

矩阵乘法

矩阵和向量相乘:

Example

$$\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}$$

$$1 \times 1 + 3 \times 5 = 16$$

$$4 \times 1 + 0 \times 5 = 4$$

$$2 \times 1 + 1 \times 5 = 7$$

3X2矩阵和2X1的向量相乘 得到 3X1的向量

Details:

$$\begin{array}{ccc} \underline{A} & \times & \underline{x} \\ \begin{array}{c} \text{matrix} \\ \text{with } m \text{ rows and } n \text{ columns} \end{array} & \times & \begin{array}{c} \text{vector} \\ \text{with } n \text{ elements} \end{array} \\ \begin{array}{c} \text{matrix} \\ (m \text{ rows, } n \text{ columns}) \end{array} & \times & \begin{array}{c} \text{matrix} \\ (n \times 1 \text{ matrix}) \\ \text{(n-dimensional vector)} \end{array} \\ \begin{array}{c} \text{matrix} \\ (m \times n \text{ matrix}) \\ \text{(m rows, n columns)} \end{array} & \times & \begin{array}{c} \text{matrix} \\ (n \times 1 \text{ matrix}) \\ \text{(n-dimensional vector)} \end{array} \\ \begin{array}{c} \text{matrix} \\ (m \times n \text{ matrix}) \\ \text{(m rows, n columns)} \end{array} & \times & \begin{array}{c} \text{matrix} \\ (n \times 1 \text{ matrix}) \\ \text{(n-dimensional vector)} \end{array} \end{array} = \begin{array}{c} \underline{y} \\ \text{vector} \\ \text{with } m \text{ elements} \end{array}$$

→ To get y_i , multiply A 's i^{th} row with elements of vector x , and add them up.

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矩阵和矩阵相乘:

Example

$$\begin{array}{l}
 \begin{array}{c} \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix} \\
 \textcircled{2 \times 3} \quad \textcircled{3 \times 2} \quad \textcircled{2 \times 2} \\
 \begin{array}{c} \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix} \\
 \textcircled{2 \times 3} \quad \textcircled{3 \times 1} \quad \textcircled{2 \times 1} \\
 \begin{array}{c} \begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix} \\
 \textcircled{2 \times 3} \quad \textcircled{3 \times 1} \quad \textcircled{2 \times 1} \end{array}
 \end{array}$$

Details:

$$\begin{array}{c}
 \underline{A} \quad \times \quad \underline{B} \quad = \quad \underline{C} \\
 \left[\begin{array}{c} \\ \\ \end{array} \right] \times \left[\begin{array}{c|c|c|c} & & & \end{array} \right] = \left[\begin{array}{c|c|c|c} & & & \end{array} \right] \\
 \textcircled{m \times n} \text{ matrix} \quad \textcircled{n \times o} \text{ matrix} \quad \textcircled{m \times o} \text{ matrix} \\
 (m \text{ rows, } n \text{ columns}) \quad (n \text{ rows, } o \text{ columns}) \quad (m \text{ rows, } o \text{ columns})
 \end{array}$$

The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B . (for $i = 1, 2, \dots, o$)

Example

$$\begin{aligned}
 &\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 4 \\ 15 & 12 \end{bmatrix} \\
 &\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \times 0 + 3 \times 3 \\ 2 \times 0 + 5 \times 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix} \\
 &\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 3 \times 2 \\ 2 \times 1 + 5 \times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}
 \end{aligned}$$

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House sizes:

$$\begin{bmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{bmatrix}$$

Have 3 competing hypotheses:

$$\begin{aligned}
 1. & h_{\theta}(x) = -40 + 0.25x \\
 2. & h_{\theta}(x) = 200 + 0.1x \\
 3. & h_{\theta}(x) = -150 + 0.4x
 \end{aligned}$$

Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix}$$

Matrix

$$\begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

Prediction of first h_{θ}

Predictions of 2nd h_{θ}

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矩阵乘法特征:

1. 矩阵乘法不服从交换律, 不可交换
2. 矩阵乘法服从结合律 $m \times n \times i \times j = m \times j$

单位矩阵:

Identity Matrix

1 is identity.

$$1 \times z = z \times 1 = z$$

for any z

Denoted I (or $I_{n \times n}$).

Examples of identity matrices:

$$[1]_{1 \times 1}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2 \times 2}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3 \times 3}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$$

Informally:

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}$$

For any matrix A ,

$$A \cdot I = I \cdot A = A$$

$m \times n$ $n \times n$ $m \times m$ $m \times n$ $m \times n$

$$I_{n \times n}$$

Note:

$$AB \neq BA \text{ in general}$$

$$AI = IA \checkmark$$

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单位矩阵和其他矩阵的乘法符合交换律