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Exact analytical solution for current flow through diode with series resistance

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A simple analytical expression is presented for the current flow in a diode driven by a voltage source through a series resistance. The proposed solution is based on the Lambert W-function. The new expression leads to an efficient method for extracting series resistance from measured current-voltage data. Experimental results are presented which validate the proposed solution and extraction method.

Introduction: The current flow I in a diode driven by a voltage source V through a series resistance R_s is governed by the KVL and diode junction equations

$$V_d = V - IR_s \tag{1}$$

$$I = I_{sat}\left(e^{V_d/\eta V_T} - 1\right) \tag{2}$$

respectively, where V_d is the diode junction voltage drop, I_{sat} is the reverse saturation current, V_T represents the thermal voltage and η is the junction ideality factor. The solution of these equations is called the generalised diode equation. Many diode and transistor circuits that have local feedback can be reduced to this model through a Thevenin equivalent circuit. Several approximate solutions have been described using rational approximations [1 – 3]. An explicit expression for $I(V)$ cannot be constructed from just the common elementary functions [4]. The desired analytical solution for the generalised diode equation was found in terms of the Lambert W-function.

Methods: Substitution of eqn. 1 into eqn. 2 gives

$$\frac{(I+I_{sat})R_s}{\eta V_T}e^{(I+I_{sat})R_s/\eta V_T} = \frac{I_{sat}R_s}{\eta V_T}e^{(V+I_{sat}R_s)/\eta V_T} \tag{3}$$

following separation of terms in I and V, respectively. The left hand side of eqn. 3 is an explicit function of current I through the quantity $w = (I + I_{sat})R_s/\eta V_T$ but not the excitation voltage V. Eqn. 3 is a transcendental equation of the form $we^w = x$. The solution of this transcendental equation is a multi-valued function $w = W_k(x)$ referred to as the Lambert W-function [5]. Although not well known in electrical engineering, this function has been thoroughly studied. The desired branch for the generalised diode equation is $k = 0$, which satisfies $W_0(x) = 0$ for $x = 0$ [5]. An explicit solution of eqn. 3 is given by

$$I(V) = \frac{\eta V_T}{R_s}W_0\left(\frac{I_{sat}R_s}{\eta V_T}e^{(V+I_{sat}R_s)/\eta V_T}\right) - I_{sat} \tag{4}$$

with $x = (I_{sat}R_s/\eta V_T)e^{(V+I_{sat}R_s)/\eta V_T}$. Several efficient methods exist for calculating $W(x)$ [6, 7]. A graph of the real valued solutions $W_0(x)$ and $W_{-1}(x)$ are shown in Fig. 1. Several features deserve mention. There is a branch point of order two at $x = -1/e$. W assumes complex values for $x < -1/e$. Asymptotic expansion gives $W_0(x) \sim \ln(x)$ for $x \to \infty$ and $W_{-1}(x) \sim \ln(-x)$ for $x \to 0^-$. There is a single real value for $x > 0$. The origin on the upper branch corresponds to $R_s = 0$. The first derivative is useful in several calculations and satisfies $dW(x)/dx = (W/1+W)(1/x)$. It has a removable singularity at the origin. Eqn. 4 can be further simplified when the diode is forward biased and $I \gg I_{sat}$, in which case

$$I(V) = \frac{\eta V_T}{R_s}W_0\left(\frac{I_o R_s}{\eta V_T}\right) \tag{5}$$

where $I_o(V) \equiv I_{sat}e^{V/\eta V_T}$ represents the diode current that would be produced by voltage V with $R_s = 0$. Eqn. 5 is a simple mapping of the behaviour for $R_s = 0$ onto the response for $R_s \neq 0$. It is worthwhile to note that the multi-valued behaviour of $W(x)$ for $-1/e < x < 0$ corresponds to stable and unstable operating points of a circuit having local positive feedback.

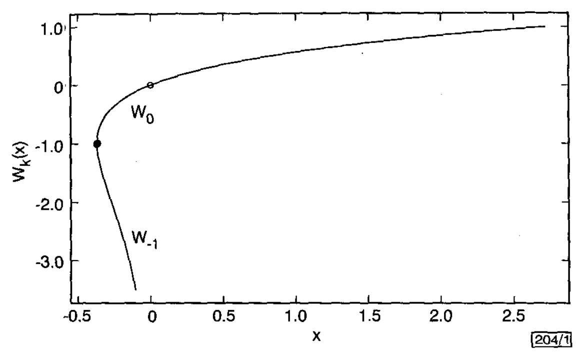

Fig. 1 *Graph of Lambert W-function showing real valued branches $W_0(x)$ and $W_{-1}(x)$*

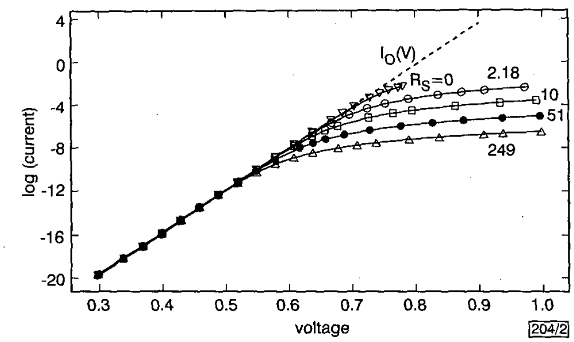

Fig. 2 *Measured and calculated current flow through diode connected transistor with preset series resistance*

Experiment and results: The proposed solution was experimentally tested using a diode connected 2N2222 transistor with five different values of series resistors R_s = 0, 2.18, 10, 51, and 249Ω. I-V measurements were performed with an HP 4145B parameter test set using a four-lead Kelvin configuration. Fig. 2 shows a log-linear plot of the measured current as noted for each R_s. The dashed line represents the extrapolated current $I_o(V)$ obtained by linear regression on the low current data. The transistor parameters were estimated to be I_{sat} = 25fA and η = 1.00. The intrinsic emitter resistance R_E was estimated from the R_s = 0 data to be R_E = 0.3 Ω, using the method described below. The solid curves in Fig. 2 were calculated from eqn. 5 with the R_E included in R_s. There is no significant difference between the calculated curves and the measured data. The slope of the log-linear plot for $I \gg I_{sat}$ is given by

$$\frac{d \ln I(V)}{dV} = \frac{1}{\eta V_T} \frac{1}{1 + w_o} \qquad (6)$$

where $w_o \equiv W_0(I_o R_s/\eta V_T)$. The slope in eqn. 6 is nearly equal to $1/\eta V_T$ for $w_o \ll 1$. The effect of series resistance is readily apparent in Fig. 2 at higher currents. There is a knee in the curve and the point where the slope decreases by half occurs at $w_o = 1$. This point corresponds to $x = I_{knee} R_s/\eta V_T = e$ and provides a simple way to estimate R_s.

Eqn. 5 offers a simple method for extracting the series resistance from measured I-V data using least-squares linear regression. Consider a set of n measured values $\{V_i, I_i\}$. The low current behaviour can be used to extrapolate the current $I_o(V)$ for values above the knee. The small effect of series resistance at low currents can be compensated using an estimate of R_s and η in eqn. 3: $I_o(V_i) = I_i e^{I_i R_s/\eta V_T}$. Several error measures can be defined for fitting eqn. 5 to the data based on $\Delta_i \equiv \ln(I_i) - \ln(w_o(V_i)\eta V_T/R_s)$. $\varepsilon = \sum_{i=1}^{n} \Delta_i^2$ is one example. The value of R_s which minimises the error ε satisfies $\partial\varepsilon/\partial R_s = 0$ or

$$f(R_s) \equiv \sum_{i=1}^{n} \Delta_i \frac{w_o}{1 + w_o} \frac{1}{R_s} = 0 \qquad (7)$$

The function $f(R_s)$ is a monotonic function of R_s. Eqn. 7 can be solved using Newton-Raphson iteration with the initial estimate $R_s = e\eta V_T/I_{knee}$. The extrapolation of the $I_o(V)$ from the low current behaviour can be refined using the revised estimate for R_s. The proposed method was applied to the measured data in Fig. 2 and the results are summarised in Table 1. Convergence is rapid, within three or four iterations to solve eqn. 7. Emitter resistance $R_E = 0.3\Omega$ was computed from the $R_s = 0$ data. The relative error in the calculated value of $R_s + R_E$ is 2% over the range of $R_s = 2.18 - 249\Omega$ investigated. The proposed method has several benefits over traditional techniques for extracting series resistance. Two prior methods use specific moments computed from the data [8 – 10]. Eqn. 5 allows an arbitrary error function to be chosen or the data to be weighted. Several traditional methods of diode parameter extraction are based on the relationship $dV/dI = R_s + \eta V_T/I$ [10 – 12]. Implicit or explicit estimation of the derivative dV/dI from the measured data is very sensitive to experimental error. Eqn. 7 employs a weighted average over the data instead.

Table 1: Summary of experimental results for parameter extraction

True	Estimated				
R_s	I_{sat} [fA]	η	R_s+R_E	Error [%]	Iterations
0.0	22.7	1.00	0.30	–	3
2.18	23.7	1.00	2.42	−2.4	3
10.01	24.9	1.00	10.08	−2.2	3
51.35	23.4	1.00	50.85	−1.5	4
249	29.6	1.01	244.5	−1.9	4

Conclusion: We present here the first published analytical solution for the current flow through a diode driven by a voltage source through a series resistance. The solution uses the Lambert W-function, an uncommon but well-characterised multivalued function. The analytical solution applies to both local negative and positive feedback around a diode or transistor. The proposed solution leads to an efficient method of extracting series resistance from measured I-V data based on least-squares linear regression.

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Influence of the feedback DAC delay on continuous-time bandpass ΣΔ coverter

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An analysis of the effect of the feedback digital-to-analogue converter (DAC) delay on the synthesis results of continuous-time ΣΔ bandpass modulators is presented. It is shown that non-null values of the feedback DAC delay can be optimal with respect to the filter gain margin.

Introduction: Continuous-time sigma-delta (ΣΔ) modulators [1] are convenient for high frequency bandpass conversion because they circumvent the frequency limit due to switched-capacitor circuitry.

Continuous-time modulators synthesis: It has been shown [2, 3] that similar behaviours could be found in both continuous and discrete-time modulators under the condition that transmittances $F(z)$ and $G(s)$ (Figs. 1 and 2) are linked according to:

$$F(z) = (1 - z^{-1}) Z_T \left\{ L^{-1} \left[\frac{G(s) e^{-ds}}{s} \right] \right\} \qquad (1)$$

where L^{-1} denotes the inverse Laplace transform, Z_T the Z transform at sampling period T, and d the propagation delay between the analogue-to-digital converter (ADC) sampling and the digital-to-analogue converter (DAC) converter [2, 4].

Consequently, eqn. 1 can be used to synthesise continuous-time modulators, i.e. to obtain $G(s)$ from $F(z)$, where d is a synthesis parameter. If $d \neq 0$, eqn. 1 may have no solution (i.e. no identification in z^{-k} terms on both sides of eqn. 1 is possible). To ensure such a solution, extra terms may be introduced so that