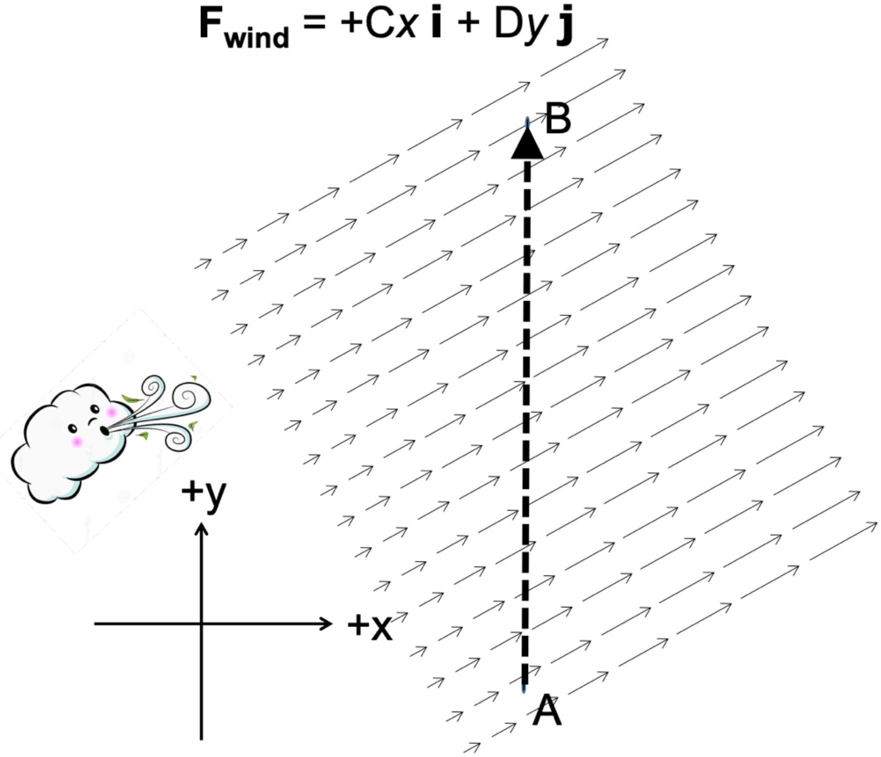
2. **KEY IDEA: Integral add up tiny bits of work to get the total work.**
So now we see that the dot product just takes the **parallel components** to find the work done as an object travels some displacement**.** But how can we calculate it when the path or the force is a more complicated function?

Let's start by considering a more complicated force. Let's say there is wind force blowing at an angle, and the force increases with position, as shown. An object moves in a straight line from A to B.

$$\mathbf{F}_{\text{wind}} = +Cx\,\mathbf{i} + Dy\,\mathbf{j}$$



a. In general (not just for this path), what is **dr** in terms of dx, dy, and the 2D cartesian unit vectors?

$\vec{dr} = dx\,\hat{i} + dy\,\hat{j}$

b. Now let's look at this path. Draw a tiny displacement vector **dr** for some portion of the path shown. For **this** path, what does **dr** simplify to?

bold letter '' vector

$\vec{dr} = dy\,\hat{j}$

c. Use the function of the wind force shown. What is $\vec{\mathbf{F}} \cdot \vec{\mathbf{dr}}$ for this force and path?

$dW = \vec{F}\cdot\vec{dr} = (Cx)(0) + (Dy)(dy)$

d. This (**F** · **dr**) is just a tiny bit of work done as the object moves a tiny displacement **dr**. Is this (**F** · **dr**) a vector anymore, once you execute the dot product?

No, because you are getting a scalar from the dot product

e. To get the total work, we just add up all the tiny bits of work, dW. That's where the integral comes in! Say that point A is at (x, y) = (0, 0) m and B is at (0, 5) m. Use: $W_{ab} = \int_a^b \mathbf{F} \cdot \mathbf{dr}$ to add up all the tiny bits of work done, so we get the total work done by the force along the path. Execute this integral to find the work done by the wind force as an object moves from point A to point B. Put you answer in terms of the positive constants ~~C_1~~ C and/or ~~C_2~~ D.

$dW = Dy\,dy$

$dW = \int_0^5 Dy\,dy = \frac{D}{2}y^2 + C = \left(\frac{D}{2}(5)^2\right) - \cancel{\left(\frac{D}{2}(0)^2\right)}$

$\left(\frac{D(5)^2}{2}\right)$

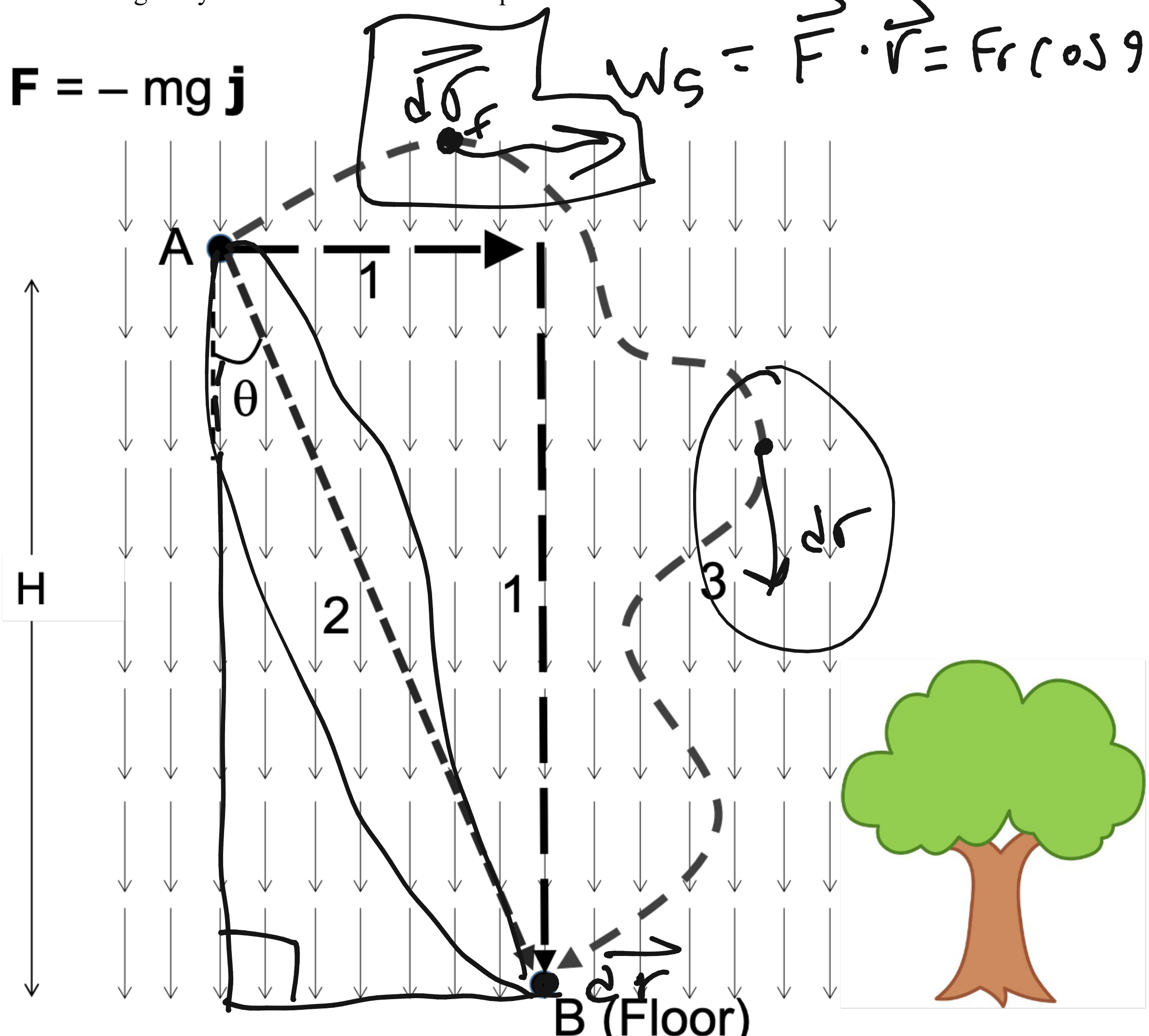
$\frac{25D}{2}$

Visualizations of the Dot Product in Vector Force Fields:

Below are three activities designed to help illustrate the integral of a dot product, such as in: $W_{ab} = \int_a^b \mathbf{F} \cdot \mathbf{dr}$.

1. **KEY IDEA: Dot product takes parallel components.** Consider the gravitational force near the surface of the Earth. It is constant and points downward, as indicated by the vector lines shown.
   Say that an object of mass m is moved from point A to point B along three different paths, also as shown. The vertical distance between the points is *H*.
   a. What is the work done by gravity along the first segment of path 1? Why?
   0, gravity isn't doing anything
   b. What is the work done by gravity along the second segment of path 1?
   $Fr\cos 90°$
   c. What is the net work done by gravity along path 1?
   $Fr\cos(0°) = mgh$
   d. What is the net work done by gravity along path 2?
   mgh
   e. Draw the tiny displacement vector **dr** at a point in path 3 along which the work done by gravity is **maximum**. Circle this vector **dr**.
   f. Then draw a second tiny displacement vector **dr** at a point in path 3 along which the work done by gravity is close to zero. Draw a square box around this vector **dr**.

$W_g = \vec{F} \cdot \vec{r} = Fr\cos 90°$



$W_g = \vec{F} \cdot \vec{r} = (-mg)\underbrace{r_y}_{-h} = mgh$