Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
C		**2206523**

A Light Purse Is a Heavy Curse: Come to Find the Optimal Trading Strategy

Summary

Gold and Bitcoin are popular volatile assets around the world now. The daily prices of these assets fluctuate to varying degrees for many reasons. Therefore, market traders often choose to trade volatile assets frequently. In order to get the highest possible total return, one trader needed us to develop mathematical model to determine each day if the trader should buy, hold, or sell their assets in their portfolio.

In this paper, we first pre-process the data provided and correct and refine the data. Then, we observe that the time series of gold and bitcoin are nonlinear and non-stationary. We decompose the time series of gold and bitcoin respectively based on Empirical Mode Decomposition. The decomposition makes the decomposed time series easy to predict and has higher accuracy. Then, we can create Directed Visibility Graphs based on these time series. After that, we map the time series into a complex network, and transform it into a prediction problem about the graph. Next, we use the link prediction based on directed visibility graph. And in this step, we give detailed calculation procedures. After the prediction model is built, we build a decision model based on Linear Programming to maximize our daily returns on that day. Finally, we obtained the final value of $1,422.20 by using the decision model to solve the problem cyclically, which is a relatively significant gain over five years.

To prove that our mathematical model can provide the best trading strategy, we set two evaluation metrics. We give computed data to show that the predicted prices of gold and bitcoin have very high accuracy. When the relative error to the real data is within 5%, the gold prices have 99.78% of the predicted data and the bitcoin prices have 81.90% of the predicted data. In addition, we analyze the sensitivity of the strategy and show how do transaction costs affect the strategy and results. At the same time, we analyze the possible causes of these phenomena based on the intuitive feeling of investment and some mathematical calculations.

The result is a comprehensive model that predicts the price of volatile assets and makes the optimal trade decision for the day. This model can help trader to gain benefits from portfolio investments. With the help of this model, trader can know whether and how much gold and bitcoin they should buy that day. Moreover, this model takes into account the investment risk and provides a sound trading strategy. We can almost guarantee the final asset value.

Finally, a one-page memorandum is provided to trader to better understand our strategies, models and results.

Keywords: Empirical Mode Decomposition, Directed Visibility Graphs, Random Walk, Link Prediction, Linear Programming, Trading Strategies

Contents

1 Introduction

1.1 Problem Background

Since 2016, the price of gold has risen significantly, which has brought about a gold investment boom. The global market volatility from the 4th quarter of 2015 created a lot of uncertainty for investments, especially the deterioration of the stock investment market environment, which made investors need to find new investment directions. In this environment, on the one hand, the safe-haven nature of gold has led to a relatively rapid rise in gold prices. On the other hand, as a decentralized and ultra-sovereign network currency, Bitcoin is favored by a wide range of investors. In 2017, the price of bitcoin rose even as high as 1,403.2%.

However, the price of Bitcoin is so volatile that investing in a combination of it and gold can effectively reduce investment risk. Trader wants to get as much out of his trading as possible and increase his total return, which requires us to build a mathematical model to get the best strategy for daily trading to help the trader adjust his trading options and invest better in the future.

1.2 Restatement of The Problem

- Develop a model that gives the best daily trading strategy based only on price data up to that day. Use this mathematical model and strategy to derive the final value of the initial $1,000 investment on 9/10/2021.
- Present evidence that our model provides the best strategy.
- Determine how sensitive the strategy is to transaction costs and explain how do transaction costs affect the strategy and results.

1.3 Our work

The topic requires us to determine the optimal strategy for a daily trade using a combination of gold and bitcoin investments using only $1,000 and to derive the final value of the initial investment on 9/10/2021, using only the price data provided.

At the same time, we need to show that this mathematical model provides the optimal solution to the trade scenario. To ensure that our strategy has a high sensitivity to changing trading prices, we add a perturbation to the transaction cost to verify the sensitivity of the model.

That is, if our model delivers significant returns and is sufficiently sensitive to changing transaction costs, then we can consider it worthy of trader's application.

2 Analysis of the Problem

Modern people have higher average savings than in the past in this era of rapid economic development. And people who are no longer satisfied with the interest returns on bank deposits are trying to increase their assets easily by investing their money. Initially, the price of a bitcoin was 1 cent. Surprisingly the price of one bitcoin is almost $69,000 at 11/01/2021. No one would not be attracted by the amazing increase of bitcoin. I'm sure many people have thought about owning some bitcoins. However, the drop in bitcoin price is just as intense. As a result, most traders did not buy bitcoins actually.

Gold is a popular financial item that is historic, stable, and retains its value. Gold will retain its value forever unless NASA finds a planet made of gold. However, the returns on gold are very little compared to bitcoin which does not meet the financial needs of modern people.

A combination of gold and bitcoin investments came into being. One needs to consider exactly how much gold and bitcoin to buy when it comes to the actual operation. On the one hand, if one buys more gold it will affect the return; on the other hand, if one buys more bitcoin one will take more risk. Obviously, a sophisticated and proven investment strategy is needed.

As the saying goes: " While there is life, there is hope. " Given the practicalities of the situation, our team believes that the priority should be to ensure that we can eventually make a profit. Only then do we consider how to make as much money as possible. The concept of prudent investment will be reflected in our entire solution.

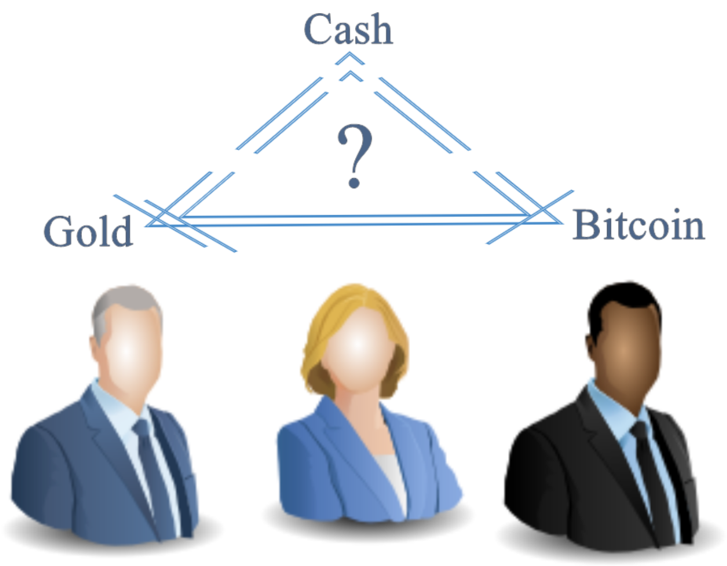

Figure 1: How to make decisions?

3 Problem1: Mathematical Model for The Best Trading Strategy and The Final Value

3.1 Data Processing

3.1.1 Checking The Validity of The Data

We found date data in the first column of the dataset that differed from the date format "month/day/year" given in the question. For these suspicious date data, we compared the gold and bitcoin price data from the source of the dataset and confirmed that they were incorrectly formatted date data. After we corrected them, they became valid and usable date data.

3.1.2 Filling In Possible Data Gaps

Checking the data shows that the bitcoin price data is complete. However, in the daily gold price data, we find that there are a small number of price data missing, and all the missing data are not adjacent. The distribution of gold price data drawn is shown in the figure, where the white line of the left image is the missing price data. Therefore, to reduce the influence on the regularity of the original data, we adopt the mean filling method, that is, the missing data is filled with the mean of the previous value and the next value of the missing data :

$$x_i = \frac{x_{i-1} + x_{i+1}}{2} \tag{1}$$

Where, x_i is the ith price number and $i \in \{\text{subscript of the missing price data}\}$. The data distribution of gold prices is shown in Figure 2 .

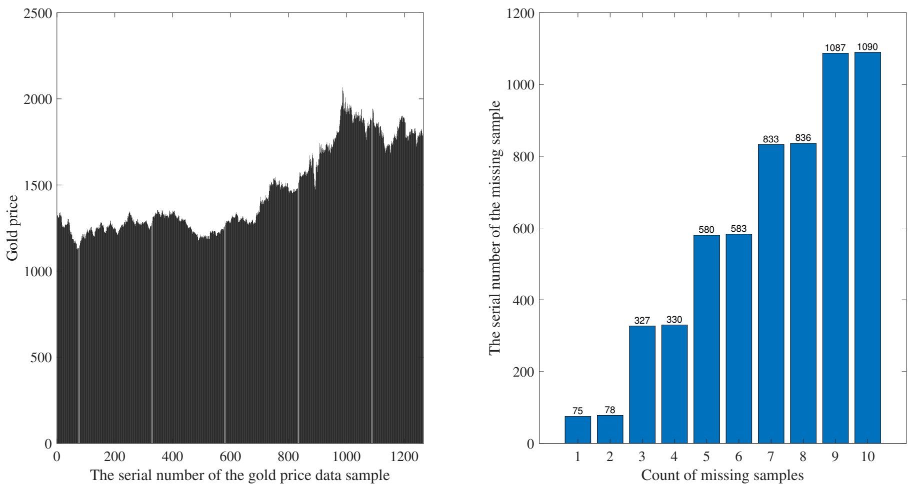

Figure 2: Distribution of Gold Price Data

3.1.3 Refine data based on problem context

Since gold can only be traded when the market is open, by looking at the daily gold price data, we find that there are holidays, in addition to weekends, that cause the gold market to close. Without loss of generality, we may assume that during the period when the gold market is closed, the price of gold remains at its price on the day before the market closes. In particular, since 9/10/2016 is a Sunday and the starting day, there is no gold price data for this day and its predecessors in the dataset. So we may consider replacing the gold price for this day with the data for the next day without affecting the subsequent model building.

At this point, we have processed all data for the five-year trading period, from 9/11/2016 to 9/10/2021.

3.2 Predictive Model Building and Solving

To get the best strategy for daily trading, we need to make use of the gold and Bitcoin price data up to that day to build the corresponding prediction model. It can predict the price of gold and bitcoin the next day. Besides, we can find a strategy which obtains as many assets as possible for investment according to the predicted results. Figure 3 shows how the prediction model is built.

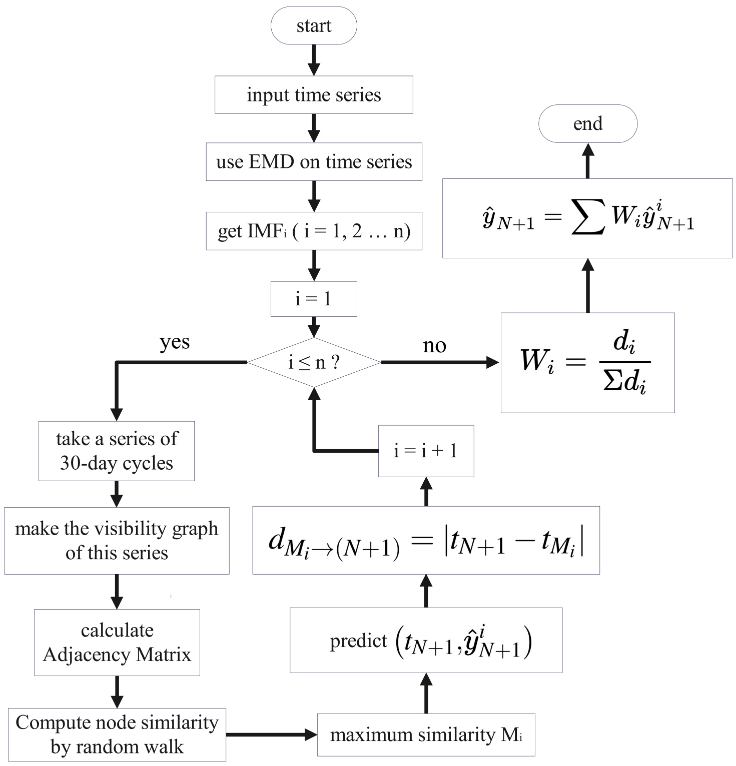

Figure 3: The Process of Predictive Model

3.2.1 Time Series Decomposition Based on EMD

- Step1:

  Empirical Mode Decomposition (EMD) is performed on two sets of time series of gold and bitcoin.

  EMD can decompose the initial time-series data to obtain a set of empirical modes with different period lengths which implies the variation pattern of the initial time series. Decomposing the series is useful and necessary because we observe that both sets of time series are not smooth, which means that direct prediction is unreliable. The decomposed series have different periods and at the same time imply the original variation pattern. The accuracy of time series prediction can be significantly improved by modeling the decomposed empirical modes and then performing general prediction.

  Using the method shown in Figure 4, we can perform an EMD on the time series of gold and bitcoin.

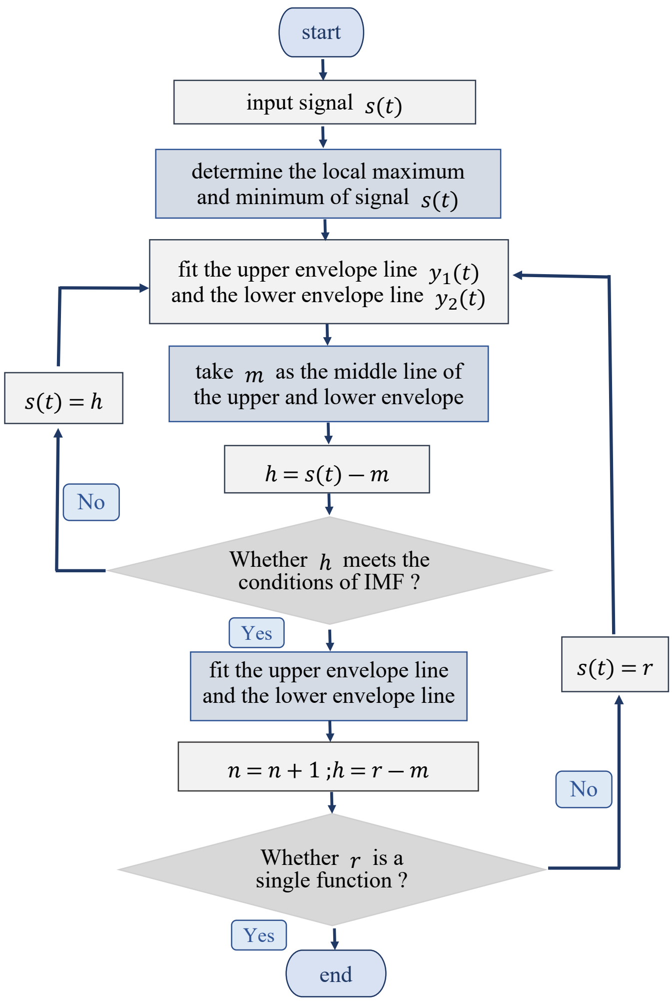

Figure 4: Empirical Mode Decomposition

- Step2:

  Now, we get the decomposed Intrinsic Mode Function (IMF). In this problem, after decomposing the time series of gold and bitcoin, we get 7 IMFs. Next, we will only discuss the series of gold, and the same method is used for the series of bitcoin. For each IMF, starting from the first IMF_1, do the following in turn.

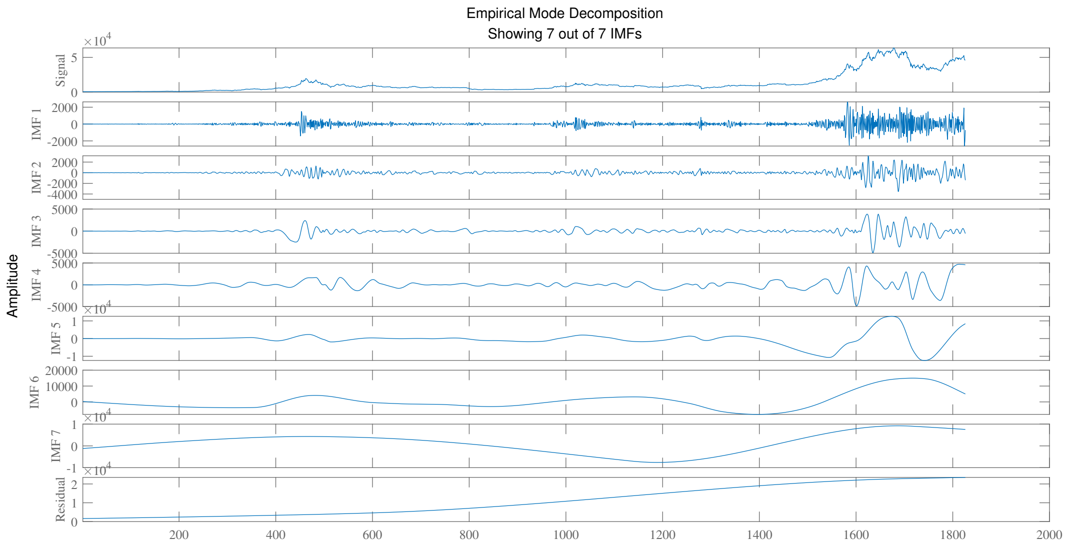

Figure 5: EMD of Bitcoin's time series

- Step3:

  Select a sub-time series of a certain length in order from back to front. Since this question is portfolio investment, we believe that risky investments usually need to wait and see 30 days or observe the price trend of the previous month before making a decision is more reasonable. Usually, cyclical patterns can be found in part within 30 days. So we will select 30 days of data to construct a directed viewable network.

3.2.2 Algorithm of The Visibility Graph

- Step1:

  Make a directional view based on the time series of gold.This is where the features of the prediction algorithm we used in this problem come into play. In this step, we map the time series into a network graph, and at this point, there are more features worth studying. Not only can we obtain information about the original time series of gold, but we can also study the network to obtain more information about the series.

  We need to iterate to make a lot of directed viewable. Let's use just one example to get a sense of how viewable works.

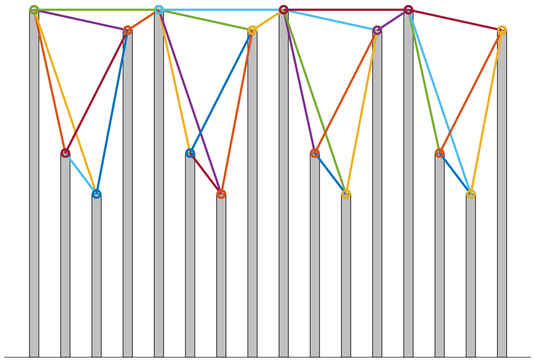

Figure 6: The Visibility Graph

Given a time series $\{x_1, x_2, ..., x_n\}$, where x_i denotes the value at moment i. The network graph consists of nodes and edges. In the viewable algorithm, each time series point is treated as a node. Edges are determined by the following rules.

$$x_c < x_b + (x_a - x_b)\frac{t_b - t_c}{t_b - t_a} \tag{2}$$

where $a < c < b$. If x_a and x_b satisfy the above relation, then a and b are connectable. Iterate through all sequence points to complete the construction of the viewable. As shown in Figure 6.

Based on Lacasa et al.'s study, we convert the time series into a Directed Visibility Graph (DVG) since time series forecasts are predicted into the future. That is, the points in the future time are not connected to the past time.

- Step2:

  The adjacency matrix can be derived directly from the DVG.

- Step3:

  Calculate the similarity matrix of the viewable network. With the viewable network made, we use random wandering to calculate the similarity between network nodes. First Local Random Walk (LRW) and then Superimposed Random Walk (SRW) are calculated. Finally, the similarity between the two points is obtained.

  The Transition Probability Matrix of Random Walk is :

$$Q'_{ij} = \begin{cases} \frac{a_{ij}}{k_i^{\text{out}}} & \text{if } (i,j) \in E \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

  Where E is the set of all edges of the viewable view. $a = (a_{ij})nxn$ is the computed adjacency matrix of the directed network graph. k_i^{out} is the out degree of node i.

Considering the time cost of computation, the prediction accuracy may be reduced if it is difficult or even impossible to walk from i to j in a random walk. Therefore, we design the random walk with the probability of returning to the starting point i and restarting the walk. In this paper, the probability is 0.2.

Then the state transfer probability matrix can be updated as :

$$Q_{ij} = (1-\alpha)Q'_{ij} + \alpha \mathbf{1}(j=i) \tag{4}$$

After t steps, the probability of reaching other nodes in the network from i is represented by the vector π_i :

$$\vec{\pi_i}(t) = Q^T \vec{\pi_i}(t-1) \tag{5}$$

Where $\pi_i(0)$ denotes a random walk starting from i which is a unit vector. In each step, the similarity between i and j is :

$$S_{ij}^{LRW}(t) = \frac{k_i^{\text{out}} k_j^{\text{in}}}{2|E|} \pi_{ij}(t) \tag{6}$$

Where $|E|$ is the total number of edges in the network and k_j^{in} is the entry degree of node j . After superimposing all random walk's processes, the total similarity between i and j is obtained as :

$$S_{ij}^{SRW}(t) = \sum_{l=1}^{t} S_{ij}^{LRW}(l) \tag{7}$$

Finally, the similarity matrix of the viewable network is derived.

- Step4:

  Find the point with the greatest similarity to the Nth point.To predict the first $(N+1)$ value, use the similarity between the last node N and all previous $(N-1)$ nodes :

$$S_N^{SRW} = \left\{S_{1N}, S_{2N}, \ldots, S_{(N-1)N}\right\} \tag{8}$$

  The maximum value in S_N^{SRW} is denoted by S_{MN} . Its corresponding node (t_M, y_M) is considered to be the node most similar to (t_N, y_N) . The n IMFs apply the above method to obtain the set of n nodes similar to the node (t_N, y_N) $(n=7)$:

$$M = \{M_1, M_2, \ldots, M_n\} \tag{9}$$

  However, there may be duplicate nodes in the similar node set M. When there is only one node in M, prediction is not possible. Therefore, for the first high-frequency IMF, 2 time-series are selected from backward to forward to get node $\{M_1, M_{n+1}\}$ similar to node (t_N, y_N) respectively.At this point, there are at least two different nodes in $M = \{M_1, M_2, \ldots, M_n, M_{n+1}\}$.

3.2.3 Link Prediction Model based on Algorithm of The Visibility Graph

- Step1:

  Use $(n+1)$ points of maximum similarity to predict $\left(t_{N+1}, \hat{y}^i_{N+1}\right)$ respectively. Using $M = \{M_1, M_2, \ldots, M_n, M_{n+1}\}$, draw parallel lines from the top of the bar chart at point (t_N, y_N) to line M_iM_{i+1} and $M_{n+1}t_N$. These parallel lines intersect at point $\{\hat{y_1}, \hat{y_2}, \ldots \hat{y_n}, \hat{y_{n+1}}\}$ on line $t = t_{N+1}$.

- Step2:

  Calculate the weight value of each prediction point and weight the calculation to get the prediction value :

  Let $d_i = |t_{N+1} - t_{M_i}|$, compute the weights :

$$w_i = \frac{d_i}{\sum_{j=1}^{n+1} d_j} \tag{10}$$

  Compute the predicted value of sequence $x(t)$ at $t = t_{N+1}$.

$$\hat{y}_{N+1} = \sum_{i=1}^{n+1} w_i \hat{y}_i \tag{11}$$

3.2.4 Predictive Model Solving

In order to make forecasts, we need 30 days of price data in each case. Therefore, we start forecasting from the 31th day (10/11/2016). A comparison of the predicted price with the real price data is shown in Figure 7 and 8. It can be seen that the predicted price curves are highly similar to the real price curves, and they have a high degree of fit, which indicates the effectiveness of our prediction model.

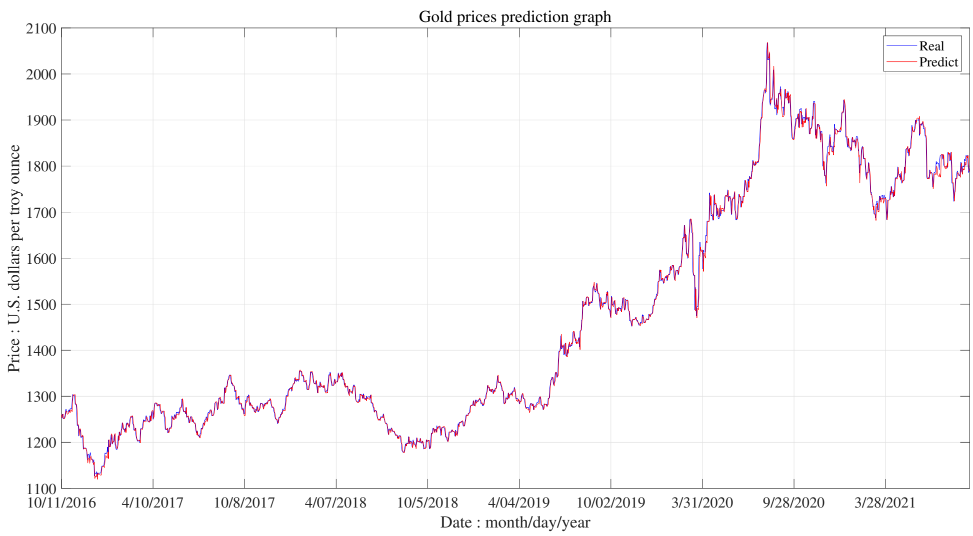

Figure 7: A Comparison of Gold Prices

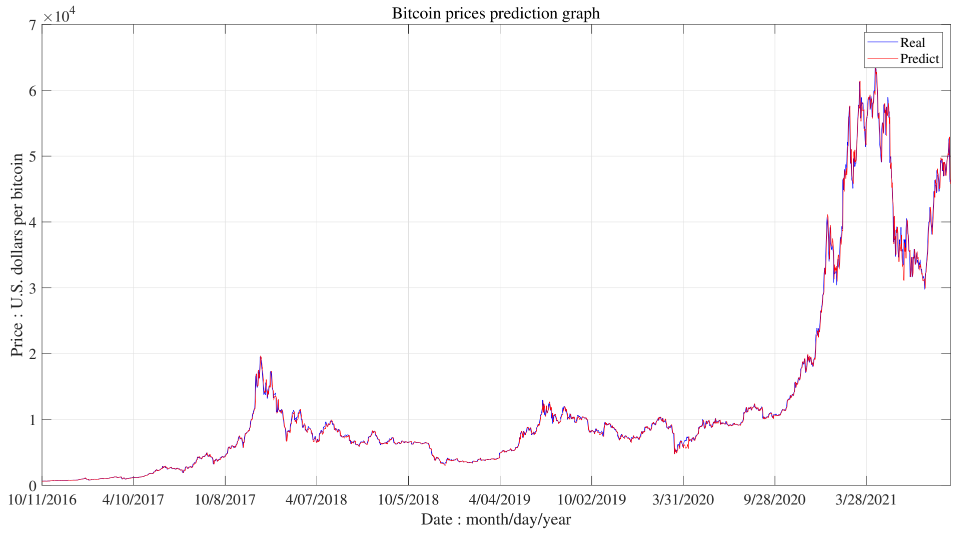

Figure 8: A Comparison of Bitcoin Prices

3.3 Decision Model Building and Solving

3.3.1 Linear Programming Based Decision Model

Using the forecast model above, for the 30th day from 9/11/2016, i.e., for every day from 10/11/2016 onwards, we have obtained the predicted price of gold and bitcoin for its next day. According to these two predicted prices, we should solve out the optimal strategy that enables us to obtain the maximum profit under the constraints of the existing objective conditions. The notation for this decision model is shown in the table below.

The main symbols we used in this decision model and the explanations of them are put in Table 1.

Table 1: Interpretation of Symbols

Symbol	Explanation
$[C,G,B]$	asset structure of the day before decision : [cash, gold, bitcoin]
$[C^*,G^*,B^*]$	asset structure of the day after decision : [cash, gold, bitcoin]
x_G, x_B	the amount of gold / bitcoin sold that day
y_G, y_B	the amount of gold / bitcoin bought that day
P_G, P_B	gold / bitcoin price of the day
P_G^0, P_B^0	the true gold / bitcoin price of tomorrow
P_G^*, P_B^*	the predicted gold / bitcoin price of tomorrow
Cin	cash received on the day
$Cout$	cash disbursed on the day
Q^*	the predicted value of tomorrow's assets
$\alpha_{gold}, \alpha_{bitcoin}$	commission cost of gold / bitcion

Our predicted assets for tomorrow are :

$$Q^* = C^* + G^* P_G^* + B^* P_B^* \tag{12}$$

And our goal is to maximize Q^*, so we can build the following linear programming model like Euation 12.

Where M is a free factor, which is not restricted by the range of solutions. We can make M sufficiently large as $M = 100$. The solution $[x_G, x_B, y_G, y_B]$ based on this model is the optimal strategy for that day.

$$\max Q^*$$
$$s.t. \begin{cases} Cout \leqslant C + Cin \\ 0 \leqslant \begin{bmatrix} x_G \\ x_B \\ y_G \\ y_B \end{bmatrix} \leqslant b \\ b = \begin{bmatrix} G \\ B \\ M \\ M \end{bmatrix} \cdot I + \begin{bmatrix} 0 \\ B \\ 0 \\ M \end{bmatrix} \cdot (1 - I) \\ I = \begin{cases} 1, & \text{if the gold market were open} \\ 0, & \text{if the gold market were closed} \end{cases} \\ Cin = x_G P_G (1 - \alpha_{gold}) + x_B P_B (1 - \alpha_{bitcoin}) \\ Cout = y_G P_G (1 + \alpha_{gold}) + y_B P_B (1 + \alpha_{bitcoin}) \end{cases} \tag{13}$$

3.3.2 Circular Solution of The Decision Model

Starting on October 11, 2016, we solved the best strategy for that day and made a decision, and so on until 9/9/2021. The final asset structure on 9/10/2021 is shown in the following table:

Table 2: The Final Asset Structure

Category	Amount	Unit
Cash	1,422.20	U.S. dollars
Gold	0	Troy ounce
Bitcoin	0	PCS
Total Price	1,422.20	U.S. dollars

You can see that over the five years, our asset value has increased by \$422.20 compared to the initial \$1,000 in cash.

Over a five-year period of 1,826 days, our mean annual rate of interest is :

$$Mari = [(\frac{1422.20}{1000})^{\frac{1}{5}} - 1] \times 100\% = 7.298\% \tag{14}$$

This shows that our investment strategy is very sound.

4 Problem2: Proof of optimality

We calculate two evaluation indicators for the forecast results by comparing the forecast results with the actual price data for a total of $n = 1796$ days from 10/11/2016 to 9/11/2016.

- Average relative error :

$$Are = \frac{1}{n}\sum_{i=1}^{n} re_i \tag{15}$$

- Accuracy of 5%error :

$$Accuracy5 = \frac{count(re \leqslant 5\%)}{n} \tag{16}$$

Where, re_i denotes the relative error of the ith sample point and $count(re \leqslant 5\%)$ denotes the number of samples with relative errors within 5%. The calculation results are as follows.

$$Are_{gold} = 0.48\%$$
$$Are_{bitcoin} = 2.96\%$$
$$Accuracy5_{gold} = 99.78\%$$
$$Accuracy5_{bitcoin} = 81.90\%$$

It is observed that our prediction accuracy for gold and bitcoin prices is very high, especially for gold, where the vast majority of the predicted prices for gold have a negligible difference from the true value. Even for Bitcoin, which has very high price volatility, we have over 80% of our predicted prices with relative errors of no more than 5% from the true price. Therefore, the validity of the prediction model is obvious, and the valid prediction results are the basis for finding the optimal strategy, on which we have achieved good results in this step.

We have achieved very effective forecasting results in the forecasting model. Since our forecasting model uses price data from the previous 30 days up to the current day, the model is updated for each day that passes. If we forecast the price data for the next two days, the previous day's forecast will be less reliable or even invalid by the next day, so we only forecast the next day's price data.

As for the decision model, we use a linear programming model to find the optimal solution, where the inequality constraint comes from objective facts. With very reliable forecast price data for the next day, the linear programming model that we use relies on the valid data we have, based on the objective of maximizing the value of the asset on the next day, obviously finds our optimal decision for that day. In other words, the result of the decision model is a locally optimal solution to our decision for each day, but it is already a globally optimal solution based on the forecast results for that day.

5 Problem3: Sensitivity Analysis

Then, we examine the impact of changes in transaction costs on our investment results. We choose to add the following perturbations to the transaction costs α_{gold} and $\alpha_{bitcoin}$ for gold and bitcoin, respectively.

$$\alpha_{gold} = \alpha_{gold} + delta$$
$$\alpha_{bitcoin} = \alpha_{bitcoin} + delta$$

Besides, $delta =$ -1% , -0.9% , ... ,1% .

The impact on the resulting result (i.e., assets of the last day) is shown in Figure 9 and 10.

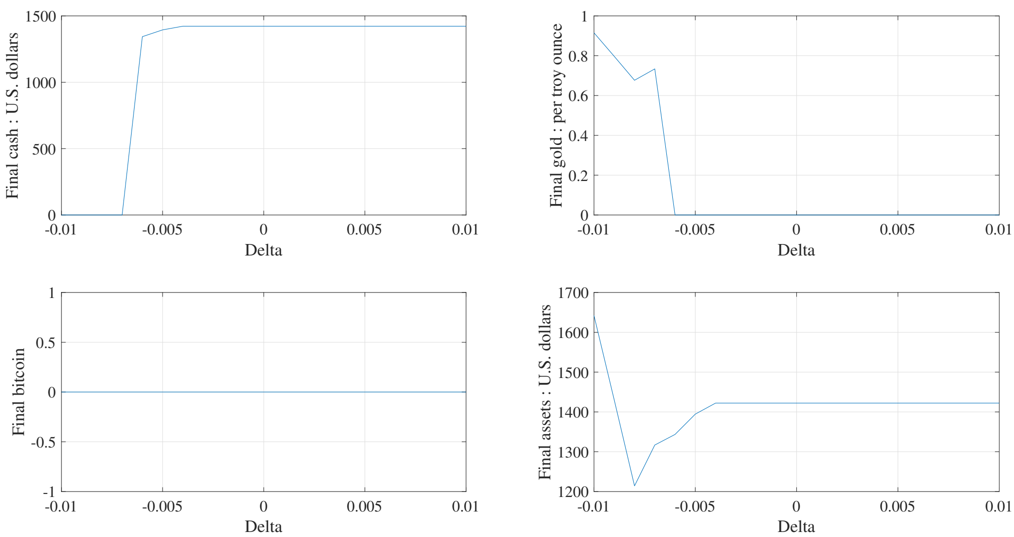

Figure 9: The effect of gold transaction cost perturbation

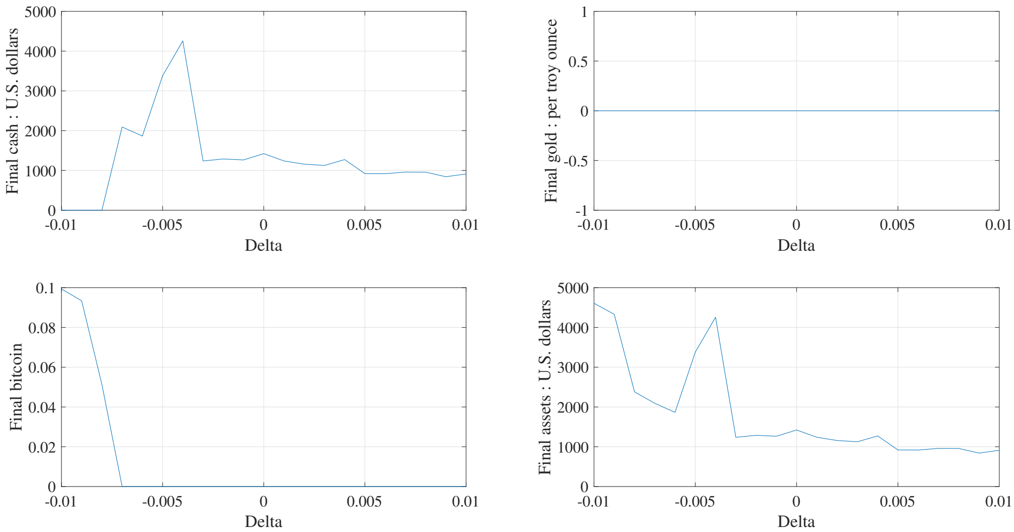

Figure 10: The effect of bitcoin transaction cost perturbation

5.1 The Effect of α_{gold} on The Composition and Value of Assets

As α_{gold} rises, the ultimate holdings of cash, gold, and bitcoin, as well as the ultimate asset value, do not change. The underlying reason may be that the raising α_{gold} leads to a decrease in the frequency of gold trading. Sometimes

In order to investigate the root cause, we recorded the daily asset holdings. And calculations were performed to verify the rationality of certain strategies. For example, continue to use the explanation of symbol in Table 1, the profit and loss coefficient of buying gold can be expressed as:

$$\beta_G = \frac{P_G^*}{(1+\alpha_{gold})P_G} \tag{17}$$

If $\beta_G > 1$, then it is profitable and we should buy gold. On the contrary , it is profitless and we wouldn't buy gold. We calculated β_G of each day and the values are all less than 1. Comparing the recorded daily gold holdings, it is true that no gold was bought from the beginning to the end. And naturally, no gold was used to sell.

Then as the α_{gold} increases, β_G will get smaller. So, from the beginning to the end there will still be no gold purchases. Eventually, the gold holdings will always be zero. Further, if gold is always untraded, then an increase in α_{gold} will not change the bitcoin trading process. Therefore, ultimately neither bitcoin nor cash holdings change, and ultimately the asset value does not change.

As α_{gold} decreases, cash final holdings gradually decrease until 0. Gold final holdings have an upward trend. Bitcoin final holdings remain unchanged. And there is no obvious pattern to the change in final asset value, but it increases as α_{gold} approaches 0.

The possible reason for such a change is that α_{gold} decreases and gold transactions become more frequent or even end up holding a lot of gold, leading to a decrease in the final cash stock and an increase in the final gold stock. According to the formula for β_G, this value will increase. If this value increases to more than 1, the purchase of gold can be profitable.

The profit and loss coefficient of selling bitcoins can be expressed as:

$$\gamma_B = \frac{P_B^*}{(1-\alpha_{bitcoin})P_B} \tag{18}$$

If $\gamma_B < 1$, then it is profitable. On the contrary , it is profitless. This shows that the change of α_{gold} do not affect the decision to sell bitcoins. So the final result of selling all bitcoins does not change.

For the α_{gold} equation close to 0, the gold's buying and selling strategy will be very similar. Eventually, the stock of gold becomes larger. The transaction cost in the process decreases and the final asset will naturally rise.

In conclusion, since the price of gold does not fluctuate so drastically, gains or losses do not change dramatically. Although the perturbations in its transaction costs have an impact on the decision-making process and thus make significant changes in the outcome, there are some patterns to these changes.

5.2 The Effect of $\alpha_{bitcion}$ on The Composition and Value of Assets

Fluctuation of Bitcoin price is very violent and its impact caused by the disturbance is complex. we will only analyze some possible reasons.

As $\alpha_{bitcion}$ rises, the final stock of both gold and bitcoin stays the same, with some minor reductions in the final cash and asset values.

The profit and loss coefficient for selling gold is expressed as :

$$\gamma_G = \frac{P_G^*}{(1-\alpha_{gold})P_G} \tag{19}$$

If $\gamma_G < 1$, then it is profitable and we should sell gold. On the contrary , it is profitless and we wouldn't sell gold.

The $\alpha_{bitcion}$ increase does not affect the decision to sell gold, so the eventual end of all gold selling does not change.

The reason why the final stock of bitcoin remains unchanged is different from the reason why the final stock of gold remains unchanged when α_{gold} increases. According to the formula of γ_B, even though $\alpha_{bitcion}$ increases, the predicted price may be much smaller than the current day's price, and the calculated result is still less than 1. In the end, the bitcoin is still sold out, resulting in the final stock of bitcoin remaining unchanged. Moreover, the final stock of both gold and bitcoin remains the same due to rising transaction costs, and the bitcoin price often fluctuates drastically, resulting in more transactions, thus making the final cash decrease.

As $\alpha_{bitcion}$ decreases, the final stock of gold remains constant. $\alpha_{bitcion}$ decreases less, the final stock of bitcoin also remains constant, the final stock of cash increases and then decreases, and the final asset value also increases and then decreases. $\alpha_{bitcion}$ decreases more, the final stock of bitcoin increases, the final stock of cash is 0, and the final asset value increases. $\alpha_{bitcion}$ decreases more, the final stock of bitcoin increases, the final stock of cash is 0, and the final asset value increases.

The $\alpha_{bitcion}$ reduction does not affect the decision to sell gold, so the eventual end of all gold selling does not change. The number of trades becomes more and the arbitrage opportunities become more when the $\alpha_{bitcion}$ drops less and the stock also remains the same, but the losses will be many when there are too many trades. $\alpha_{bitcion}$ declined more and the stock also increased.

The profit and loss coefficient for buging bitcoin is expressed as :

$$\beta_B = \frac{P_B^*}{(1+\alpha_{bitcoin})P_B} \tag{20}$$

If $\beta_B > 1$, then it is profitable and we should buy bitcoin. On the contrary , it is profitless and we wouldn't buy bitcion.

Then as the $\alpha_{bitcion}$ decreases, the larger β_B is. we are more likely to spend a large amount of cash on bitcoins. This leads to a decrease in the final cash stock until 0. But the final price of bitcoin is higher and the stock increases, so the final total asset value also increases.

In conclusion, Since the price of Bitcoin is very volatile and the gains or losses always change dramatically, the perturbations in its transaction cost not only have an impact on the decision-making process, but also make a significant change in the final outcome. However, there are patterns to some of these perturbations.

6 Conclusion

After the linear programming-based decision model was developed, we found the best strategy for each day of trade and we analyzed the sensitivity of the strategy. After accumulating day by day, we end up with a price of $1,422.20. Perhaps this number does not sound as gratifying as the rise in bitcoin, but we can guarantee that the trader can gain profit.

7 Memorandum

Dear trader,

We are writing to introduce you to a mathematical model which provides an optimal trading strategy. Many people want to add extra income by trading volatile assets now. You have wisely chosen to invest in a combination of gold and bitcoin which balances risk and profit. We know you need a sophisticated and proven investment strategy and we are committed to helping you gain profit.

We have been asked to develop a model that uses only the past stream of daily prices to date to determine each day if the trader should buy, hold, or sell them We will start with $1,000 on 9/11/2016 and use the five-year trading period from 9/11/2016 to 9/10/2021. The final price we arrive at is $1,422.20. Perhaps this number does not sound as gratifying as the rise in Bitcoin, but we are confident that you will be able to profit. Then, we will walk you through the development of the model and advise you trading strategies on further.

To get the best strategy for daily trading, we need to make full use of the gold and bitcoin price data as of the current day to build the corresponding prediction model which can predict the gold and bitcoin prices for the next day. Besides, we can then look for strategies based on the prediction result which is to get as many assets as possible to continue trading. So you will see two models - a prediction model and a decision model. We have tested and found that the accuracy of the prediction is very high. And the result of the decision model is a locally optimal solution to our daily trading decision, but already a global (5 years) optimal solution based on the results of the day's prediction.

We hope that this model will provide you with solid ideas and methods about portfolio investing, and as much as possible, ensure that you can achieve a substantial total return with this strategy. While the price increase of bitcoin is attractive, you risk losing money if its price drops significantly. The bitcoin price plummeted from above $20,000 at 2017 the end of the year. Until 2018 the end of the year, bitcoin fell to $3,200 and fell by about 85%. It's clear that bitcoin is very risky. As an average investors, we should be more cautious about investing than gambling on risky ground. Therefore we offer you this more reliable and mature investment strategy. We hope you will be able to make the desired gains in your future trading.

If there are any further questions or problems regarding this model, please contact us and we will do whatever we can to explain and improve the model.

We are looking forward to your good news.

Yours Sincerely,
Team 2206523

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Appendices

Here are programmes we used in our model as follow.

MATLAB code:

```
clear;clc;tic;
[Bn,~,Bc]=xlsread('BCHAIN-MKPRU','A2:B1827');
[Gn,~,Gc]=xlsread('LBMA-GOLD','A2:B1266');
Gn(isnan(Gn))=0;
figure(1);subplot 121;
bar(1:length(Gn),Gn,'k');
xlabel('The serial number of the gold price data sample');
ylabel('Gold price');kk=[];
for i=1:1265
if Gn(i)==0
Gn(i)=(Gn(i-1)+Gn(i+1))/2;
kk=[kk,i];
end
end
subplot 122;
b=bar(1:length(kk),kk);
xtips = b.XEndPoints;
ytips = b.YEndPoints;
labels = string(b.YData);
xlabel('Count of missing samples');
ylabel('The serial number of the missing sample');
text(xtips,ytips,labels,'HorizontalAlignment','center',...
'VerticalAlignment','bottom');
Gcc=cell(1265,1);
for i=1:1265
tchar=char(Gc(i,1));
if length(tchar)>8
Gcc{i}=tchar(3:end);
else
Gcc{i}=tchar;
end
end
Gcn=zeros(1265,1);
for i=1:1265
Gcn(i)=datenum(Gcc(i),'mm/dd/yy');
end
dn=Gcn(1):Gcn(end);
Bi=[[dn(1)-1;dn'],Bn];BG=[Bi,zeros(1826,2)];
for i=1:1826
for j=1:1265
if BG(i,1)==Gcn(j)
BG(i,3)=Gn(j);
BG(i,4)=1;
end
end
end
BG(1,3)=BG(2,3);
```

```
for i=1:1826
if BG(i,3)==0
BG(i,3)=BG(i-1,3);
end
end
Bitcoin=BG(1:500,2);gold=BG(1:500,3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;clc;tic;
t=load('data.mat').BG;t1=t(:,2);
plot(1:length(t1),t1);hold on
[imf, residual, info] =emd(t1,'Interpolation','pchip','Display',1);
[mm,nn]=size(imf);ttstar=zeros(mm,1);
for cc=1:length(t1)-29
E=zeros(30,30);
d=zeros(nn+1,1);
xh=zeros(nn+2,1);
for ab=1:nn
E=dvg(imf(cc:cc+29,ab));
K=sum(E,2);Kin=sum(E);Ks=sum(Kin);
Q=zeros(length(K));
for i=1:length(K)-1
Q(i,:)=E(i,:)/K(i);
end
alpha=0.2;
Q=Q*(1-alpha)+alpha*eye(length(K));
Pai=eye(length(K));Pai=Q'*Pai;
for i=2:length(K)
Pai=Q'*Pai+Pai;
toc;
end
S=zeros(length(K),length(K));
for i=1:length(K)
for j=1:length(K)
S(j,i)=(K(i)*Kin(j)/(2*Ks))*Pai(j,i);
end
end
[m,n]=max(S(length(K),:));
d(ab)=30-n;
xh(ab)=cc+30-d(ab);
end
d(nn+1,1)=d(1);
xh(nn+1,1)=cc+30-d(nn+1,1);xh(nn+2,1)=cc+29;tt=zeros(nn+1,1);
for i=1:nn+1
if xh(i,1)==xh(i+1,1)
tt(i,1)=t1(cc+29);
else
tt(i,1)=(t1(xh(i,1))-t1(xh(i+1,1)))/(xh(i,1)-xh(i+1,1))+t1(cc+29);
end
w(i)=d(i,1)/sum(d);
end
ttstar(cc+30,1)=sum(tt'.*w);
end
plot(ttstar,'r');
%%%%%%%%%%%%%%%%%%%%
function E=dvg(x)
```

```
n=length(x);
E=zeros(n,n);
for i=1:n
for j=i:n
if j==i+1
E(i,j)=1;continue;
end
for k=i+1:j-1
y(k)=x(j)+(x(i)-x(j))*(j-k)/(j-i);
if y(k)<=x(k)
E(i,j)=0;break;
else
E(i,j)=1;
end
end
end
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;clc;tic;
%%%%%%%%%%%%%%%%%%%%%%%
bite=load('bite.mat').ttstar;bite(1827)=[];
hj=load('gold.mat').ttstar;hj(1827)=[];BG=load('bg.mat').BG;
delta=-0.01:0.001:0.01;
%%%%%%%%%%%%%%%%%%%%%%%%%
HG=zeros(21,4);
for j=1:length(delta)
C=1000;
G=0;
B=0;
for i=30:1825
PG=BG(i,3);PB=BG(i,2);PGn=hj(i+1);PBn=bite(i+1);aG=0.01+delta(j);
aB=0.02;c=[-PGn+PG*(1-aG),-PBn+PB*(1-aB),PGn-PG*(1+aG),PBn-PB*(1+aB)];
A=[-PG*(1-aG),-PB*(1-aB),PG*(1+aG),PB*(1+aB)];b=C;
lb=zeros(4,1);
if BG(i,4)==-3
ub=[0,B,0,1000];
else
ub=[G,B,1000,1000];
end
t=linprog(-c,A,b,[],[],lb,ub);
m=t(1);n=t(2);x=t(3);y=t(4);
C=C+m*PG*(1-aG)+n*PB*(1-aB)-x*PG*(1+aG)-y*PB*(1+aB);
G=G-m+x;B=B-n+y;
R=BG(i+1,3)*G+BG(i+1,2)*B+C;
disp([1,j,i]);toc;
end
HG(j,:)=[C,G,B,R];
toc;
end
%%%%%%%%%%%%%%%%%%%%%%%
HB=zeros(21,4);
for j=1:length(delta)
C=1000;G=0;B=0;
for i=30:1825
```

```
PG=BG(i,3);PB=BG(i,2);PGn=hj(i+1);PBn=bite(i+1);
aG=0.01;aB=0.02+delta(j);
c=[-PGn+PG*(1-aG),-PBn+PB*(1-aB),PGn-PG*(1+aG),PBn-PB*(1+aB)];
A=[-PG*(1-aG),-PB*(1-aB),PG*(1+aG),PB*(1+aB)];b=C;lb=zeros(4,1);
if BG(i,4)==0
ub=[0,B,0,1000];
else
ub=[G,B,1000,1000];
end
t=linprog(-c,A,b,[],[],lb,ub);
m=t(1);n=t(2);x=t(3);y=t(4);
C=C+m*PG*(1-aG)+n*PB*(1-aB)-x*PG*(1+aG)-y*PB*(1+aB);G=G-m+x;B=B-n+y;
R=BG(i+1,3)*G+BG(i+1,2)*B+C;
disp([2,j,i]);toc;
end
HB(j,:)=[C,G,B,R];toc;
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;clc;tic;
bite=load('bite.mat').ttstar;bite(1827)=[];
hj=load('gold.mat').ttstar;hj(1827)=[];BG=load('bg.mat').BG;
%%%%%%%%%%%%%%%%%%%%%%
nn=31:1826;
figure(1);
plot(nn,BG(31:end,3),'b');hold on
plot(nn,hj(31:end),'r');
set(gca,'XTick',31:181:1826);
set(gca,'XTicklabel',{'10/11/2016','4/10/2017','10/8/2017',...
    '4/07/2018','10/5/2018','4/04/2019','10/02/2019','3/31/2020',...
    '9/28/2020','3/28/2021'})
xlim([31,1826]);
xlabel('Date : month/day/year');
ylabel('Price : U.S. dollars per troy ounce');
legend('Real','Predict');
title('Gold prices prediction graph');
%%%%%%%%%%%%%%%%%%%%%
nn=31:1826;
figure(2);
plot(nn,BG(31:end,2),'b');hold on
plot(nn,bite(31:end),'r');
set(gca,'XTick',31:181:1826);
set(gca,'XTicklabel',{'10/11/2016','4/10/2017','10/8/2017',...
    '4/07/2018','10/5/2018','4/04/2019','10/02/2019','3/31/2020',...
    '9/28/2020','3/28/2021'})
xlim([31,1826]);
xlabel('Date : month/day/year');
ylabel('Price : U.S. dollars per bitcoin');
legend('Real','Predict');title('Bitcoin prices prediction graph');
sseg=0;sseb=0;
for i=31:1826
sseg=sseg+(hj(i)-BG(i,3))^2;
sseb=sseb+(bite(i)-BG(i,2))^2;
end
mseg=sseg/(1826-31+1);mseb=sseb/(1826-31+1);
[~,areg,accuracy5g]=fscores(hj(31:end),BG(31:end,3));
```

```
[~,areb,accuracy5b]=fscores(bite(31:end),BG(31:end,2));
%%%%%%%%%%%%%%%%%%
function[ape,mape,accuracy5]=fscores(y_e,y)
n=length(y);
ape=abs(y_e-y)./y;
mape=sum(ape)/n;
k=0;
for i=1:n
if ape(i)<=0.05
k=k+1;
end
accuracy5=k/n;
end
end
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear;clc;tic;
bite=load('bite.mat').ttstar;bite(1827)=[];
hj=load('gold.mat').ttstar;hj(1827)=[];
BG=load('bg.mat').BG;HG=load('resultnew.mat').HG;
HB=load('resultnew.mat').HB;
%%%%%%%%%%%%%%%%%%%%
delta=-0.01:0.001:0.01;
figure(1);%Influence of gold transaction disturbance
subplot(2,2,1);
plot(delta,HG(:,1));
xlabel('Delta');ylabel('Final cash : U.S. dollars');
subplot(2,2,2);plot(delta,HG(:,2));
xlabel('Delta');ylabel('Final gold : per troy ounce');
subplot(2,2,3);plot(delta,HG(:,3));
xlabel('Delta');ylabel('Final bitcoin');
subplot(2,2,4);plot(delta,HG(:,4));
xlabel('Delta');ylabel('Final assets : U.S. dollars');
%%%%%%%%%%%%%%%%%%%
figure(2);%Influence of bitcoin transaction disturbance
subplot(2,2,1);plot(delta,HB(:,1));
xlabel('Delta');ylabel('Final cash : U.S. dollars');
subplot(2,2,2);plot(delta,HB(:,2));
xlabel('Delta');ylabel('Final gold : per troy ounce');
subplot(2,2,3);plot(delta,HB(:,3));
xlabel('Delta');ylabel('Final bitcoin');
subplot(2,2,4);plot(delta,HB(:,4));
xlabel('Delta');ylabel('Final assets : U.S. dollars');
```