The second order PDE

$$\frac{\partial}{\partial z}\left[-r_p^2 D_g \frac{\partial C}{\partial z}\right] = -2 r_p k_{het} C \tag{1}$$

implies

$$-r_p^2 D_g \frac{\partial^2 C}{\partial z^2} = -2 r_p k_{het} C \tag{2}$$

implies

$$\frac{\partial^2 C}{\partial z^2} = \left(\frac{2k_{het}}{r_p D_g}\right) C \tag{3}$$

which, by its form, gives the solution

$$\boxed{C = A_1 \mathrm{exp}(z\sqrt{2k_{het}/D_g r_p}) + A_2 \mathrm{exp}(-z\sqrt{2k_{het}/D_g r_p})} \tag{4}$$

because

$$C' = \sqrt{\frac{2k_{het}}{r_p D_g}} A_1 \mathrm{exp}(z\sqrt{2k_{het}/D_g r_p}) - \sqrt{\frac{2k_{het}}{r_p D_g}} A_2 \mathrm{exp}(-z\sqrt{2k_{het}/D_g r_p}), \tag{5}$$

leads to

$$\begin{aligned} C'' &= \frac{2k_{het}}{r_p D_g} A_1 \mathrm{exp}(z\sqrt{2k_{het}/D_g r_p}) + \frac{2k_{het}}{r_p D_g} A_2 \mathrm{exp}(-z\sqrt{2k_{het}/D_g r_p}) \\ &= \frac{2k_{het}}{r_p D_g} C = C''. \end{aligned} \tag{6}$$