Ninepoint circle

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1 Centroid

On $\triangle ABC$, let D, E, F be the midpoints of BC, CA, AB respectively.

1. Show that $BC \parallel EF$.
2. Show that $\triangle ABC \sim \triangle DEF$.
3. Show that AD, BE, CF meet at one point, say, G.
4. Show that $AG/GD = BG/GE = CG/GF = 2$.

2 Orthocentre

On $\triangle ABC$, let AD, BE, CF be the altitudes.

1. Show that $BCEF$ is cyclic. Where is the centre of this circle?
2. Show that $\triangle ABC \sim \triangle AEF$.
3. Show that AD, BE, CF meet at one point, say, H.
4. Let M be the midpoint of BC. Show that $ME = MF$. Show that these are tangents to the circumcircle of $\triangle AEF$.

3 Nine point circle

Let AD, BE, CF be altitudes of the triangle concurrent at orthocentre H. O is the circumcentre of $\triangle ABC$. Let M be the midpoint of BC. Let L be the midpoint of AH.

1. Show that M, L lies on the circumcircle of $\triangle DEF$.
2. Let X be the reflection of H over BC. Show that X is on the circumcircle of $\triangle ABC$.
3. Let Y be the reflection of H over M. Show that Y is on the circumcircle of $\triangle ABC$.

4. Let the radius of the circumcircle of $\triangle ABC$ be R. Let the radius of the circumcircle of $\triangle DEF$ be r. What is R/r?

5. Where is the circumcentre of the circumcircle of $\triangle DEF$?

6. What's the ninepoint circle of $\triangle HBC$?

4 Euler line

Continue the diagram from the Nine point circle section.

1. Show that AY is a diameter.

2. Show that $AH = 2OM$.

3. Hence show that O, H, G are collinear, where G is the centroid.

4. Find the ratio $HN : NG : GO$.

5 Problems

1. Let $ABCD$ be a cyclic quadrilateral with AB as the diameter. Let AC, BD intersect at P. Let AD, BC intersect at Q. Let the tangent at C, D intersect at R. Prove that P, Q, R collinear. Say 2 more interesting facts about this diagram.

2. Do the Euler line section, but for when $\angle BAC > 90°$.

6 Q-point

Continue the diagram from the Nine point circle section.

1. Let line HY meet the circle again at Q. Show that $AQEF$ cyclic.

2. Show that AQ, EF, BC are concurrent at a point K.

3. Show that $KQFB$ is cyclic. Show that $KQEC$ is cyclic.

4. Let KH meet the circumcircle $AEHF$ again at J. Show that A, J, M are collinear.

5. Show that $HJBC$ cyclic.

6. Show that AM is the *symmedian* of triangle AEF.

7. Show that $KFJC$ cyclic. Show that $KEJB$ cyclic.

8. Show that $KQJM$ cyclic. Show that $KQHD$ cyclic.

9. Show that the midpoint of KA lies on the circumcircle of $\triangle QDJ$.

7 Generalisation

Let $EFBC$ be a cyclic quadrilateral. Let BF and EC intersect at A. Let the circumcircle of $\triangle AFE$ and $\triangle ABC$ intersect again at Q.

1. Show that AQ, EF, and CB are concurrent at K.
2. Show that $KQFB$ is cyclic and that $KQEC$ is cyclic.
3. Let H be the intersection of FC and BE. Let M be the center of the circumcircle of $EFBC$. Show that M, H, and Q are collinear.
4. Show that $r^2 = MH \times MQ$, where r is the radius of the circumcircle of $EFBC$.
5. Show that $AH \perp KM$.
6. Show that $AM \perp KH$.
7. Show that $QBME$ is cyclic and $QFMC$ is cyclic.
8. Show that QM is the angle bisector of $\angle BQE$.