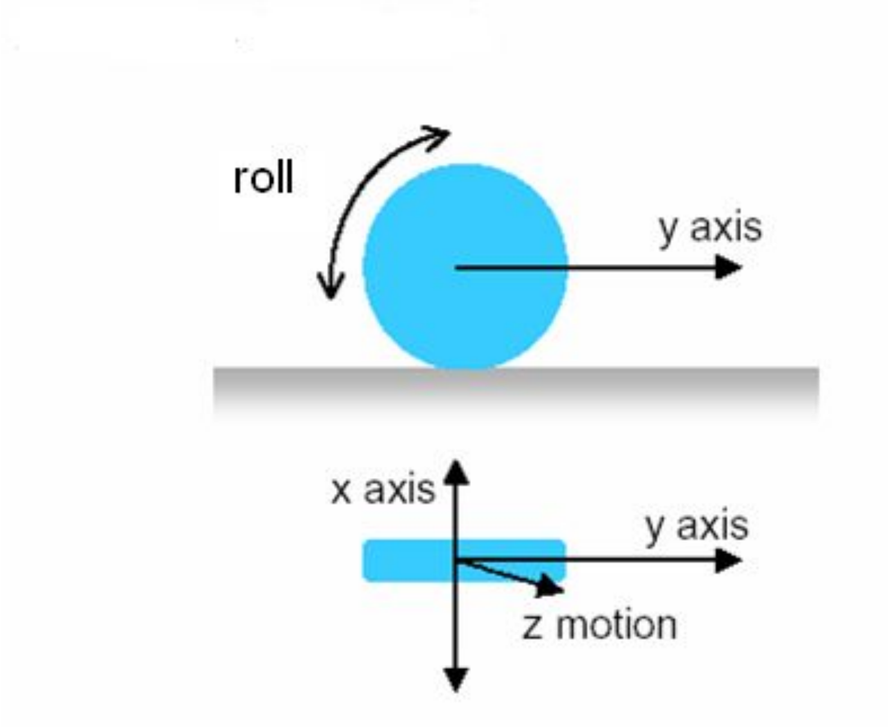

Off-centered orientable wheel
(Castor wheel)




Swedish wheel: omnidirectional property



# Wheels



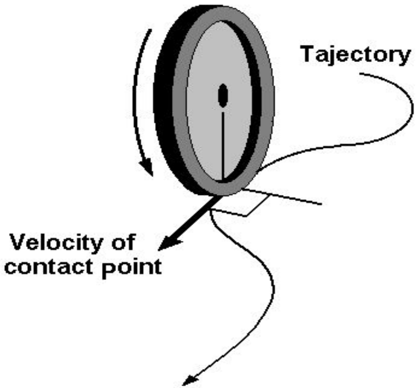

Rolling motion

Lateral slip

# Idealized Rolling Wheel

- **Assumptions**

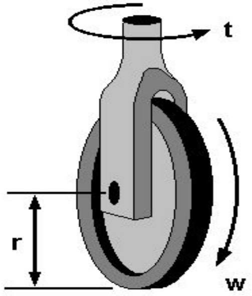
1. The robot is built from rigid mechanisms.
2. No slip occurs in the orthogonal direction of rolling (non-slipping).
3. No translational slip occurs between the wheel and the floor (pure rolling).
4. The robot contains at most one steering link per wheel.
5. All steering axes are perpendicular to the floor.

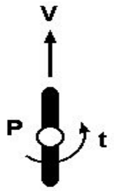


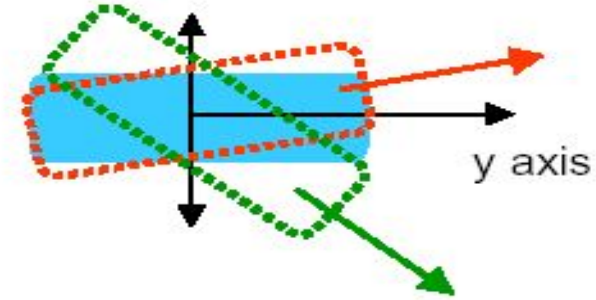

**Non-slipping and pure rolling**

# Steered Wheel

- **Steered wheel**
  - The orientation of the rotation axis can be controlled



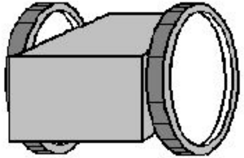



# Robot wheel parameters

- For low velocities, rolling is a reasonable wheel model.
  - This is the model that will be considered in the kinematics models of WMR
- Wheel parameters:
  - r = wheel radius
  - v = wheel linear velocity
  - w = wheel angular velocity
  - t = steering velocity

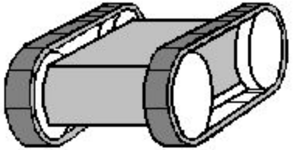
# Examples of WMR

Example



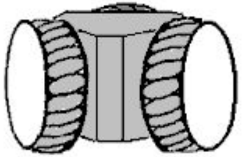
Bi-wheel type robot

- Smooth motion
- Risk of slipping
- Some times use roller-ball to make balance



Caterpillar type robot

- Exact straight motion
- Robust to slipping
- Inexact modeling of turning

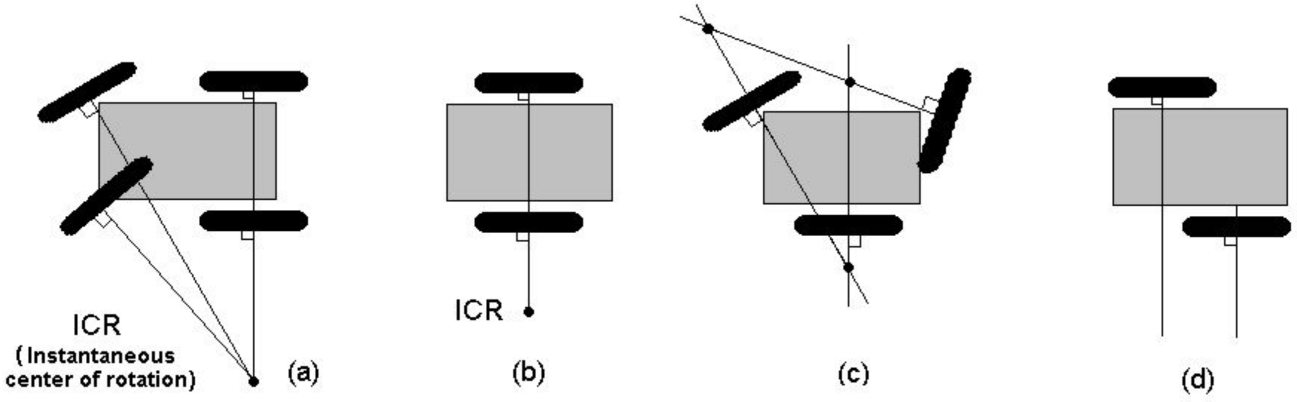


Omnidirectional robot

- Free motion
- Complex structure
- Weakness of the frame

# Mobile Robot Locomotion

- Instantaneous center of rotation (ICR) or Instantaneous center of curvature (ICC)
    - A cross point of all axes of the wheels



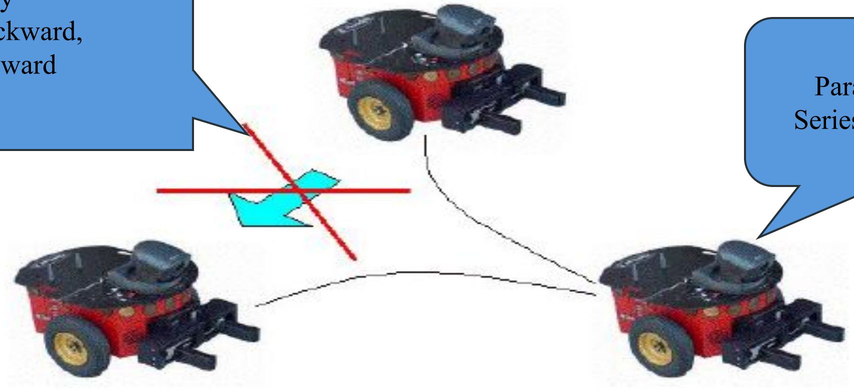
# Non-holonomic constraint

**Physical Meaning?**

***So what does that mean?***

**Your robot can move in some directions (forward and backward), but not others (sideward).**

The robot can instantly move forward and backward, but can not move sideward

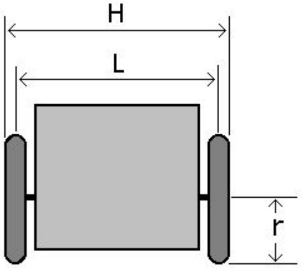


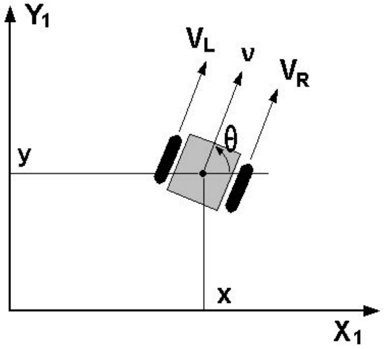
Parallel parking, Series of maneuvers

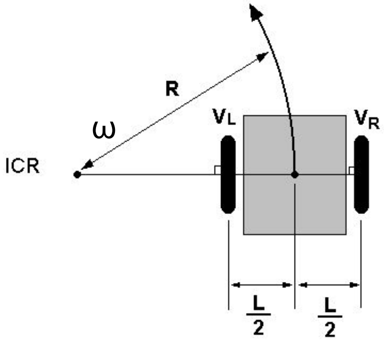
# Mobile Robot Locomotion

- Differential Drive
    - two driving wheels (plus roller-ball for balance)
    - simplest drive mechanism
    - sensitive to the relative velocity of the two wheels (small error result in different trajectories, not just speed)
- Steered wheels (tricycle, bicycles, wagon)
    - Steering wheel + rear wheels
    - cannot turn ±90º
    - limited radius of curvature
- Synchronous Drive
- Omni-directional
- Car Drive (Ackerman Steering)

# Differential Drive




- Posture of the robot

$$P = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

(x,y) : Position of the robot

$\theta$ : Orientation of the robot

- Control input

$$U = \begin{pmatrix} v \\ w \end{pmatrix}$$

*v* : Linear velocity of the **robot**

*w* : Angular velocity of the **robot**

**(notice: not for each wheel)**

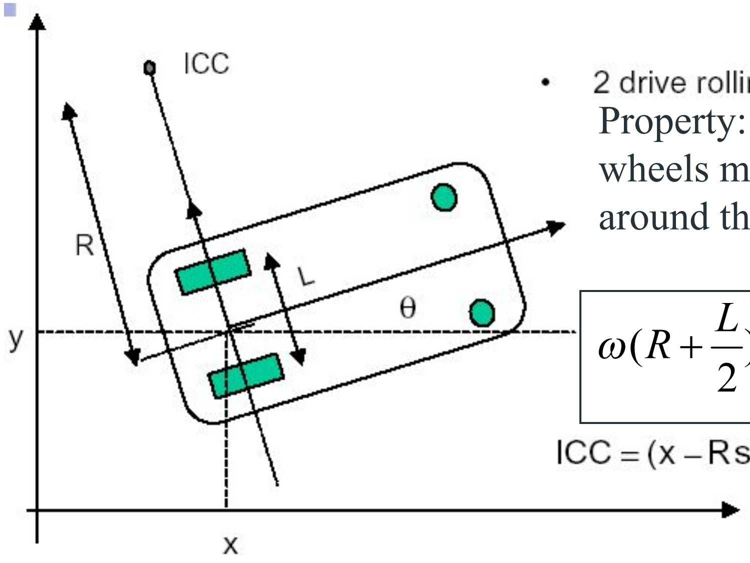
# Differential Drive

$V_R(t)$ – linear velocity of right wheel

$V_L(t)$ – linear velocity of left wheel

r – nominal radius of each wheel

R – instantaneous curvature radius of the robot trajectory (distance from ICC to the midpoint between the two wheels).




- 2 drive rolling wheels

Property: At each time instant, the left and right wheels must follow a trajectory that moves around the ICC at the same angular rate ω, i.e.,

$$\omega\left(R+\frac{L}{2}\right)=V_R \qquad \omega\left(R-\frac{L}{2}\right)=V_L$$

$$ICC=(x-R\sin\theta,\ y+R\cos\theta)$$

# Differential Drive

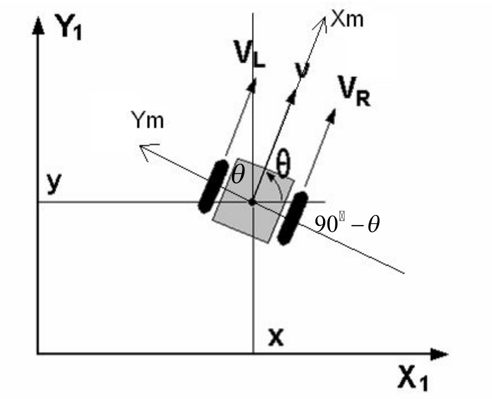
- Relation between the control input and speed of wheels

$$V_L = r\,\omega_L \qquad V_R = r\,\omega_R$$

$$\omega = \frac{V_R - V_L}{L} \qquad v = \frac{V_R + V_L}{2}$$

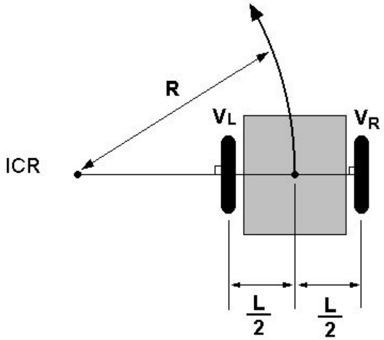
- Kinematic model in world frame

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix}$$



# Basic Motion Control

- Instantaneous center of rotation




$$(V_R - V_L)/\ L \ = \ V_R\ /(R + \frac{L}{2})$$

$$R = \frac{L}{2}\,\frac{V_R + V_L}{V_R - V_L}$$

R : Radius of rotation

- Straight motion

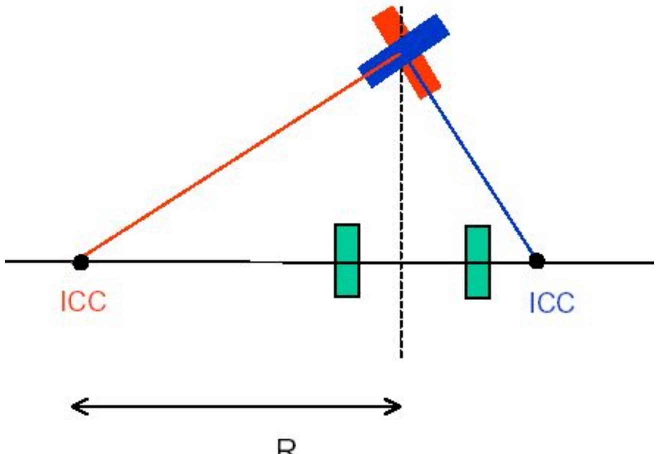
  R = Infinity → $V_R = V_L$

- Rotational motion

  R = 0 → $V_R = -V_L$

# Tricycle

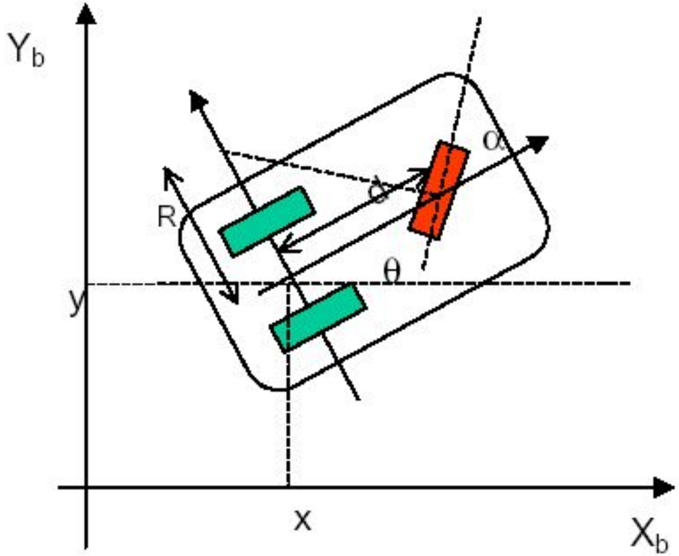
- Three wheels and odometers on the two rear wheels
- Steering and power are provided through the front wheel
- control variables:
    - steering direction $\alpha(t)$
    - angular velocity of steering wheel $w_s(t)$




The ICC must lie on the line that passes through, and is perpendicular to the fixed rear wheels

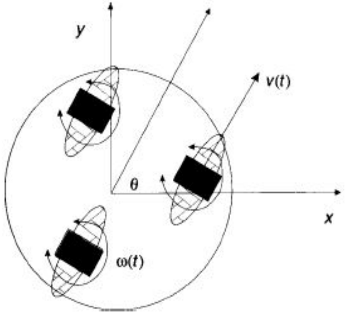
# Tricycle

- If the steering wheel is set to an angle α(t) from the straight-line direction, the tricycle will rotate with angular velocity $\omega$(t) about ICC lying a distance R along the line perpendicular to and passing through the rear wheels.



# Synchronous Drive

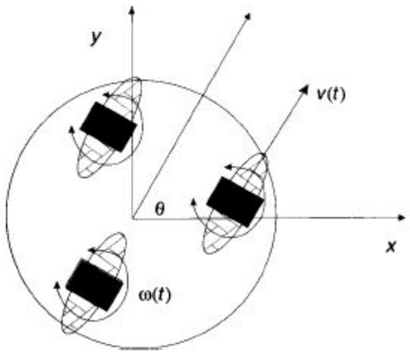
- Typical configuration
  - Three steered wheels arranged as vertices of an equilateral triangle surmounted by a cylindrical platform
  - All of the three wheels point in the same direction and turn at the same rate
  - This is typically achieved through the use of a complex collection of belts that physically link the wheels together
  - All the wheels turn and drive in unison
  - Two independent motors, one rolls all wheels forward, one rotate them for turning
- This leads to a holonomic behavior
- The synchro drive robot has the ability to control the orientation $\theta$ of their pose directly.


**Holonomic System**

# Synchronous Drive

- Control variables (independent)
  - v(t), ω(t)



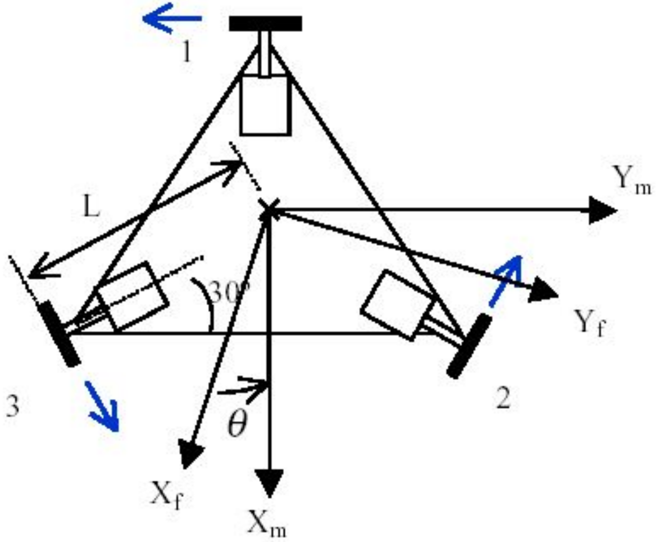

$$x(t) = \int_0^t v(\sigma)\cos(\theta(\sigma))d\sigma$$

$$y(t) = \int_0^t v(\sigma)\sin(\theta(\sigma))d\sigma$$

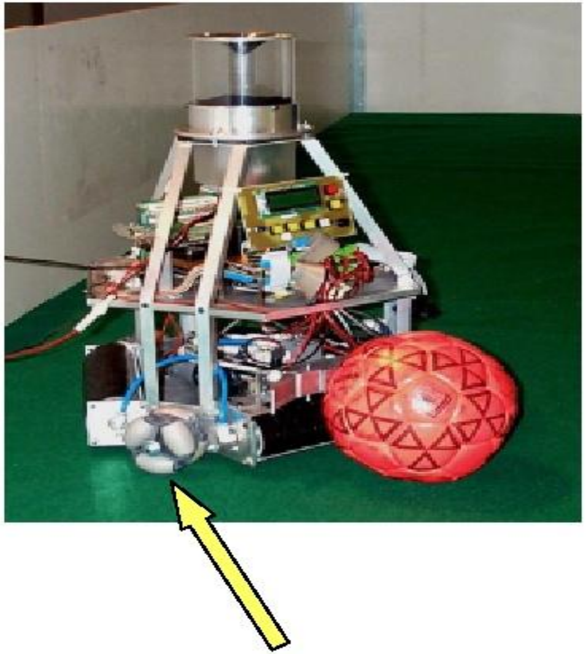
$$\theta(t) = \int_0^t w(\sigma)d\sigma$$

- The ICC is always at infinity
- Changing the orientation of the wheels manipulates the direction of ICC

# Omnidirectional Drive

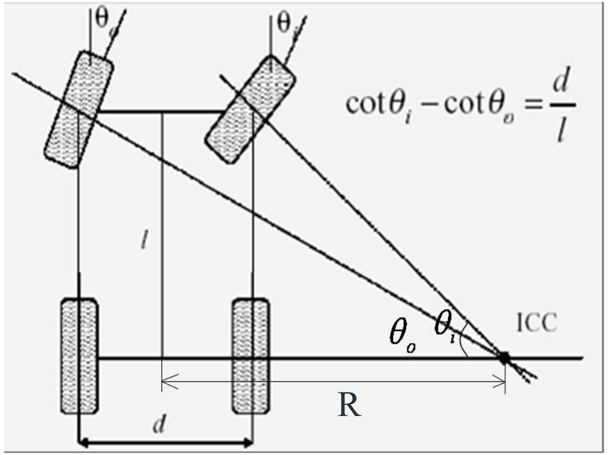



Holonomic System



Swedish Wheel

# Car Drive (Ackerman Steering)



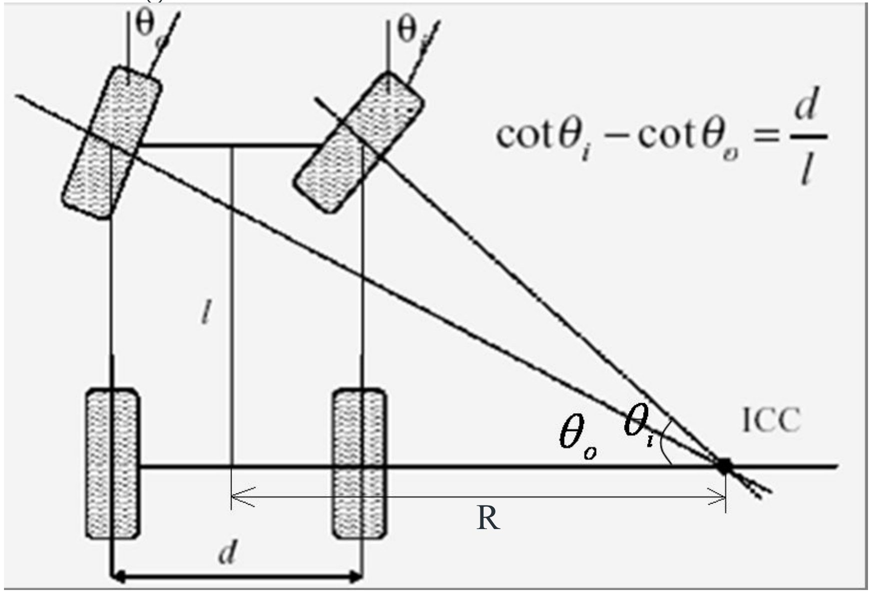

- Used in motor vehicles, the inside front wheel is rotated slightly sharper than the outside wheel (reduces tire slippage).
- Ackerman steering provides a fairly accurate dead-reckoning solution while supporting traction and ground clearance.
- Generally the method of choice for outdoor autonomous vehicles.

# Ackerman Steering

- The Ackerman Steering equation:

$$\cot\theta_i - \cot\theta_o = -\frac{d}{l}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$




$$\cot\theta_i - \cot\theta_o$$

$$= \frac{R - d/2}{l} - \frac{R + d/2}{l}$$

$$= -\frac{d}{l}$$

# Summary

- Mobot: Mobile Robot
- Classification of wheels
  - Fixed wheel
  - Centered orientable wheel
  - Off-centered orientable wheel (Caster Wheel)
  - Swedish wheel
- Mobile Robot Locomotion
  - 5 types of driving (steering) methods
- Basic Control

# Thank you, enjoy the design!