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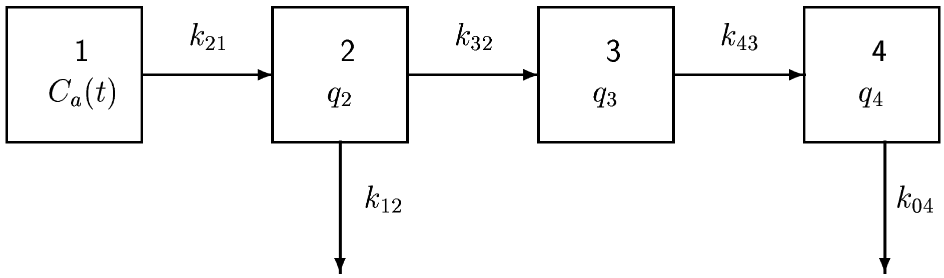
NO FLOW GLUCOSE MODEL FOR NEUROLOGY



$$q_1(t) = V_B\ C_a(t) \tag{1}$$

$$\frac{dq_2(t)}{dt} = k_{21}\ q_1(t) - (k_{12} + k_{32})\ q_2(t) \tag{2}$$

$$\frac{dq_3(t)}{dt} = k_{32}\ q_2(t) - k_{43}\ q_3(t) \tag{3}$$

$$\frac{dq_4(t)}{dt} = k_{43}\ q_3(t) - k_{04}\ q_4(t) \tag{4}$$

$$k_{22} = k_{12} + k_{32} \tag{5}$$

Solutions to the differential equations are as follows:

$$q_1(t) = V_B\ C_a(t) \tag{6}$$

$$q_2(t) = V_B\ k_{21}\ e^{-k_{22}t} \otimes C_a(t) \tag{7}$$

$$q_3(t) = \frac{V_B\ k_{21}\ k_{32}}{(k_{22} - k_{43})}\ [\ e^{-k_{43}t} - e^{-k_{22}t}] \otimes C_a(t) \tag{8}$$

$$q_4(t) = V_B\ k_{21}\ k_{32}\ k_{43} \left\{ \left[\frac{e^{-k_{22}t}}{(k_{04} - k_{22})(k_{43} - k_{22})} \right] + \left[\frac{e^{-k_{43}t}}{(k_{22} - k_{43})(k_{04} - k_{43})} \right] + \left[\frac{e^{-k_{04}t}}{(k_{22} - k_{04})(k_{43} - k_{04})} \right] \right\} \otimes C_a(t) \tag{9}$$

where the $\otimes$ denotes convolution. Total PET activity (PET counts/ml) in a region of interest is given by

$$q_{pet}(t) = V_B C_a(t) + q_2(t) + q_3(t) + q_4(t) \tag{10}$$

where V_B is the fractional blood volume.

The equation for the extravascular activity can be written as

$$
\begin{aligned}
q_e(t) &= q_2(t) + q_3(t) + q_4(t) && (11)\\
q_e(t) &= [Ae^{-k_{22}t} + Be^{-k_{43}t} + Ce^{-k_{04}t}] \otimes C_a(t) && (12)\\
A &= V_B\ k_{21}\Big[1 + \frac{k_{32}}{(k_{43}-k_{22})} + \frac{k_{32}k_{43}}{(k_{43}-k_{22})(k_{04}-k_{22})} && (13)\\
B &= \frac{V_B\ k_{21}\ k_{32}\ k_{04}}{(k_{22}-k_{43})(k_{04}-k_{43})} && (14)\\
C &= \frac{V_B\ k_{21}\ k_{32}\ k_{43}}{(k_{22}-k_{04})(k_{43}-k_{04})} && (15)
\end{aligned}
$$

VARIABLE	DESCRIPTION	UNITS
$C_a(t)$	concentration of activity in blood	(PET counts/ml blood)
$q_i(t)$	activity in compartment i normalized to PET volume	(PET counts/ml)
K_1	rate of movement of FDG from vascular to extravascular space $= V_B k_{21}$	(ml blood/ml sec)
k_{ij}	rate constant for kinetics to compartment i from compartment j as shown in diagram	(1/sec)
V_B	blood volume	(ml blood/ml)

References

[1] Huang, S-C., M. E. Phelps, E. J. Hoffman, K. Sideris, C. J. Selin, and D. E. Kuhl. Non-invasive determination of local cerebral metabolic rate of glucose in man. Am. J. Physiol. 238:E69-E82, 1980.

[2] Marquardt, D. W., An algorithm for least-square estimation of nonlinear parameters. J. SIAM 11:431-441, 1963.