Q3

	①	②	③	④	⑤
X	(0,1)	(1,1)	(1,0)	(1,-1)	(0,-1)

Distance

① & ② $\sqrt{(0-1)^2+(1-1)^2} = 1$ ①'s closest neighbor ~~X~~ ② ③

① & ③ $\sqrt{(0-1)^2+(1-0)^2} = \sqrt{2}$

① & ④ $\sqrt{(0-1)^2+(1-(-1))^2} = \sqrt{5}$

① & ⑤ $\sqrt{(0-0)^2+(1-(-1))^2} = 2$

② & ③ $\sqrt{(1-1)^2+(1-0)^2} = 1$ ②'s cloest neighht ① and ③

② & ④ $\sqrt{(1-1)^2+(1-(-1))^2} = 2$.

② & ⑤ $\sqrt{(1-0)^2+(1-(-1))^2} = \sqrt{5}$.

③ & ④ $\sqrt{(1-1)^2+(0-(-1))^2} = 1$ ③'s cloest neighbor. ② and ④.

③ & ⑤ $\sqrt{(1-0)^2+(0-(-1))^2} = \sqrt{2}$.

④ & ⑤ $\sqrt{(1-0)^2+(-1-(-1))^2} = 1$ ④'s cloest neigbor ③ and ~~④~~ ⑤

⑤'s cloest neighbor. ③ ~~④~~ and ④

$$\hat{w} = \arg\min \sum_{i=1}^{m} \| x^{(i)} - \sum_{j=1}^{m} w_{i,j} x^{(j)} \|^2$$

$$\cancel{\| x_i - \sum_j w_{ij} x_j \|^2 + \alpha \sum_j w_{ij}^2}$$

$RSS_i = \| \sum_j w_{ij} (\vec{x}_j - \vec{x}_i) \|^2 = \| \sum_j w_{ij} \vec{z}_j \|^2.$

We have Z_i for each RSS_i

$Z_1 = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$ $\quad w_1 = \begin{pmatrix} w_{12} \\ w_{13} \end{pmatrix}$

$Z_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ $\quad w_2 = \begin{pmatrix} w_{21} \\ w_{23} \end{pmatrix}$

$Z_3 = \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}$ $\quad w_3 = \begin{pmatrix} w_{32} \\ w_{34} \end{pmatrix}.$

$Z_4 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$ $\quad w_4 = \begin{pmatrix} w_{43} \\ w_{45} \end{pmatrix}.$

$Z_5 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}.$ $\quad w_5 = \begin{pmatrix} w_{53} \\ w_{54} \end{pmatrix}.$

$RSS_1 = w_i^T G_i w_1$ where. $G_i = ZZ^T$

$L(w_i \lambda) = w_i^T G_i w_i - \lambda(1^T w - 1).$

$\frac{\partial L}{\partial w_i} = 2G_i w_i - \lambda 1 = 0$

$\frac{\partial L}{\partial \lambda} = 1^T w_i - 1 = 0$

$w_1 = \frac{\lambda}{2} G_i^{-1} 1$

we have. G_i^{-1} and.

and in w_i $\sum_1^2 w_{ij} = 1$ we can slove. w and λ.

$G_1 = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$ $G_1^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$ $G^{-1} I = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\begin{pmatrix} w_{12} \\ w_{13} \end{pmatrix} = \lambda \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\begin{cases} w_{12} = 2\lambda. \\ 1 - w_{12} = 3\lambda \end{cases}$ $\begin{cases} \lambda = \frac{1}{5} \\ w_{12} = \frac{2}{5} \\ w_{13} = \frac{3}{5} \end{cases}$

$G_2 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$ $G_2^{-1} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$ $\begin{cases} \lambda = \frac{1}{2} \\ w_{21} = \frac{1}{2} \\ w_{23} = \frac{1}{2} \end{cases}$

$G^{-1} I = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$G_3 = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{pmatrix} w_{32} \\ w_{34} \end{pmatrix} = \begin{matrix} \lambda^* \\ \lambda^* \end{matrix}$

$2w_{34} = \lambda^* = 0$

$w_{34} = 0 \quad w_{32} = 1$

$G_4 = \begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$ $\begin{cases} w_{43} = \frac{1}{2} \\ w_{45} = \frac{1}{2} \end{cases}$

$G_5 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $G_5^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ $G_5^{-1} I = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$w_{54} = 1$

$w_{53} = 0$

$$M = (I-W)^T(I-W) = \begin{bmatrix} 1.52 & -0.6 & -0.2 & 0.3 & 0 \\ -0.6 & 1.5 & -1.5 & 0.25 & 0 \\ -0.2 & -1.5 & 2 & -0.5 & 0 \\ 0.3 & 0.25 & -0.5 & 1.5 & -1.5 \\ 0 & 0 & 0 & -1.5 & 2 \end{bmatrix}$$

find the second smallest eigenvalue : 0.1239

eigenvector $\begin{bmatrix} 0.1539 \\ 0.0419 \\ -0.1523 \\ -0.7611 \\ -0.6085 \end{bmatrix}$

$\begin{bmatrix} 0.1539 \\ 0.0419 \\ -0.1523 \\ -0.7611 \\ -0.6085 \end{bmatrix}$ is the answer!