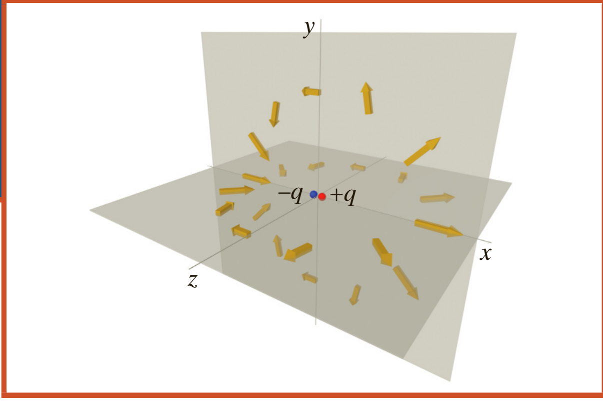
CHAPTER

13

Electric Field



OBJECTIVES

After studying this chapter you should be able to

- Mathematically relate electric field and force.
- Calculate the 3D electric field at a particular location due to a collection of point charges.
- Explain the approximations made in deriving expressions for the electric field of a dipole, and use these approximate expressions appropriately.
- Graphically represent the magnitude and direction of the electric field of a dipole with arrows, at locations in a plane containing the dipole.
- Create a computational model to compute and display the electric field of a collection of point charges in 3D, and predict the motion of a charged particle that interacts with this field.

13.1 NEW CONCEPTS

Two important new ideas will form the core of our study of electric and magnetic interactions. The first is the concept of electric and magnetic fields. This concept is more abstract than the concept of force, which we used extensively in our study of modern mechanics. The reason we want to incorporate the idea of "field" into our models of the world is that this concept turns out to be a very powerful one, which allows us to explain and predict important phenomena that would otherwise be inaccessible to us.

The second important idea is a more sophisticated and complex model of matter. In our previous study of mechanics and thermal physics it was usually adequate to model a solid as an array of electrically neutral microscopic masses (atoms) connected by springs (chemical bonds). As we consider electric and magnetic interactions in more depth, we will find that we need to consider the individual charged particles—electrons and nuclei—that make up ordinary matter.

The material in this chapter lays the foundation for all succeeding chapters, so it is worth taking time to understand it thoroughly. In addition, if you did not use volume 1 of this textbook in your previous study of physics, you should work through the summary from Chapter 1 on 3D vectors, vector notation, and computational modeling available at no charge on the student website, `www.wiley.com/college/chabay`.

13.2 ELECTRIC CHARGE AND FORCE

In this section we briefly review concepts familiar to you from your previous studies.

Point Charges

There are two kinds of electric charge, which are called positive and negative. Particles with like charges repel each other (two positive or two negative particles); particles with unlike charges (positive and negative) attract each other. By "point particle" we mean an object whose radius is very small compared to the distance between it and all other objects of interest, so we can treat the object as if all its charge and mass were concentrated at a single mathematical point. Small particles such as protons and electrons can almost always be considered to be point particles.

The Coulomb Force Law for Point Particles

The electromagnetic interaction is one of the four fundamental physical interactions (see Chapter 3, The Fundamental Interactions). The electric force law, called Coulomb's law, describes the magnitude of the electric force between two point-like electrically charged particles:

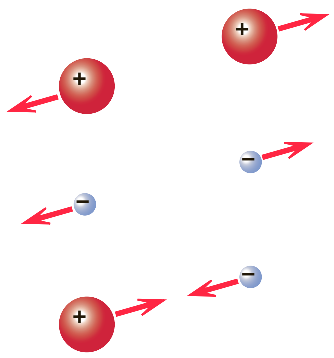


Figure 13.1 Two protons repel each other; two electrons repel each other; a proton and an electron attract each other.

$$|\vec{F}| = F = \frac{1}{4\pi\varepsilon_0}\frac{|Q_1Q_2|}{r^2}$$

where Q_1 and Q_2 are the magnitudes of the electric charge of objects 1 and 2, and r is the distance between the objects. As indicated in Figure 13.1:

- The electric force acts along a line between two point-like objects.
- Like charges repel; unlike charges attract.
- Two charged objects interact even if they are some distance apart.

Units and Constants

The SI unit of electric charge is the coulomb, abbreviated C. The charge of one proton is 1.6×10^{-19} C in SI units. The constant e is often used to represent this amount of positive charge: $e = +1.6 \times 10^{-19}$ C. An electron has a charge of $-e$.

The constant $1/4\pi\varepsilon_0$ has the value $9 \times 10^9\ \mathrm{N \cdot m^2/C^2}$. We write this constant in this way, instead of using the letter k, for two reasons. Since the letter k is often used for other quantities, we avoid confusion by not using it here. Also, the constant $\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$ will appear by itself in important situations.

Charged Particles

There are many microscopic particles, some of which are electrically charged and hence interact with each other through the electric interaction. The characteristics of some charged particles are shown in the table below.

Particle	Mass	Charge	Radius
Electron	9×10^{-31} kg	$-e\,(-1.6 \times 10^{-19}\,\mathrm{C})$	? (too small to measure)
Positron	9×10^{-31} kg	$+e\,(+1.6 \times 10^{-19}\,\mathrm{C})$	?
Proton	1.7×10^{-27} kg	$+e$	$\sim 1 \times 10^{-15}$ m
Antiproton	1.7×10^{-27} kg	$-e$	$\sim 1 \times 10^{-15}$ m
Muon	1.88×10^{-28} kg	$+e\,(\mu^+)$ or $-e\,(\mu^-)$	?
Pion	2.48×10^{-28} kg	$+e\,(\pi^+)$ or $-e\,(\pi^-)$	$\sim 1 \times 10^{-15}$ m

This is not an exhaustive list; you may also have learned about other charged particles, such as the W and Δ particles, and other particles that are short-lived and not commonly encountered in everyday circumstances. Positrons and antiprotons are *antimatter*. A positron is an antielectron; if a positron and an electron encounter each other they will annihilate, releasing all their energy as high-energy photons. Antimatter is therefore not found in ordinary matter.

Ordinary matter is composed of protons, electrons, and neutrons (which have about the same size and mass as protons but are uncharged). However, some other charged particles do play a role in everyday processes, as we will see, for example, in our study of sparks in a later chapter.

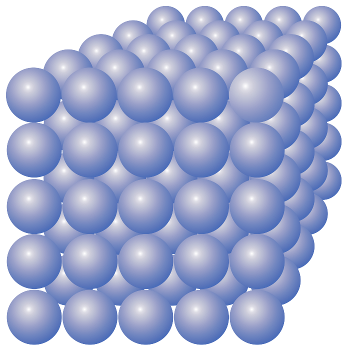


Figure 13.2 A metal lattice. The radius of a single atom is on the order of 1×10^{-10} m, and in a cube that is 1 cm on a side, there are on the order of 1×10^{23} atoms!

Size and Structure of Atoms

As you have learned in chemistry or from other sources, matter is made of tiny atoms. In a solid metal, atoms are arranged in a regular three-dimensional array, called a lattice (Figure 13.2). A cubic centimeter of solid metal in which atoms are packed right next to each other contains around 1×10^{23} atoms, which is an astronomically large number.

A neutral atom has equal numbers of protons and electrons. The protons and neutrons are all found in the nucleus at the center of the atom. The electrons are spread out in a cloud surrounding the nucleus.

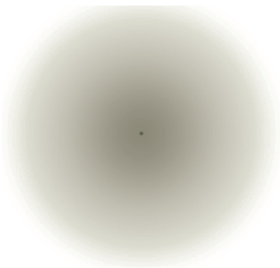


Figure 13.3 The electron cloud of an iron atom. The radius of the electron cloud is approximately 1×10^{-10} m. On this scale the tiny nucleus, located at the center of the cloud, would not actually be visible.

Each atom consists of a cloud of electrons continually in motion around a central “nucleus” made of protons and neutrons. If we imagine taking a snapshot of an iron atom, with a nucleus of 26 protons and 30 neutrons surrounded by 26 moving electrons, it might look something like Figure 13.3. In this figure the nucleus is hardly visible, because it is much smaller than the electron cloud, whose radius is on the order of 1×10^{-10} m.

The nucleus of the iron atom, depicted in Figure 13.4, has a radius of roughly 4×10^{-15} m, about 25,000 times smaller than the tiny electron cloud. If an iron atom were the size of a football field, the nucleus would have a radius of only 4 mm! Yet almost all of the mass of an atom is in the nucleus, because the mass of a proton or neutron is about 2000 times larger than the mass of an electron.

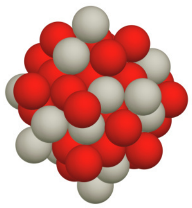


Figure 13.4 The nucleus of a iron atom contains 26 protons and 30 neutrons. Its radius is approximately 4×10^{-15} m.

> **Checkpoint 1** What is the approximate radius of the electron cloud of a typical atom? Which of the following charged particles are constituents of ordinary matter? Protons, positrons, electrons, antiprotons, muons

13.3 THE CONCEPT OF “ELECTRIC FIELD”

Consider the following thought experiment. Having evacuated the air from the room (to avoid collisions with air molecules), you hold a proton in front of you and release it. There are no other objects nearby. You observe that the proton begins to move downward, picking up speed (accelerating) at the rate of 9.8 m/s each second (Figure 13.5). Recall that at speeds much less than the speed of light,

$$\frac{d\vec{p}}{dt} \approx m\frac{d\vec{v}}{dt} = m\vec{a}$$

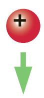


Figure 13.5 Acceleration of a proton.

QUESTION What do you think is responsible for this change in the velocity of the proton?

You probably inferred that the gravitational interaction of the Earth and the proton caused the downward acceleration of the proton. That is a reasonable explanation for this observation.

Now suppose that at a later time, you release another proton in the same location. This time you observe that the proton begins to move to the right, picking up speed at a rate of 1×10^{11} m/s^2 (Figure 13.6).

Figure 13.6 Acceleration of another proton at a later time.

QUESTION What might be responsible for this change in the velocity of the proton?

This acceleration cannot be due to a gravitational interaction of the proton with the Earth, since the magnitude of the effect is too large, and the direction is not appropriate. Could it be due to a gravitational interaction with a nearby black hole? No, because if there were a black hole very nearby we would not be here to observe anything! Could it be an interaction via the strong (nuclear) force? No, because the strong force is a very short range force, and there are no other objects near enough.

It is, however, plausible that the interaction causing the acceleration of the proton could be an electric interaction, since electric interactions can have large effects and can occur over rather large distances.

QUESTION What charged objects might be responsible for this interaction, and where might they be?

There are many possible configurations of charged particles that might produce the observed effect. As indicated in Figure 13.7, there might be positively charged objects to your left. Alternatively, there could be negative charges to your right. Perhaps there are both—you can't draw a definite conclusion from a single observation. There are many possible arrangements of charges in space that could produce the observed effect.

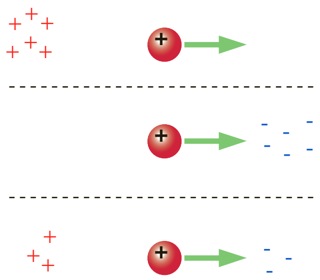


Figure 13.7 Three possible arrangements of charged particles that might be responsible for the observed high acceleration of a proton.

(If, however, you made several observations of the proton over some time period, you might note a change in the proton's acceleration. For example, if you noticed that the acceleration of the proton increased as it moved to the right, you might suspect that there was a negative charge to your right, which would have an increasingly large effect on the proton as it moved closer to the charge.)

QUESTION On the basis of your observations so far, can you predict what you would observe if you released an electron instead of a proton at the same location (Figure 13.8)?

Figure 13.8 A (negatively charged) electron.

An electron would accelerate to the left rather than to the right, since it has a negative charge. Its acceleration would be greater than 1×10^{11} m/s^2, because the mass of the electron is much less than the mass of the proton.

QUESTION What would happen if you placed an alpha particle (a helium nucleus, with charge $+2e$) at the observation location (Figure 13.9)?

Figure 13.9 An alpha particle with two (positively charged) protons and two (uncharged) neutrons.

Since the alpha particle has a positive charge, it would accelerate to the right as the proton did. However, because the charge of the alpha particle is twice the charge of the proton, and its mass is about four times that of a proton, the magnitude of its acceleration would be one-half of 1×10^{11} m/s^2.

QUESTION If you released a neutron at the observation location, what would you expect to observe (Figure 13.10)?

Figure 13.10 An (uncharged) neutron.

Since the neutron has no electric charge, it would experience no electric force, and should simply accelerate toward the Earth at 9.8 m/s^2.

QUESTION Finally, suppose we do not put any particle at the observation location. Is there anything there?

Since we know that if we were to put a charged particle at that location, it would experience a force, it seems that in a certain sense there is something there,