Math 252 – §5.5 Question 4

$f(x_0)$	$f(0)$		0
$f(x_1)$	$f(1)$	A	3.1
$f(x_2)$	$f(2)$	B	4.5
$f(x_3)$	$f(3)$	C	4.3
$f(x_4)$	$f(4)$	D	6.7
$f(x_5)$	$f(5)$	E	6.0
$f(x_6)$	$f(6)$		0

Figure 1: *
Distances across the region at 1 cm intervals

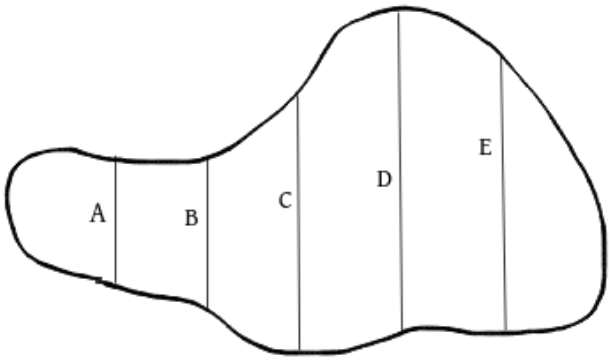

Figure 2: *
Vertical distances measured across the intervals

Interpreting the Table and Applying Simpson's Rule:

The table provides measurements across the region. To apply Simpson's Rule, we must identify our interval and parameters:

$$a = 0, \quad b = 6, \quad n = 6 \text{ (even, as required)}$$
$$\Delta x = \frac{b-a}{n} = \frac{6-0}{6} = 1 \text{ cm}$$
$$x_k = a + k\Delta x = 0, 1, 2, 3, 4, 5, 6 \quad \text{for } k = 0, 1, \ldots, 6$$

The table gives us the values A, B, C, D, E which represent $f(1) = 3.1$, $f(2) = 4.5$, $f(3) = 4.3$, $f(4) = 6.7$, $f(5) = 6.0$. The endpoints are $f(0) = 0$ and $f(6) = 0$ (region touches the x-axis at both ends).

$$\int_0^6 f(x)\,dx \approx \frac{\Delta x}{3}\Big[f(0) + 4f(1) + 2f(2) + 4f(3) + 2f(4) + 4f(5) + f(6)\Big]$$

$$= \frac{1}{3}\big[0 + 4(3.1) + 2(4.5) + 4(4.3) + 2(6.7) + 4(6.0) + 0\big]$$
$$= \frac{1}{3}(12.4 + 9.0 + 17.2 + 13.4 + 24.0) = \frac{76.0}{3} = \mathbf{25.33} \text{ cm}^2$$

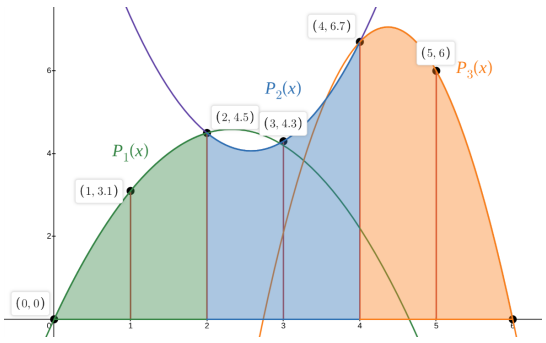

Figure 3: *
Three parabolas (each spanning two sub-intervals) illustrating Simpson's Rule approximation. Interactive version available at desmos.com/calculator/zeb7iuxly2.