CS 3843 Midterm Exam One Solution Fall 2013

Name (Last)________________, (First)__________________

You may use a calculator and one sheet of notes on this exam, but no other materials and no computer.

This test has a full score of 100 points. Show all the major steps in your work to receive partial credits.

Problem 1 (20 points)

(a) (4 points) Convert to decimal: 0xA9C4

Sol: 0xA9C4 = $10\times16^3+9\times16^2+12\times16+4= 43460$

(b) (4 points) Convert to binary: 0xC7EF

Sol: 0xC7EF = $(1100,0111, 1110, 1111)_2$

(c) (4 points) Convert from decimal to hexadecimal: 2617

Sol: $2617 = 10\times16^2+ 3\times16+9=$ 0xA39

Convert to binary= $(1010, 0011, 1001)_2$

(d) (4 points) Convert from decimal to binary: 937

Sol: $937 = 3\times16^2+10\times16+9=$0x3A9 = 0011,1010,1001

Show how to use shift, adding and/or subtracting to efficiently multiply *x* by the following numbers:

(e) (4 points) $(0011,1110)_2$

Sol: $x*(0011,1110)_2=x*(0100,0000 - 0000,0010)_2 = (x<<6) - (x<<1)$

Problem 2 (20 points) For each of the following values of *K*, find ways to express *x*K* using only the specified number of operations, where we consider both additions and subtractions to have comparable cost.

K	shifts	Add/Subs	Expression (5 points each)
28	2	1	*(x<<5)-(x<<2)*
63	1	1	*(x<<6)-x*
-12	2	1	*(x<<2)-(x<<4)*
97	2	2	*(x<<7)-(x<<5)+x or (x<<6)+(x<<5)+x*

Problem 3 (15 points)

Assume x = 0xFDC4 and y=0x79A8, what is the following value:

Sol:

x = 0xFDC4 = $(1111, 1101, 1100, 0100)_2$
~x = ~0xFDC4 = $(0000, 0010, 0011, 1011)_2$
y = 0x79A8 = $(0111, 1001, 1010, 1000)_2$
~y = ~0x79A8 = $(1000, 0110, 0101, 0111)_2$

(a) (2.5 points) ~*x* & *y*

~*x* = ~0xFDC4 = $(0000, 0010, 0011, 1011)_2$
y = 0x79A8 = $(0111, 1001, 1010, 1000)_2$
~x & y = $(0000, 0000, 0010, 1000)_2$ = 0x28

(b) (2.5 points) *x* | ~*y*

x = 0xFDC4 = $(1111, 1101, 1100, 0100)_2$
~y = ~0x79A8 = $(1000, 0110, 0101, 0111)_2$
x | ~*y* = $(1111,1111,1101,0111)_2$ = 0xFFD7

(c) (2.5 points) ~*x* & ~*y*

~x = ~0xFDC4 = $(0000, 0010, 0011, 1011)_2$
~y = ~0x79A8 = $(1000, 0110, 0101, 0111)_2$
~*x* | ~*y* = $(0000, 0010, 0001, 0011)_2$ = 0x213

(d) (2.5 points) *x* && *y*

x && *y* = TURE && TRUE = TRUE =0x1

(e) (2.5 points) *x* &! *y*

x &! *y* = *x* & !TRUE = *x* & FALSE = *x* & 0 = 0

(f) (2.5 points) *x* ||~ *y*

x ||~ *y* = TRUE || TRUE = TRUE= 0x1

Problem 4 (18 points)

(a) (6 points) Use an 10-bit word, find the binary representation of -456 in

i) (2 points) two's complement

ii) (2 points) one's complement

iii) (2 points) sign-magnitude

Sol:

$456=(01,1100,1000)_2= N$

i) $N^*=(10,0011,1000)_2$

ii) $\overline{N} = (10,0011,0111)_2$

iii) $N =(11,1100,1000)_2$

(b) (6 points) Repeat the above question using a 12-bit word.

Sol: $456=(0001,1100,1000)_2= N$

i) $N^*=(1110,0011,1000)_2$

ii) $\overline{N} = (1110,0011,0111)_2$

iii) $N =(1001,1100,1000)_2$

(c) (3 points) Assume that x, y, and z are unsigned 11-bit integers. Suppose x = 925, y = 536, and $z = x + y$. What is the value of z?

Sol: $z= x+y = (x+y) \bmod 2^{11}=(925+536) \bmod 2^{11} = 1461$

(d) (3 points) Assume that x, y, and z are two's complement 11-bit integers. Suppose x = 925, y = 536, and $z = x + y$. What is the value of z?

Sol: $z=x+y= 1461 > 2^{10}-1= 1023$ use N^* to represent $-N$

$N = 2^{11}-N^*=2048 - 1461 = 587$

Therefore z is $N^*= -587$

Problem 5 (27 points) Consider a hypothetical 10-bit IEEE floating point representation:

s (1 bit)	exp (5 bits)	frac (4 bits)

a) (3 points) What is the bias?

b) (3 points) How many different values can be represented with 10 bits?

c) (3 points) What is the smallest positive normalized value?

d) (3 points) What is the largest positive normalized value?

e) (3 points) What is the largest positive denormalized value?

f) (3 points) What is the floating-point representation for -1.0?

g) (3 points) What is the floating-point representation for 3.0?

Sol:

a) bias = $2^{k-1}-1=2^{5-1}-1=2^4-1=15$

b) $2^{10}=1024$

c) $V = (-1)^s \times M \times 2^E$, s=0: positive value,

$E = \text{exp-bias} = (00001)_2-15=1-15=14$

$M=1+\text{frac}\times 2^{-n}=1+(0000)\times 2^{-4}=1.0$

smallest normalized positive value = $1.0*2^{-14}=1/(2^{14})$

d) largest normalized positive value

s = 0, exp!=00000 and exp!=11111, $\text{exp}=(11110)_2=16+8+4+2=30$

E=exp-bias=30-15=15, frac = $(1111)_2$

$M = 1 + \text{frac} \times 2^{-4}=1+0.1111=1.1111=1+1/2+1/4+1/8+1/16=29/16$

Therefore, $V = (-1)^0\times 29/16\times 2^E=29/16\times 2^{15}$

$= (1.1111)\times 2^{15}=(1111,1000,0000,0000)_2$

(e) largest denormalized value

s = 0, exp =00000, frac = 1111

$M=\text{frac}\times 2^4=0.111$, E=1-bias=1-15=14;

$V = M\times 2^E=0.1111\times 2^{-14}=(00, 0000, 0000, 0000, 1111)_2$

f) $V = -1.0= (-1)^1\times 1.0\times 2^{-0}$

s = 1, M = 1.0, E=0

E = 0 = exp - bias => exp = bias = 15 = $(01111)_2$

$M = 1 + \text{frac}\times 2^{-4}=1.0$ => frac = $(0000)_2$

Therefore, 1.0 = s k=5 n=4

s	k=5	n=4
1	01111	0000

e) V= 3.0 so s = 0

$V = (-1)^s\times M\times 2^E= 3.0$ => $1.5\times 2^1= M\times 2^E$ => M =1.5 and E = 1.0

$M = 1 + \text{frac}\times 2^{-4}=(1.1000)_2$ => frac = 1000

E = 1.0 = exp - Bias => exp = Bias + E = 15+1=16 => exp = 1,0000

Therefore, 3.0 = s k=5 n=4

s	k=5	n=4
0	10000	1000