Midterm 1: Patrick Kim Section 1.1: Systems of Linear Equations

Definitions Linear equation a x + a x + ... + a x = b Organization of coefficients and variables with a solution 'b'

System of linear equations Collection of multiple linear equations

Solution of a system (s , s , ... ,s) List of numbers that make each equation a true statement when the s values are substituted for the x variables

Solution set Set of all possible solutions of a linear system

Equivalent linear systems 2 linear systems with the same solution set

Consistent system 1 solution or infinitely many solutions

Inconsistent system No solution for a specific input

Existence Does a solution set exist?

Uniqueness If a solution exists, is there more than one solution?

Key Notes A system of linear equations has either: No solution Exactly one solution Infinitely many solutions

Matrix notation Rectangular format that contains info of a linear system

Example system 1x - 2x + x = 0 0x + 2x - 8x = 8 5x + 0x - 5x = 10

Coefficient matrix

Augmented matrix

Size of a matrix m x n m: rows n: columns

Row reduction operations Replacement Eliminating elements (making them 0) by comparing two rows and scaling one of them Interchange Swapping rows Scaling Usually done to make a leading entry into one

Goal of row reduction: to create an echelon form or RREF Triangle of 0's

Section 1.2: Row Reduction and Echelon Forms

Definitions Non-zero row/column Row or column with at least one nonzero entry

Zero row/column Row or column with all zeros

Leading entry Leftmost nonzero entry in a row

Row reduced echelon form (RREF) A simplified matrix that represents a potential solution set for a linear system Each matrix has only one RREF

Pivot position Location in a matrix that corresponds to a leading 1 in RREF

Pivot column Column that contains a pivot position

Basic/leading variables Variables that correspond to a pivot Basic variables have an exact value for a solution set

Free variables Variables that do not correspond to any pivots and pivot columns Can be assigned any value for a consistent linear system

Overdetermined system # of rows > # of columns System of linear equations with more equations than unknowns Can be consistent Can have a unique solution

Underdetermined system # of columns > # of rows System of linear equations with more unknowns than equations Can never have a unique solution (always a free variable) If the system is consistent -> infinite solutions If the system is inconsistent -> no solution

Key Notes Echelon Form of a Matrix 3 Properties: 1. All zero rows are at the bottom 2. Each leading entry (non-zero entry) of a row is to the right of any leading entries in the row above it (if any) 3. Below a leading entry, all entries are 0

RREF All leading entries are 1's There are 0's above and below each leading 1 A matrix can be in neither echelon form nor RREF This means that more row reduction needs to be done

Uniqueness of the RREF Each matrix is row equivalent (has the same solution set) to one and only one reduced echelon matrix A matrix has only one RREF matrix Inconsistent systems have empty solution sets

Existence and Uniqueness Theorem A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot column No row of the form: [0 0 0 0 0 | b] with b non-zero If a linear system is consistent, the solution set has either: Unique solution (no free variables) Infinitely many solutions (at least one free variable)