

You may use a calculator and one sheet of notes on this exam, but no other materials and no computer.

This test has a full score of 110 points. Answer question worth 100 points or more. The exam will be graded for a maximum score of 100 points. Show all the major steps in your work to receive partial credits.

Problem 1 (24 points)

(a) (4 points) Convert to decimal: 0x4F9

Sol: $0x4F9 = 4 \times 16^2 + 15 \times 16 + 9 = 1273$

(b) (4 points) Convert to binary: 0xC9D = 1100,1001,1101

(c) (4 points) Convert from decimal to hexadecimal: 1834

Sol: $1834 = 7 \times 16^2 + 2 \times 16 + 10 = 0x72A$

(d) (4 points) Convert from decimal to binary: 1834

Sol: $1834 = 0111,0010,1010$

Show how to use shift, adding and/or subtracting to efficiently multiply the following numbers:

(e) (4 points) 159

Sol: $x \times 159 = x \times (128 + 32 - 1) = x \times (2^7 + 2^5 - 1) = (x \ll 7) + (x \ll 5) - x$

Alternative solution:

$$x \times 159 = x \times (128 + 16 + 8 + 4 + 2 + 1) = (x \ll 7) + (x \ll 4) + (x \ll 3) + (x \ll 2) + (x \ll 1) + x$$

(f) (4 points) $(0111,1110)_2$

Sol: $x \times (0111,1110)_2 = x \times [(1000,000)_2 - 2] = x \times (2^7 - 2^1) = (x \ll 7) - (x \ll 1)$

Problem 2 (12 points)

Assume $x = 0x45$ and $y = 0xF8$, what is the following value:

(a) (2 points) $x \& y$

Sol: $x = 0100,0101$

$y = 1111,1000$

$$x \& y = 0100,0000 = 0x40$$

(b) (2 points) $x / y = 1111,1101 = 0xFD$

(c) (2 points) $\sim x \& \sim y$

$$\sim x = 1011,1010$$

$$\sim y = 0000,0111$$

$$\sim x \& \sim y = 0000,0010 = 0x2$$

(d) (2 points) $x \&\& y$

Sol: $x \&\& y = \text{TRUE} \&\& \text{TRUE} = \text{TRUE} = 0x1$

(e) (2 points) $!x \& y$

Sol: $!x \& y = !\text{TRUE} \& y = \text{FALSE} \& y = 0x0 \& y = 0x0$

(f) (2 points) $\sim x // y$

Sol: $\sim x // y = \text{TRUE} // \text{TRUE} = \text{TRUE} = 0x1$

Problem 3 (20 points)

(a) (10 points) Use an 8-bit word, find the binary representation of -47 in

i) (4 points) two's complement

ii) (3 points) one's complement

iii) (4 points) sign-magnitude

Sol: $N=47 = (0010,1111)_2$

Its two's complement $N^* = 1101,0001$ and one's complement $1101,0000$.

Its sign-magnitude is $1010,1111$.

(b) (10 points) Repeat the above question using a 12-bit word.

Sol: Its two's complement $N^* = 1111,1101,0001$ and one's complement $1111,1101,0000$.

Its sign-magnitude is $1000,0010,1111$.

Problem 4 (30 points)

Determine the output of the following code segment (without running it):

Assume that a short is represented by 7 bits and an int is represented by 12 bits. What is the output generated by the following code segment:

```
int x = 63;
int y = -63;
short sx = (short) x;
short sy = (short) y;
printf("%d , %d, %d, %d\n", x, y, (int)sx, (int)sy); (10 points)
printf("%x, %x, %x, %x\n", x, y, (int)sx, (int)sy); (10 points)
printf("%u, %u, %u, %u\n", x, y, (int)sx, (int)sy); (10 points)
```

You may use a calculator to generate the values, but you must show you calculated them.

Sol: $63 = 32 + 31 = (11,1111)_2$

$\text{int } x = 0000,0011,1111 = 0x3F$

$\text{int } y = 1111,1100,0001 = 0xFC1$ its unsigned number is $2^{12} - 63 = 4096 - 63 = 4033$

$sx = 011,1111 = 0x3F$

$sy = 100,0001 = 0x41 = -[011,1111] = -63$

$(\text{int}) sx = 0000,0011,1111 = 0x3F = 63$

$(\text{int}) sy = 1111,1100,0001 = 0xFC1 = -63$ its unsigned number is 4033.

(a) 63, -63, 63, -63

(b) 0x3F, 0xFC1, 0x3F, 0xFC1

(c) 63, 4033, 63, 4033

Problem 5 (24 points)

Consider a hypothetical 8-bit IEEE floating point representation:

```
-----
| s | exp ( 3 bits) | frac (4 bits) |
-----
```

- (4 points) What is the bias?
- (4 points) How many different values can be represented with 8 bits?
- (4 points) What is the smallest positive normalized value?
- (4 points) What is the largest positive normalized value?
- (4 points) What is the largest positive denormalized value?
- (4 points) What is the floating-point representation for 1.0?

Sol: $k = 3$ and $n = 4$

a) $Bias = 2^{k-1} - 1 = 2^{3-1} - 1 = 3$

b) $2^8 = 256$

c) $exp = 001$ $frac = 0000$

$$M = 1 + frac \times 2^{-n} = 1.0 \quad E = exp - Bias = 1 - 3 = -2$$

$$V = (-1)^S \times M \times 2^E = 1 \times 1.0 \times 2^{-2} = (.01)_2 = \frac{1}{4}$$

d) $exp = 110$, $frac = 1111$

$$M = 1 + frac \times 2^{-n} = 1.1111 \quad E = exp - Bias = 6 - 3 = 3$$

$$V = (-1)^S \times M \times 2^E = 1 \times 1.1111 \times 2^3 = (1111.1)_2 = \frac{31}{2}$$

e) $exp = 000$, $frac = 1111$

$$M = frac \times 2^{-n} = .1111 \quad E = 1 - Bias = 1 - 3 = -2$$

$$V = (-1)^S \times M \times 2^E = 1 \times .1111 \times 2^{-2} = (.001111)_2 = \frac{8+4+2+1}{64} = \frac{15}{64}$$

f) $1 = 1.0 \times 2^1 = M \times 2^E$

$$M = 1.0 = 1 + frac \times 2^{-n} \Rightarrow frac = 0000$$

$$exp = E + Bias = 0 + 3 = 3 \Rightarrow exp = 011$$

So its floating-point representation is 0 011 0000