Fast MAS Classification via Supervised Learning

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Abstract

We explore whether machine learning models can be used to approximate the Maximal Admissible Set (MAS) of constrained dynamical systems using data. We begin with a second-order linear system with a known polyhedral MAS and train separate linear and logistic regression models to reconstruct its boundaries. These models serve as a proof of concept, demonstrating that even basic supervised learning methods can approximate constraint boundaries when given appropriate data. Building on this, we propose a neural network classifier that predicts whether a point lies within the MAS in high-dimensional systems with complex admissibility regions, without explicit knowledge of the constraints.

1 Introduction and Problem Definition

The Maximal Admissible Set (MAS) of a stable Linear Time-Invariant (LTI) system defines the set of all initial conditions and constant inputs that guarantee the system's output remains within predefined constraints for all time. MAS is central to constraint management and forms the foundation of many set-theoretic control methods, including constrained model predictive control and safety filtering.

In low-dimensional systems, the MAS can be easily visualized and evaluated. For instance, in two-dimensional cases with a finite number of constraints, one can directly plot the set and check admissibility of a point by evaluating a small number of inequalities. However, in realistic high-dimensional systems—with hundreds of states and thousands of constraints—explicitly plotting the MAS becomes infeasible, and checking all constraints for each point is computationally prohibitive.

This creates a practical bottleneck: many control applications require real-time evaluation of whether a given state lies within the MAS. In high dimensions, repeatedly checking thousands of constraints becomes too slow for online use.

To address this challenge, we investigate whether machine learning can offer faster alternatives for approximating the MAS. As a first step, we consider a low-dimensional linear system with a finite number of constraints and attempt to reconstruct its MAS using linear and logistic regression. These models allow us to directly approximate the MAS boundaries using labeled data and serve as a foundation for more advanced methods.

Building on this, we then shift our focus to high-dimensional systems where the MAS cannot be easily evaluated. For such systems, we propose training a neural network classifier that takes a state vector as input and predicts whether it lies within the MAS. This bypasses the need to evaluate each constraint individually, enabling fast admissibility checks suitable for real-time applications.

*Use footnote for providing further information about author (webpage, alternative address)—*not* for acknowledging funding agencies.

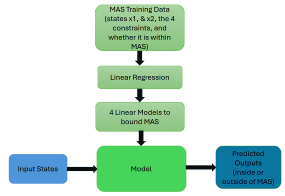

Figure 1: Linear Regression Block Diagram

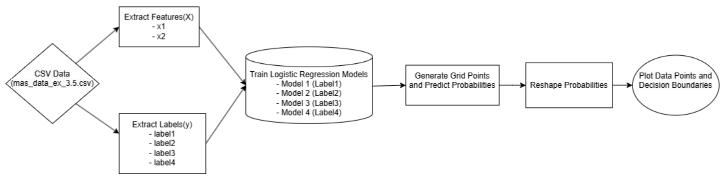

Figure 2: Logistic Regression Block Diagram

2 Related Work and Bibliography

The Maximal Admissible Set (MAS) was formally introduced in Gilbert and Tan's foundational work [2]. Their method allows for finite determination of the MAS for asymptotically stable, observable Linear Time-Invariant (LTI) systems and construction of inner approximations in cases of Lyapunov stability or lack of observability. While computing the MAS for general nonlinear systems is difficult, methods for estimating it for certain classes of nonlinear systems can be found in [1, 3]. A data-driven formulation of MAS was given in [4] where output predictors from the behavioral system theory and subspace predictive control literiture were levereged to formulate a data-driven version of MAS. It is shown that the proposed set is polytopic and has finite compplexity and reside in a higher dimensinal space. Across all these methods, a core challenge remains: efficient real-time evaluation of MAS membership in high-dimensional systems. In many control applications, the system must quickly determine whether a state is admissible—i.e., whether it lies within the MAS—without evaluating thousands of inequalities.

3 Model and Training Algorithm

3.1 Linear Regression

One of the models trained was a linear regression model. This model trains on generated MAS data. This data has two input states and four constraints to meet. If it meets all the constraints, it is within the MAS. A model was trained for each of the four constraints creating four linear boundaries. After training on the data the models form a boundary that should be able to predict whether a new set of state is within the MAS(1) or outside of the MAS(0).

3.2 Logistic Regression

The second model we used was a logistic regression model. This model also trains on pregenerated MAS data. This data again has two input states and four constraints to meet. The label of each constraint is stored as a binary value for whether it is negative(1) or positive(0). We then run a logistic regression model for each constraint's label using the input as the X features and the binary values as y. Given the labels from the constraints, a certain data point will fall within the MAS if all the binary labels are 1, or they are negative solutions out of the constraint equations. The produced logistic regression model can now accurately predict whether a new set of states falls within the MAS or not.

4 Dataset

Our dataset was generated synthetically in MATLAB to simulate the Maximal Admissible Set (MAS) for constrained linear systems. The initial experiments were conducted on a two-state LTI system with input and state constraints specifically example 3.5 of [2] represented by a polyhedral MAS defined by four linear inequalities. These inequalities were derived analytically using Algorithm 3.2 introduced in [2].

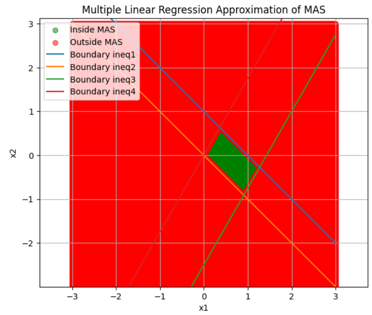

Figure 3: MAS approximation using multiple linear regression.

To construct the dataset, we sampled a uniform grid of state values (x_1, x_2) within a fixed bounding box and evaluated each of the four MAS constraints for every point. For each sample, we recorded the input features x_1 and x_2, the values of each inequality evaluated at that point, a binary label for each constraint indicating whether it was satisfied (1) or not (0), and a final binary label `inside_MAS`, which is set to 1 if the point satisfies all constraints and thus lies within the MAS.

To simulate real-world complexity, we plan to extend this approach to high-dimensional systems. In particular, we are developing a version with a 50-dimensional state space and 1000 randomly generated linear inequality constraints, where the MAS can no longer be visualized or explicitly represented. In such settings, it is infeasible to check constraint satisfaction analytically or visualize the admissible region. Instead, we will rely on a combination of random state sampling and constraint evaluation to label data points, and then train a neural network to predict MAS membership.

5 Experimental Evaluation

5.1 Evaluation Methodology

To evaluate the accuracy of our models in approximating the MAS, we use both analytical and data-driven baselines. For the low-dimensional example system, we apply Algorithm 3.2 [2] to exactly compute the polyhedral MAS. This allows us to obtain the ground-truth constraint inequalities that define the MAS boundaries. Using these inequalities, we generate labeled data to train and evaluate linear and logistic regression models. Since the true MAS is known in this case, we can directly compare the learned boundaries to the actual constraints and assess the accuracy of each model in reconstructing the MAS geometry.

We will elaborate on the performance metrics, visualization, and comparisons in detail for the high dimensional caseonce we complete the experimental phase of the project.

5.2 Results

We begin by computing the true Maximal Admissible Set (MAS) for a two-dimensional system using Algorithm 3.2 [2]. This algorithm provides an exact polyhedral description of the MAS for stable LTI systems with linear state and input constraints. The resulting MAS serves as ground truth for evaluating our regression-based approximations. As shown in Figure 3, the linear regression models closely approximate the true MAS boundaries. Next, we applied multiple logistic regression, this time treating each inequality as a binary classification task. Figure 4 shows that logistic regression is also able to recover the MAS boundaries exactly.

5.3 Discussion

Our hypothesis is supported. Both linear and logistic regression models were able to exactly reconstruct the maximal output admissible set. This outcome is attributed to the linear structure of the constraints defining the MAS. Further discussion and analysis involving neural network models will be provided in the next report following additional experiments.

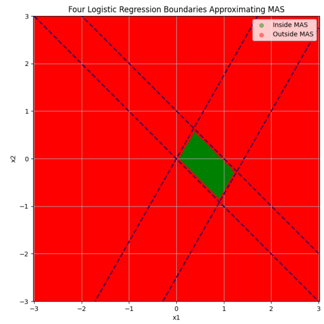

Figure 4: MAS approximation using multiple logistic regression.

6 Next Steps

Currently, we are working with a second-order system subject to only four constraints. While linear and logistic regression methods have proven effective and straightforward for this case, they lack the capacity to scale with increasing complexity. As we aim to extend our approach to higher-order systems with more constraints, a more expressive model becomes necessary. To address this, we propose developing a neural network capable of predicting whether a given input lies within the Maximal Output Admissible Set (MAS) for high-dimensional systems

7 Code

Stay tuned for the final report to see our code!

8 Student Roles

Farzan: Farzan is in charge of the big idea planning, as it is his research. He has been working to ensure that our plans are feasible and that they align with a common goal of his research.

Jack and David: As Computer Science students in the group, they have been in charge of taking the ideas and putting them to the keyboard (i.e., coding the work). They have been and will be responsible for creating the program and the neural network that we intend to build to build a system with many more than just 4 constraints (possibly even hundreds).

9 Conclusion

Our initial results using linear and logistic regression demonstrate that a second-order system with four constraints is easily solvable. Both models successfully predict whether a given set of inputs lies within the maximal output admissible set (MAS), as shown in the accompanying figures. These early experiments confirm the feasibility of our approach. In our final report, we will extend this work by implementing a neural network to handle more complex, higher-dimensional systems and evaluate its performance in identifying admissible regions.

References

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