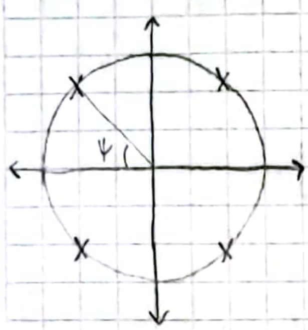
TAREA SEMANAL N° 7

(2)

Butter orden 2 $\quad F_c = 1\,\mathrm{KHz}$

(a) $F_s = 100\,\mathrm{kHz}$



$$\Psi = \frac{\pi}{4} \qquad Q = \frac{1}{2\cos\left(\frac{\pi}{4}\right)} = 0{,}707$$

$$T(s) = \frac{\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} \qquad F_B(z) = k\,\frac{z-1}{z+1}$$

$$T(z) = \frac{\omega_0^2}{k^2\frac{(z-1)^2}{(z+1)^2} + k\frac{z-1}{z+1}\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(z) = \frac{(z+1)^2\,\omega_0^2}{k^2(z^2-2z+1) + k(z^2-1)\frac{\omega_0}{Q} + \omega_0^2(z^2+2z+1)}$$

$$= \frac{(z+1)^2\,\omega_0^2}{z^2\left(k^2 + k\frac{\omega_0}{Q} + \omega_0^2\right) + z\left(2\omega_0^2 - 2k^2\right) + k^2 - k\frac{\omega_0}{Q} + \omega_0^2}$$

$F_s = 100\,\mathrm{kHz} \longrightarrow k = 200\,\mathrm{kHz} \qquad \omega_0 = 1\,\mathrm{kHz}$

NORMA $\begin{cases} \omega_0 = 0{,}01 \\ \omega_s = 1 \\ k = 2 \end{cases}$

$F_s = 100\,\mathrm{kHz}$
$$T(z) = \frac{z^2 + 2z + 1}{z^2 \cdot 4{,}028 - z \cdot 7{,}9998 + 3{,}972}$$

$F_s = 10\,\mathrm{kHz}$
$$T(z) = \frac{z^2 + 2z + 1}{z^2 \cdot 4{,}293 - z\,7{,}98 + 3{,}727}$$

NORMA $\begin{cases} \omega_0 = 0{,}1 \\ \omega_s = 1 \\ k = 2 \end{cases}$

(c)

$$T_2(s) = T_1(s)\Big|_{s=\frac{1}{s}} = \frac{s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T_2(z) = \frac{k^2\frac{(z-1)^2}{(z+1)^2}}{k^2\frac{(z-1)^2}{(z+1)^2} + k\frac{z-1}{z+1}\frac{\omega_0}{Q} + \omega_0^2}$$

NORMA $\begin{cases} \omega_0 = 0{,}06 \\ \omega_s = 1 \\ k = 2 \end{cases}$

$$T_2(z) = \frac{k^2(z-1)^2}{z^2\left(k^2 + k\frac{\omega_0}{Q} + \omega_0^2\right) + z\left(2\omega_0^2 - 2k^2\right) + k^2 - k\frac{\omega_0}{Q} + \omega_0^2}$$

NORMA $\begin{cases} \omega_0 = 0{,}6 \\ \omega_s = 1 \\ k = 2 \end{cases}$

$F_s = 100\,\mathrm{kHz}$
$$T_2(z) = \frac{4z^2 - 8z^2 + 4}{z^2 \cdot 4{,}17 - z\,7{,}9928 + 3{,}83}$$

$$\frac{\omega_0}{s + \omega_0}$$

$F_s = 10\,\mathrm{kHz}$
$$T_2(z) = \frac{4z^2 - 8z^2 + 4}{z^2 \cdot 6{,}057 - z\,7{,}22 + 2{,}66}$$

$$\frac{\omega_0}{k\frac{(z-1)+z\omega_0+\omega_0}{z+1}} = \frac{\omega_0(z+1)}{z(k+\omega_0) + \omega_0 - k}$$

NOTA

③
- Transferencia
- Plano PZK
- Módulo y fase

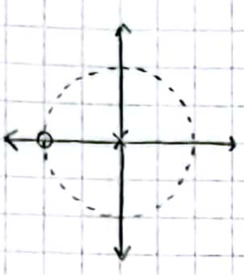
ⓐ $h_1(k) = (1, 1) \qquad h_2(k) = (1, 1, 1)$

- $h_1(k) \longrightarrow Y(k) = X(k) + X(k-1) \xrightarrow{Z} Y(z) = X(z) + X(z) \cdot z^{-1}$

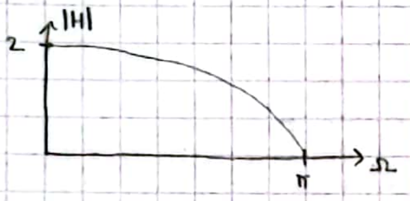
salida act — Muestra act — Muestra anterior

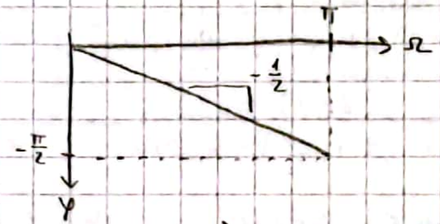
$$\frac{Y(z)}{X(z)} = z^{-1} + 1$$

$$H(z) = \frac{z+1}{z}$$



$$H(z)\Big|_{z=e^{j\Omega}} = \frac{e^{j\Omega} + e^{j0}}{e^{j\Omega}} = e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} + e^{-j\frac{\Omega}{2}}\right) = \underbrace{e^{-j\frac{\Omega}{2}}}_{\text{FASE}} \cdot \underbrace{2\cos(\Omega/2)}_{\text{MÓDULO}}$$

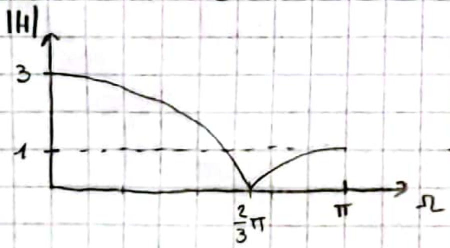


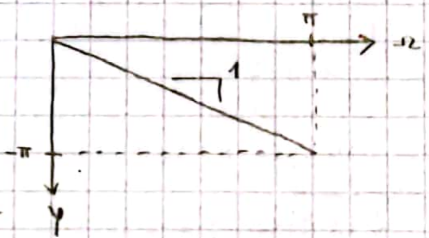


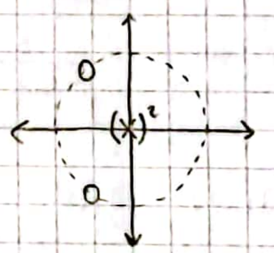
- $h_2(k) \longrightarrow Y(k) = X(k) + X(k-1) + X(k-2) \xrightarrow{Z} Y(z) = X(z)\left(1 + z^{-1} + z^{-2}\right)$

$$\frac{Y(z)}{X(z)} = \frac{z^2 + z + 1}{z^2} \longrightarrow -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j$$

$$H(e^{j\Omega}) = \frac{e^{j2\Omega} + e^{j\Omega} + e^{j0}}{e^{j2\Omega}} = e^{-j\Omega}\left(1 + 2\cos(\Omega)\right)$$





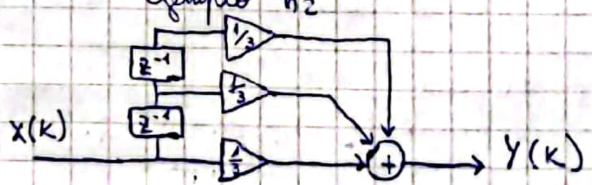


ⓑ Como la salida es la suma de entradas pasadas y presente, habría que dividir por el número de muestras

Ejemplo h_2

$$h_1(k) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right)$$

$$h_2(k) = \left(\tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}\right)$$



ⓒ $\pi \longrightarrow F_s/2 \qquad \Omega_0 = 2 \cdot tg^{-1}\left(\omega_0 / F_s \cdot 2\right)$

$2\pi/3 \longrightarrow F_s/3 \qquad tg\left(\frac{\frac{2\pi}{3}}{2}\right) = \frac{\omega_0}{2F_s}$ $\omega_0 \rightarrow 2\pi\, 50$

$$\left[F_s = 90{,}69\ Hz\right]$$

NOTA

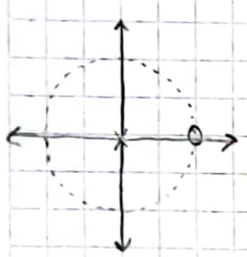
(b)

$h_1(k) = (1, -1)$ $\qquad$ $h_2(k) = (1, 0, -1)$

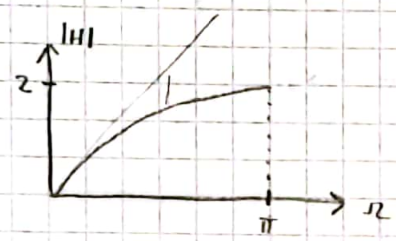
- $h_1(k) = (1, -1)$

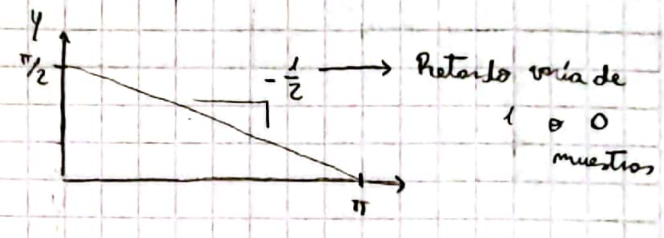
$Y(k) = X(k) - X(k-1) \xrightarrow{Z} Y(z) = X(z) - X(z) \cdot z^{-1}$

$$H(z) = \frac{z - 1}{z}$$



$$H(e^{j\Omega}) = e^{-j\frac{\Omega}{2}}\left(e^{j\frac{\Omega}{2}} - e^{-j\frac{\Omega}{2}}\right) = e^{-j\frac{\Omega}{2}} \cdot \text{sen}\left(\frac{\Omega}{2}\right) \cdot 2j$$



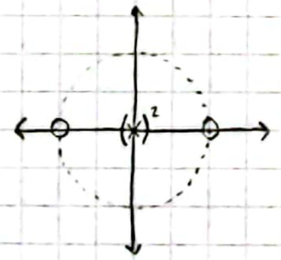


$$\left.\begin{aligned} 2\,\text{sen}\left(\frac{\Omega}{2}\right) &= \Omega - 0{,}05\,\Omega \\ 2\,\text{sen}\left(\frac{\Omega}{2}\right) &= \Omega \cdot 0{,}95 \\ \text{sen}\left(\frac{\Omega}{2}\right) &= \Omega \cdot 0{,}475 \end{aligned}\right\} \Omega = 1{,}077$$

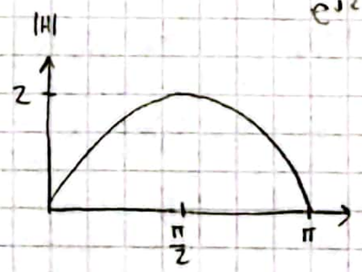
- $h_2(k) = (1, 0, -1)$

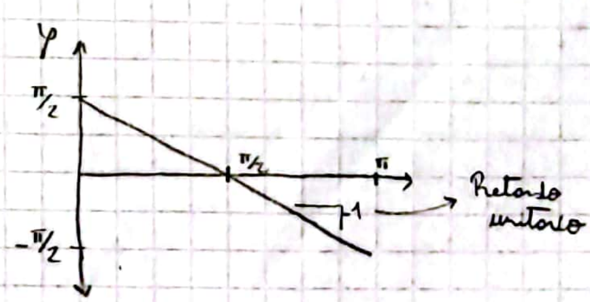
$Y(k) = X(k) - X(k-2) \xrightarrow{Z} Y(z) = X(z) - X(z) \cdot z^{-2}$

$$H(z) = \frac{z^2 - 1}{z^2}$$



$$H(e^{j\Omega}) = \frac{e^{j2\Omega} - e^{j0}}{e^{j2\Omega}} = e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right) = e^{-j\Omega}\, 2\,\text{sen}(\Omega) \cdot j$$





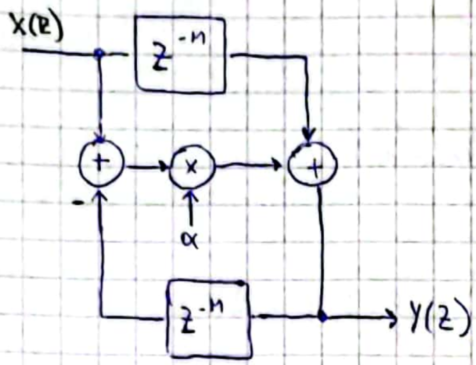
1,953

$$2\,\text{sen}(\Omega) = \Omega - 0{,}05\,\Omega$$

$\downarrow$

$$\Omega = 1{,}953$$

(2)



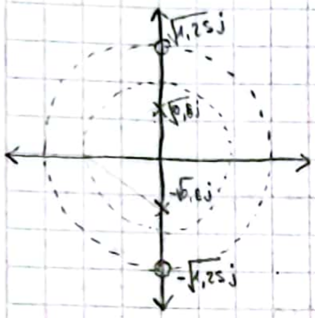
(a) $n=2 \quad \alpha=0{,}8$

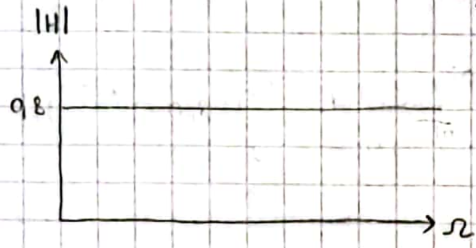
$$\left(X(z)-Y(z)\,z^{-2}\right)\alpha + X(z)\cdot z^{-2} = Y(z)$$

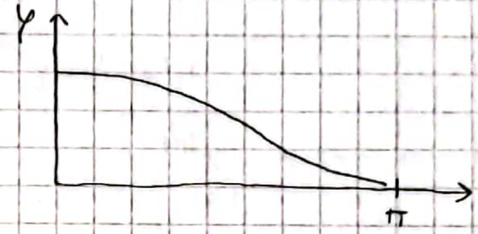
$$Y(z) + Y(z)\cdot z^{-2}\alpha = \alpha X(z) + X(z)\cdot z^{-2}$$

$$\frac{Y(z)}{X(z)} = \frac{\alpha + z^{-2}}{1 + z^{-2}\alpha} = \frac{z^2\cdot\alpha + 1}{z^2 + \alpha}$$

$$H(z) = \alpha\,\frac{z^2 + 1/\alpha}{z^2 + \alpha} = 0{,}8\,\frac{z^2 + 1{,}25}{z^2 + 0{,}2}$$







(b)

$$H(z) = \alpha\,\frac{z^4 + 1/\alpha}{z^4 + \alpha}$$

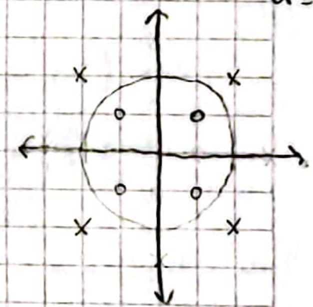
$$\sqrt[4]{-\tfrac{1}{\alpha}} \longrightarrow \sqrt[4]{\tfrac{1}{\alpha}}\cdot e^{j\frac{\pi}{4}}$$

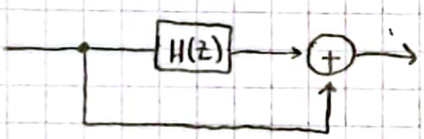
$$\sqrt[4]{\tfrac{1}{\alpha}}\cdot e^{j\frac{3}{4}\pi}$$

$$\sqrt[4]{\tfrac{1}{\alpha}}\cdot e^{j\frac{5}{4}\pi}$$

$$\sqrt[4]{\tfrac{1}{\alpha}}\cdot e^{j\frac{7}{4}\pi}$$

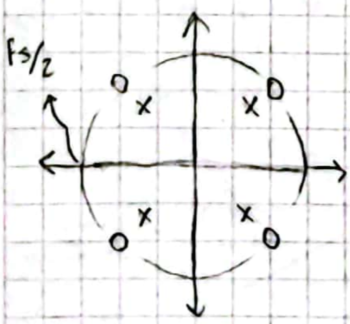
$\alpha=0{,}8$





$$H_2(z) = \frac{\alpha z^4 + 1 + z^4 + \alpha}{z^4 + \alpha} = \frac{z^4(\alpha+1) + 1 + \alpha}{z^4 + \alpha}$$

$$H_2(z) = (\alpha+1)\,\frac{z^4 + 1}{z^4 + \alpha} \overset{\alpha=0{,}8}{\Longrightarrow} H_2(z) = 1{,}8\cdot\frac{z^4 + 1}{z^4 + 0{,}8}$$



Quiero cero en $F_s/8$ y $F_s\cdot 3/8$ $\qquad 125\cdot 3 = 375$ ✓

$$\frac{F_s}{8} = 125 \Rightarrow F_s = 1000 \;✓$$

$$\frac{3F_s}{8} = 375 \;✓$$

NOTA