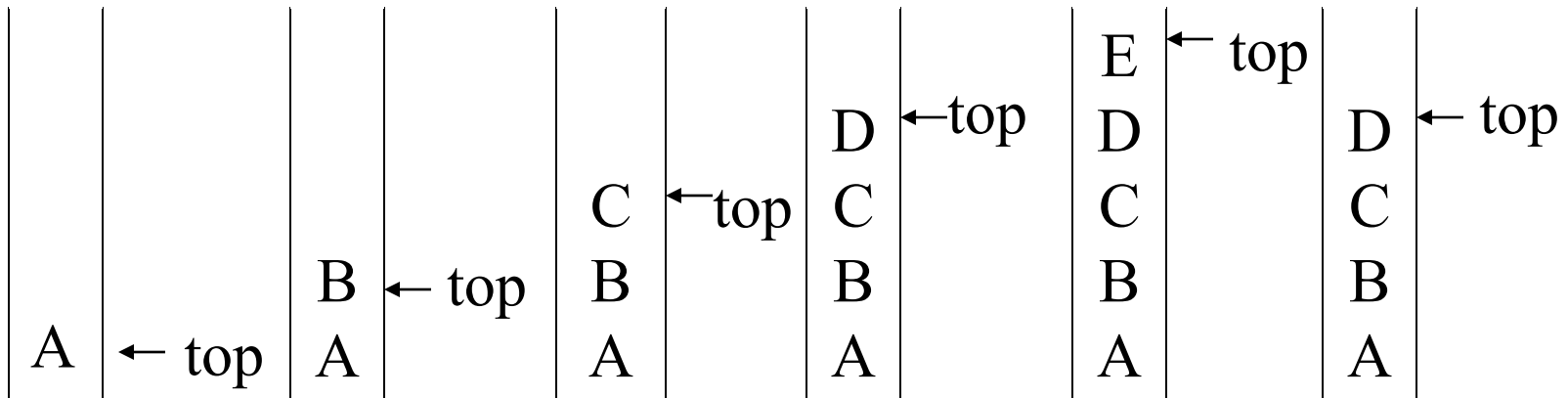


CHAPTER 3

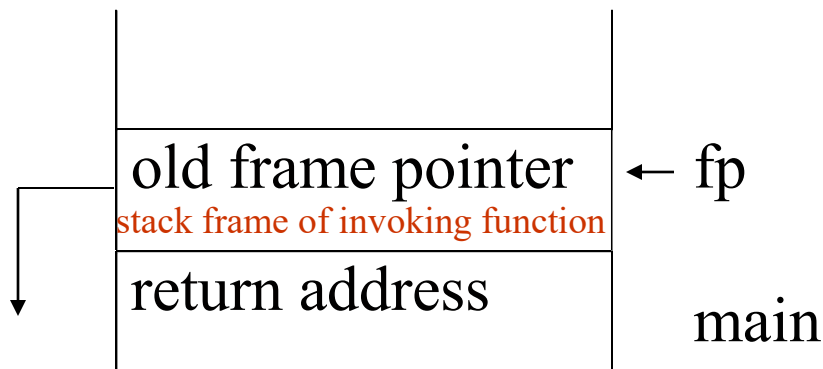
STACKS AND QUEUES

stack: a Last-In-First-Out (LIFO/FILO) list



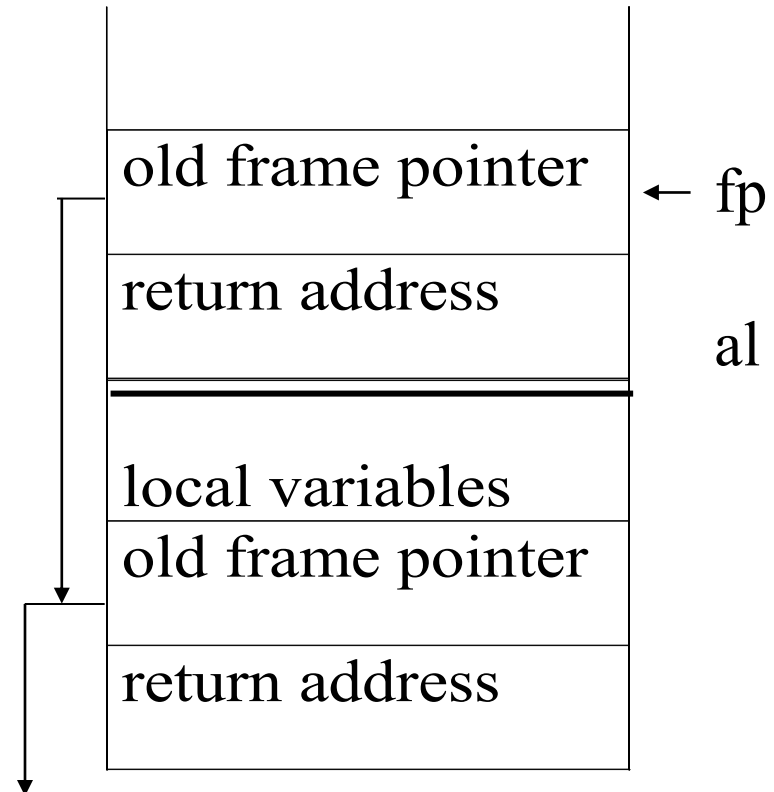
an application of stack: stack frame of function call (activation record)

fp: a pointer to current stack frame



system stack **before** **a1** is invoked

(a)



system stack **after** **a1** is invoked

(b)

System stack after function call **a1**

abstract data type for stack

structure *Stack* is

objects: a finite ordered list with zero or more elements.

functions:

for all $stack \in Stack$, $item \in element$, $max_stack_size \in \text{positive integer}$

Stack CreateS(max_stack_size) ::=

create an empty stack whose maximum size is max_stack_size

Boolean IsFull($stack$, max_stack_size) ::=

if (number of elements in $stack == max_stack_size$)
return TRUE
else return FALSE

Stack Push($stack$, $item$) ::=

if (IsFull($stack$)) $stack_full$
else insert $item$ into top of $stack$ and **return**



Boolean IsEmpty(*stack*) ::=

if(*stack* == CreateS(*max_stack_size*))

return TRUE

else return FALSE

Element Pop(*stack*) ::=

if(IsEmpty(*stack*)) **return**

else remove and return the *item* on the top
of the stack.

Abstract data type *Stack*

Catalan Number

假設有 n 筆資料，依序執行push，中間可穿插pop，試問其合法的排列組合有多少種可能？

$$\frac{\binom{2n}{n}}{n+1}$$

Implementation: using array

Stack CreateS(max_stack_size) ::=

```
#define MAX_STACK_SIZE [1..n] /* maximum stack size */
```

```
typedef struct {
```

```
    int key;
```

```
    /* other fields */
```

```
    } element;
```

```
element stack[MAX_STACK_SIZE];
```

```
int top = 0;
```

Boolean IsEmpty(Stack) ::= top == 0;

Boolean IsFull(Stack) ::= top == MAX_STACK_SIZE;


Add to a stack (Push)

```
void push(int *top, element item)
{
    /* add an item to the global stack */
    if (*top == MAX_STACK_SIZE) {
        stack_full( );
        return;
    }
    stack[++*top] = item;
}
```

← **top=top+1**
***top=item**

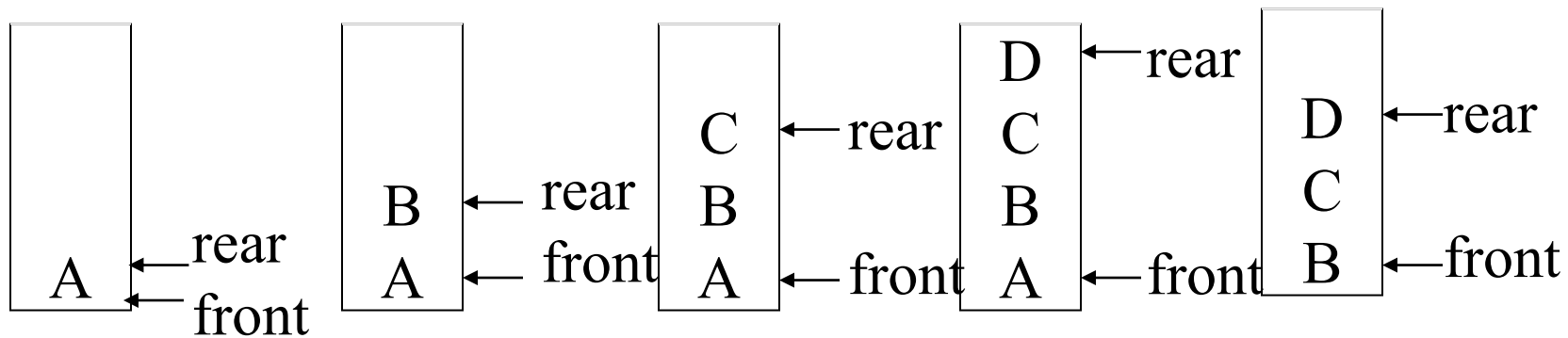
Delete from a stack (Pop)

```
element pop(int *top)
{
    /* return the top element from the stack */
    if (*top == 0)
        return stack_empty( ); /* returns and error key */
    return stack[(*top)--];
}
```



***top=item
top=top-1**

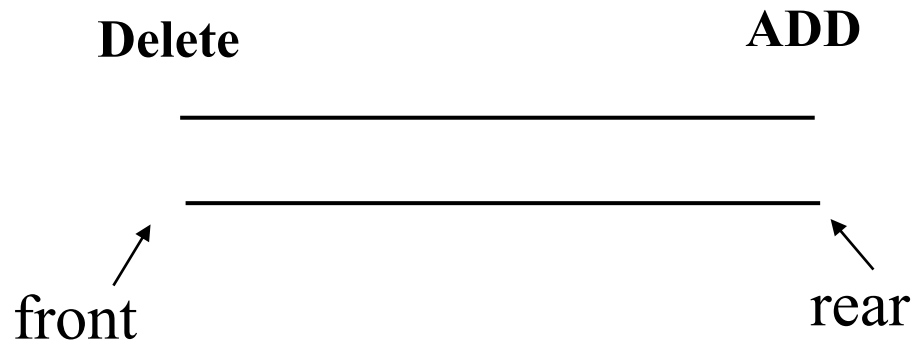
Queue: a First-In-First-Out (FIFO/LILO) list



- (1) 插入元素(Add) – Rear 端
- (2) 删除元素>Delete) – Front 端

Different Queue

- (1) FIFO Queue
- (2) Priority Queue
- (3) Double-ended Queue
- (4) Double-ended Priority Queue



Application: Job scheduling

front	rear	Q[0]	Q[1]	Q[2]	Q[3]	Comments
-1	-1					queue is empty
-1	0	J1				Job 1 is added
-1	1	J1	J2			Job 2 is added
-1	2	J1	J2	J3		Job 3 is added
0	2		J2	J3		Job 1 is deleted
1	2			J3		Job 2 is deleted

***Figure 3.5:** Insertion and deletion from a sequential queue (p.108)

Abstract data type of queue

structure *Queue* is

objects: a finite ordered list with zero or more elements.

functions:

for all $queue \in Queue$, $item \in element$,

$max_queue_size \in \text{positive integer}$

Queue CreateQ(max_queue_size) ::=

create an empty queue whose maximum size is

max_queue_size

Boolean IsFullQ($queue$, max_queue_size) ::=

if (number of elements in $queue == max_queue_size$)

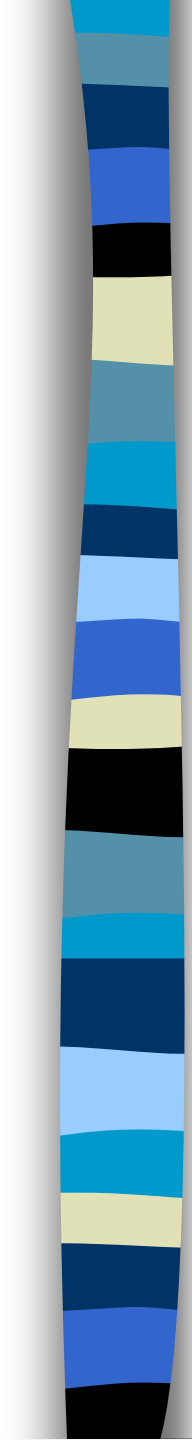
return *TRUE*

else return *FALSE*

Queue AddQ($queue$, $item$) ::=

if (IsFullQ($queue$)) $queue_full$

else insert $item$ at rear of $queue$ and return $queue$



Boolean IsEmptyQ(*queue*) ::=

if (*queue* == CreateQ(*max_queue_size*))

return *TRUE*

else return *FALSE*

Element DeleteQ(*queue*) ::=

if (IsEmptyQ(*queue*)) **return**

else remove and return the *item* at front of queue.

Abstract data type (*Queue*)

Implementation 1: using array

```
Queue CreateQ(max_queue_size) ::=  
# define MAX_QUEUE_SIZE [1...n]  
typedef struct {  
    int key;  
    /* other fields */  
} element;  
element queue[MAX_QUEUE_SIZE];  
int rear = 0;  
int front = 0;
```

Add to a queue

```
void addq(int *rear, element item)
{
    /* add an item to the queue */
    if (*rear == MAX_QUEUE_SIZE) {
        queue_full( );
        return;
    }
    queue [++*rear] = item;
}
```

Add to a queue

Delete from a queue

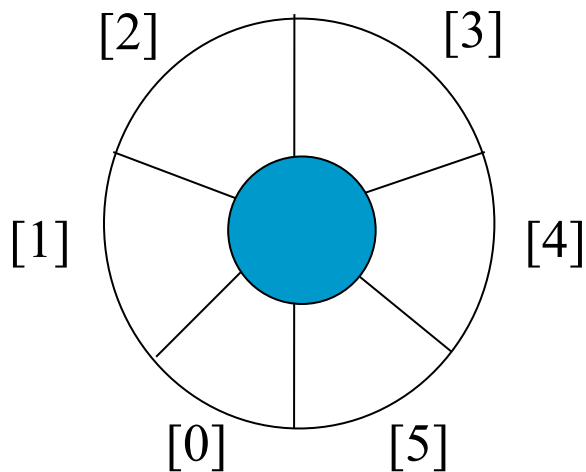
```
element deleteq(int *front, int rear)
{
    /* remove element at the front of the queue */
    if ( *front == * rear)
        return queue_empty( );    /* return an error key */
    else
        return queue [++ *front];
}
```

Delete from a queue

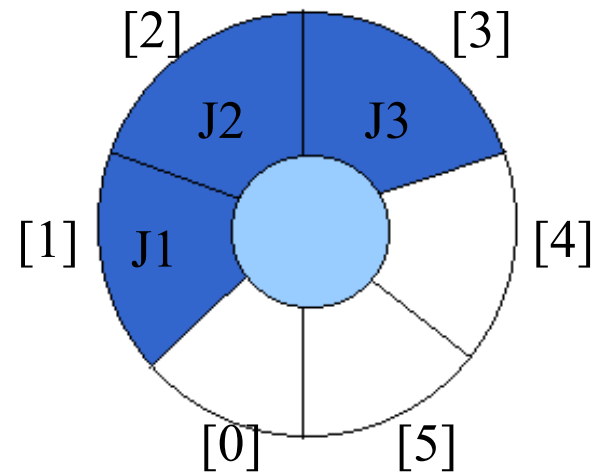
problem: there may be available space when IsFullQ is true, i.e., movement is required.

Implementation 2: regard an array as a circular queue

EMPTY QUEUE



front = 0
rear = 0



front = 0
rear = 3

Empty and nonempty circular queues

Create a circular queue

```
Queue CreateQ(max_queue_size) ::=  
# define MAX_QUEUE_SIZE [0...n-1]  
typedef struct {  
    int key;  
    /* other fields */  
} element;  
element queue[MAX_QUEUE_SIZE];  
int rear = 0;  
int front = 0;
```

Add to a circular queue

```
void addq(int front, int *rear, element item)
{
    /* add an item to the queue */
    *rear = (*rear + 1) % MAX_QUEUE_SIZE;
    if (*front == *rear) then queue_full( );
    return;
}
queue[*rear] = item;
}
```

Add to a circular queue

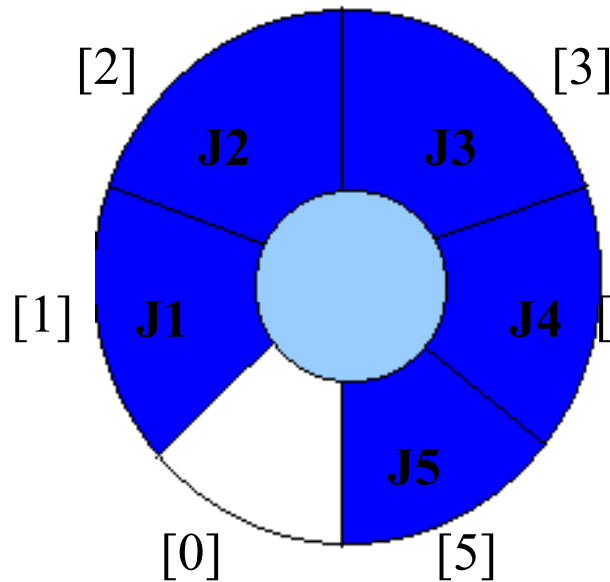
Delete from a circular queue

```
element deleteq(int* front, int rear)
{
    element item;
    /* remove front element from the queue and put it in item */
    if (*front ==* rear) return queue_empty( );
        /* queue_empty returns an error key */
    else
        *front = (*front+1) % MAX_QUEUE_SIZE;
    return queue[*front];
}
```

Delete from a circular queue

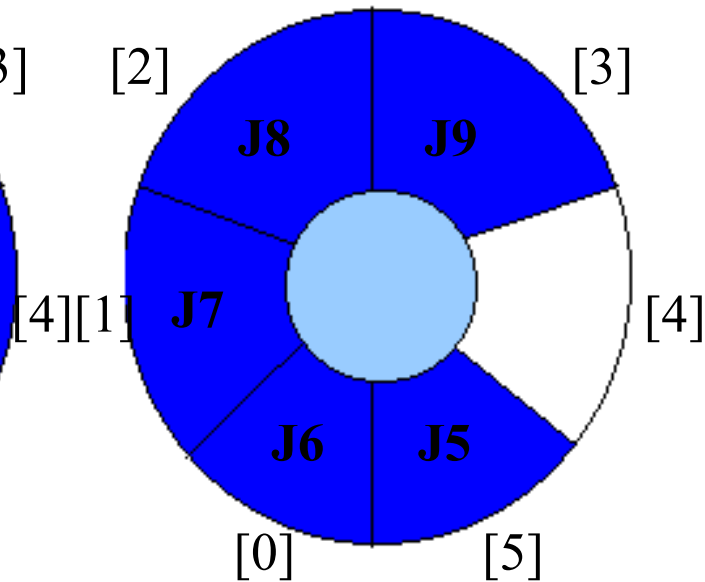
Problem: one space is left when queue is full

FULL QUEUE



front = 0
rear = 5

FULL QUEUE



front = 4
rear = 3

Full circular queues and then we remove the item

Create a circular queue (with n spaces used)

```
Queue CreateQ(max_queue_size) ::=  
# define MAX_QUEUE_SIZE [0...n-1]  
typedef struct {  
    int key;  
    /* other fields */  
} element;  
element queue[MAX_QUEUE_SIZE];  
int rear = 0;  
int front = 0;  
boolean tag = 0
```

Add to a circular queue (with n spaces used)

```
void addq(int front, int *rear, element item)
{
    if (*rear==*front) and tag =1) then queue_full();
    else
        *rear = (*rear +1) % MAX_QUEUE_SIZE;
        queue[*rear]=item
        if (* front == *rear) tag=1;
}
```

Add to a circular queue

Delete from a circular queue (with n spaces used)

```
element deleteq(int* front, int rear)
{
    element item;
    if ((*front==*rear) and (tag=0)) then queue_empty();
    else
        *front = (*front+1) % MAX_QUEUE_SIZE;
        return queue[*front];
        if (*front==*rear) then tag=0;
}
```

Delete from a circular queue

Evaluation of Expressions

$$X = a / b - c + d * e - a * c$$

$$a = 4, b = c = 2, d = e = 3$$

Interpretation 1:

$$((4/2)-2)+(3*3)-(4*2)=0 + 8+9=1$$

Interpretation 2:


$$(4/(2-2+3))*(3-4)*2=(4/3)*(-1)*2=-2.66666\cdots$$

How to generate the machine instructions
corresponding to a given expression?

precedence rule + associative rule

Token	Operator	Precedence ¹	Associativity
() [] -> .	function call array element struct or union member	17	left-to-right
-- ++	increment, decrement ²	16	left-to-right
-- ++ ! - - + & * sizeof	decrement, increment ³ logical not one's complement unary minus or plus address or indirection size (in bytes)	15	right-to-left
(type)	type cast	14	right-to-left
* / %	mutiplicative	13	Left-to-right

+ -	binary add or subtract	12	left-to-right
<< >>	shift	11	left-to-right
> >= < <=	relational	10	left-to-right
== !=	equality	9	left-to-right
&	bitwise and	8	left-to-right
^	bitwise exclusive or	7	left-to-right
	bitwise or	6	left-to-right
&&	logical and	5	left-to-right
⊠	logical or	4	left-to-right

?:	conditional	3	right-to-left
= += -= /= *= %= <<= >>= &= ^= 	assignment	2	right-to-left
,	comma	1	left-to-right

- 1.The precedence column is taken from Harbison and Steele.
- 2.Postfix form
- 3.prefix form

***Figure 3.12:** Precedence hierarchy for C (p.119)

user

compiler

Infix	Postfix
2+3*4	234*+
a*b+5	ab*5+
(1+2)*7	12+7*
a*b/c	ab*c/
(a/(b-c+d))*(e-a)*c	abc-d+/ea-*c*
a/b-c+d*e-a*c	ab/c-de*ac*-

*Figure 3.13: Infix and postfix notation (p.120)


Postfix: no parentheses, no precedence

Infix to Postfix Conversion (Intuitive Algorithm)

- (1) Fully parenthesize expression

$$a / b - c + d * e - a * c \rightarrow$$
$$((((a / b) - c) + (d * e)) - a * c))$$

- (2) All operators replace their corresponding right parentheses.

$$((((a / b) - c) + (d * e)) - a * c))$$


- (3) Delete all parentheses.

$$ab/c-de^*+ac^*-$$

two passes

Infix	Prefix
$a*b/c$	<u>$/*abc$</u>
$a/b-c+d*e-a*c$	<u>$-+-/abc*de*ac$</u>
$a*(b+c)/d-g$	<u>$-/*a+bcdg$</u>

(1) evaluation

(2) transformation

*Figure 3.17: Infix and postfix expressions (p.127)