$L_D = \sum \lambda_i - \frac{1}{2}\sum_i \sum_j \lambda_i \lambda_j y_i y_j \langle \vec{x_i}, \vec{x_j} \rangle - \mu(\lambda_1 + \lambda_2 - \lambda_3 - \lambda_4)$

$(x_1 x_1) = 2 \quad (x_2 x_2) = 2 \quad (x_3 x_3) = 2 \quad (x_4 x_4) = 2 \quad (x_1 x_2) = -2 \quad (x_1 x_3) = 0$

$(x_1, x_4) = 0 \quad (x_2, x_3) = 0 \quad (x_2, x_4) = 0 \quad (x_3, x_4) = -2$

$\frac{\partial L_D}{\partial \lambda_1} = 1 - \frac{1}{2} \times 2 \times \lambda_1 \times 2 - \frac{1}{2} \times 2 \times \lambda_2 \times -2 \underset{\hat{\mu}}{=} 0 \Rightarrow 1 - 2\lambda_1 + 2\lambda_2 \overset{\mu}{\hat{=}} 0$ ①

$\frac{\partial L_D}{\partial \lambda_2} = 1 - \frac{1}{2} \times 2 \times \lambda_2 \times 2 - \frac{1}{2} \times 2 \times \lambda_1 \times -2 \underset{\hat{\mu}}{=} 0 \Rightarrow 1 - 2\lambda_2 + 2\lambda_1 \overset{\mu}{\hat{=}} 0$ ②

$\frac{\partial L_D}{\partial \lambda_3} = 1 - \frac{1}{2} \times 2 \times \lambda_3 \times 2 - \frac{1}{2} \times 2 \times \lambda_4 \times -2 \underset{\hat{\mu}}{=} 0 \Rightarrow 1 - 2\lambda_3 + 2\lambda_4 \overset{+\mu}{\hat{=}} 0$ ③

$\frac{\partial L_D}{\partial \lambda_4} = 1 - \frac{1}{2} \times 2 \times \lambda_4 \times 2 - \frac{1}{2} \times 2 \times \lambda_3 \times -2 \underset{\hat{\mu}}{=} 0 \Rightarrow 1 - 2\lambda_4 + 2\lambda_3 \overset{+\mu}{\hat{=}} 0$ ④

from ① and ②, we have $\mu = 1$

from ③ and ④, we have $\mu = -1$. Hence no solution.

Case 2 set. $\lambda_1 = 0$ then $\lambda_2 - \lambda_3 - \lambda_4 = 0$

$$\left.\begin{aligned} &\frac{\partial L_D}{\partial \lambda_2} = 1 - 2\lambda_2 - \mu = 0 \\ &\frac{\partial L_D}{\partial \lambda_3} = 1 - 2\lambda_3 + 2\lambda_4 + \mu = 0 \\ &\frac{\partial L_D}{\partial \lambda_4} = 1 - 2\lambda_4 + 2\lambda_3 + \mu = 0 \end{aligned}\right\} \mu = -1, \quad \lambda_2 = 1, \quad \lambda_3 = \lambda_4 = \frac{1}{2}$$

$\frac{\partial^2 L_D}{\partial^2 \lambda_2} = -2 < 0$

so $\lambda_2 = 1$ max at that point. $Ld(\ ) = \cancel{4}\ 1$

In Case 3, ~~Case 4~~ and ~~Ca~~: set $\lambda_2 = 0$ then $\lambda_1 - \lambda_3 - \lambda_4 = 0$

~~We have a solution~~

solution $\begin{cases} \mu = -1 \\ \lambda_1 = 1 \\ \lambda_3 = \lambda_4 = \frac{1}{2} \end{cases}$ $\quad \frac{\partial^2 L_D}{\partial^2 \lambda_1} = -2 < 0$ max

$L(d) = \cancel{4}\ 1$

Case 4 $\lambda_3=0$, $\lambda_1+\lambda_2-\lambda_4=0$

Solution $\begin{cases}\mu=1\\ \lambda_4=1\\ \lambda_1=\lambda_2=\frac{1}{2}\end{cases}$ $\qquad \dfrac{\partial^2 L_D}{\partial^2\lambda^2}=-2<0$ max

Case 5 $\lambda_4=0$ $\lambda_1+\lambda_2-\lambda_3=0$

Solution $\begin{cases}\mu=1\\ \mu_3=1\\ \lambda_1=\lambda_2=\frac{1}{2}\end{cases}$ $\qquad \dfrac{\partial^2 L_D}{\partial\lambda^2}=-2<0$ max

Compare each case, case 1 has no solution. but case 2 to case 5 has the solution $L_d=1$.

for case 2, solution. $w=1\times1\times\begin{pmatrix}-1\\-1\end{pmatrix}+\frac{1}{2}\times(-1)\times\begin{pmatrix}-1\\1\end{pmatrix}+\frac{1}{2}\times(-1)\times\begin{pmatrix}1\\-1\end{pmatrix}$

$=\begin{pmatrix}-1\\-1\end{pmatrix} \Rightarrow (w,x_2)+b=1. \Rightarrow b=-1$

for case 3, solution $w=\begin{pmatrix}-1\\-1\end{pmatrix}\Rightarrow(w,x_1)+b=-1\Rightarrow b=-1.$

for cas 4, solution: $w=1\times-1\times\begin{pmatrix}-1\\1\end{pmatrix}+\frac{1}{2}\times1\times\begin{pmatrix}1\\1\end{pmatrix}+\frac{1}{2}\times1\times\begin{pmatrix}-1\\-1\end{pmatrix}$

$=\begin{pmatrix}1\\-1\end{pmatrix}\Rightarrow(w,x_2)+b=1\Rightarrow b=-1.$

for case 5, solution $w=\begin{pmatrix}-1\\1\end{pmatrix}\Rightarrow(w,x_1)+b=-1\Rightarrow b=-1.$

~~for since.~~

Also for each case $0\le\lambda_i\le C$, Hence, for each case $\varepsilon_i=0$

A A A A B B B B A A B B C C

1 2 3 4 5 6 7 8 9 10 11 12 13 14

① $x \le 1$

$H_1 = -\frac{1}{1}\log_2 1 = 0$

$G_1 = 1 - 1 = 0$

Misclass : 0

$x > 1$

$H_2 = -\frac{5}{13}\log_2\frac{5}{13} - \frac{6}{13}\log_2\frac{6}{13} - \frac{2}{13}\log_2\frac{2}{13} = 1.46$

$G_2 = [1 - (\frac{5}{13})^2 - (\frac{6}{13})^2 - (\frac{2}{13})^2] = 0.6154$

Missclass : $1 - \max(\frac{5}{13}, \frac{6}{13}, \frac{2}{13}) = \frac{7}{13}$

$H = \frac{1}{14}H_1 + \frac{13}{14}H_2$
$= 1.3557$

$G = \frac{1}{14}G_1 + \frac{13}{14}G_2$
$= 0.5714$
$2G = 1.1428$

$M: \frac{1}{14}\times 0 + \frac{13}{14}\times\frac{7}{13}$
$= 0.5$
$2M = 1$

② $x \le 2$

$H_1 = -\frac{2}{2}\log_2\frac{1}{2} = 0$

$G_1 = 1 - (\frac{2}{2})^2 = 0$

Misclass: 0

$x > 2$

$H_2 = -\frac{4}{12}\log_2\frac{4}{12} - \frac{6}{12}\log_2\frac{6}{12} - \frac{2}{12}\log_2\frac{2}{12} = 1.4591$

$G_2 = [1 - \frac{4^2}{12} - \frac{6^2}{12} - \frac{2^2}{12}] = 0.61$

Misclass: $1 - \max(\frac{4}{12}, \frac{6}{12}, \frac{2}{12}) = \frac{1}{2}$

$H = \frac{2}{14}H_1 + \frac{12}{14}H_2$
$= 1.2507$

$G = \frac{2}{14}G + \frac{12}{14}\times 0.61 = 0.5229$
$2G = 1.0458$

$M: \frac{12}{14}\times\frac{1}{2} = \frac{6}{14}$
$2M = \frac{12}{14}$

③ $x \le 3$

$H_1 = 0$

$G_1 = 0$

$M_1 = 0$

$x > 2$

$H_2 = -\frac{3}{11}\log_2\frac{3}{11} - \frac{6}{11}\log_2\frac{6}{11} - \frac{2}{11}\log_2\frac{2}{11} = 1.4354$

$G_2 = [1 - \frac{3^2}{11} - \frac{6^2}{11} - \frac{2^2}{11}] = 0.595$

$M \quad 1 - \max(\frac{3}{11}, \frac{6}{11}, \frac{2}{11}) = \frac{5}{11}$

$H = \frac{11}{14}H_2 = 1.1278$

$G = 0.595 \times \frac{11}{14} = 0.4675$

$2G = 0.935$

$2M = \frac{10}{14}$

④ $\cancel{H} \le x \le 4$

$H_1 = 0$

$G_1 = 0$

$M_1 = 0$

$x > 4$

$H_2 = -\frac{2}{10}\log_2\frac{2}{10} - \frac{6}{10}\log_2\frac{6}{10} - \frac{2}{10}\log_2\frac{2}{10} = 1.3710$

$G_2 = (1 - \frac{2}{10}^2 - \frac{6}{10}^2 - \frac{2}{10}^2) = 0.56$

$M_2 = 1 - \max(\frac{2}{10}, \frac{6}{10}, \frac{2}{10}) = \frac{4}{10}$

$H = \frac{10}{14} \times 1.3710$
$= 0.9793$

$G = \frac{10}{14} \times 0.56$
$= 0.4$

$2G = 0.8$

$M = \frac{10}{14} \times \frac{4}{10} = \frac{4}{14}$

$2M = \frac{4}{7}$

⑤ $x \le 5$

$H_1 = -\frac{4}{5}\log\frac{4}{5} - \frac{1}{5}\log_2\frac{1}{5}$
$= 0.7219$

$G_1 = 0.32$

$C = 0.2$

$x > 5$

$H_2 = \frac{-2}{9}\log_2\frac{2}{9} - \frac{5}{9}\log_2\frac{5}{9} - \frac{2}{9}\log_2\frac{2}{9} = \cancel{0.4533}\ 1.4355$

$G_2 = 0.5926$

$C_2 = 0.4444$

$H = \frac{5}{14}H_1 + \frac{9}{14}H_2$
$= 1.1806$

$G_2 = 0.4952$

$2G = 0.9904$

$M = 0.3571$

$2M = 0.7142$

⑥ $x \le 6$

$H_1 = 0.9183$

$G_1 = 0.4444$

$C_1 = 0.3333$

$H = 1.2507$

$x > 6$

$H_2 = 1.5$

$G_2 = 0.625$

$C_2 = 0.5$

$G = 0.5476$ $C = 0.4286$

$2G = 1.0952$ $2C = 0.8572$

⑦ $x \le 7$

$H_1 = 0.9852$

$G_1 = 0.4898$

$C = 0.4286$

$H = 1.2710$

$x > 7$

$H_2 = 1.5567$

$G_2 = 0.6531$

$C_2 = 0.5714$

$G = 0.5715$ $C = 0.5$

$2G = 1.1430$ $2C = 1$

$x \le 8$	$x > 8$		
$H_1 = 1$	$H_2 = 1.5850$	$H = 1.2507$	
$G_1 = 0.5$	$G_2 = 0.6667$	$G = 0.5714$	$2G = 1.1428$
$C_1 = 0.5$	$C_2 = 0.6667$	$C = 0.5714$	$2C = 1.1428$

$x \le 9$	$x > 9$		
$H_1 = 0.9911$	$H_2 = 1.5219$	$H = \cancel{1.2894}\ 1.1807$	
$G_1 = 0.4938$	$G_2 = 0.64$	$G = \cancel{0.5917}\ 0.546$	$2G = 1.092$
$C_1 = 0.4444$	$C_2 = 0.6$	$C = 0.5$	$2C = 1$

$x \le 10$	$x > 10$		
$H_1 = 0.97$	$H_2 = 1$	$H = 0.9786$	
$G_1 = 0.48$	$G_2 = 0.5$	$G = 0.4857$	$2G = \cancel{1.4}\ 0.97124$
$C_1 = 0.4$	$C_2 = 0.5$	$C = 0.4286$	$2C = 0.8572$

$x \le 11$	$x > 11$		
$H_1 = 0.994$	$H_2 = 0.9183$	$H = 0.9778$	
$G_1 = 0.4959$	$G_2 = 0.444$	$G = 0.4849$	$2G = 0.9698$
$C_1 = 0.4545$	$C_2 = 0.3333$	$C = 0.4285$	$2C = 0.8570$

$x \le 12$	$x > 12$	$H = \frac{12}{14}$	
$H_1 = 1$	$H_2 = 0$	$G = \cancel{0.5}\ \frac{6}{14}$	$2G = \frac{12}{14}$
$G_1 = 0.5$	$G_2 = 0$	$C = \frac{6}{14}$	$2C = \frac{12}{14}$
$C_1 = 0.5$	$C_2 = 0$		

$x \le 13$ $\qquad$ $x > 13$

$H_1 = 1.3143$ $\qquad$ $H_2 = 0$ $\qquad$ $H = 1.22$

$G_1 = 0.5680$ $\qquad$ $G_2 = 0$ $\qquad$ $G = 0.5274$ $\qquad$ $\Sigma G = 1.0548$

$C_1 = 0.5385$ $\qquad$ $C_2 = 0$ $\qquad$ $C = 0.5$ $\qquad$ $\Sigma C = 1$

b) 1) Learn from a) we already know the split position should be between 12 and 13 with min Shennon Entropy

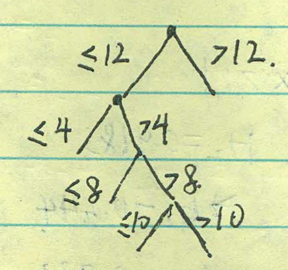
2) there's no need to split 13 and 14.
then, lets check the shennon Entropy for. 1 ~ 12.

A	A	A	A	B	B	B	B	A	A	B	B
1	2	3	4	5	6	7	8	9	10	11	12

0.91   0.81   0.69   0.54   0.80   0.92   0.9793   1   0.97   0.81   0.91

should break between 4 and 5.

The decision tree should look like that.



3) there's no need to separate 1~4.
consider 5~12

B	B	B	B	A	A	B	B
5	6	7	8	9	10	11	12

0.76   0.69   0.69   0.5   0.80   0.69   0.76

should split between 8 and 9.

4)

A	A	B	B
9	10	11	12

0.69   0   0.69   should split between 10, 11