Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
The sums used to compare the value. Annuity is used for count the future value in current value. Also lot of mathematic method to solve these ort of problems such as perturbation method, derivitive method, infinite limits and integration bounds

For the English:
perturbation, derivitive(derivative?), annuity, geometry sequence, arithmetic sequence, infinite convergence, differentiate

Lec 12. SUMS:

● **Tons of equations so I cut off most of them and just write down the big picture of method**

- Annuity

DEF: an year \$m – payment annuity pays, \$m ath the start of each year for n years

Assumption : fixed interest rate P

Total current value for dozens years =

I is [0 ~ n-1] ∑\$m/(1+p)^i which is sum of 'geometry sequence'

we derive this formula through 'Perturbation method' easily

S = 1 + x + x^2 ... + x^n-1 // xS = x + x^2 ... + x^n

subtract two equation => S = (1 – x^n)/(1 – x) if |x|<1 and n goes infinite,

the formula will be 1/1 – x and the value gonna be,

V = \$M(1+P)/P (P = 1 + x)

getting harder one

what about ∑ ix^i [i is 1~n]

1) perturbation method

S = (x - (n+1)x^n+1 + nx^n+2)/(1 – x)^2

2) derivitive method

for x != 1, we can differentiate both side and the we multiple additional x so make formula complete

Also if n goes infinite => x/(1 – x)^2

ex) annuity that pays im\$ at the end year I(1 ~ infinite) is worth m(1+p)/p^2

∑i = n(n+1)/2 // ∑i^2 = n(n+1)(2n+1)/6

GUESS: ∀n, ∑i^2 = an^3 + bn^2 + cn + d

we make 4 variable and 4 equation

plugin: n = 0 -> d = 0

n = 1 -> a+b+c = 1

n = 2 -> 8a+4b+c2 = 5

n = 3 = 27a+9b+3c = 14 a = 1/3, b = 1/2, c = 1/6, b = 0

$\sum\sqrt{i}$ Integration Bound for $\sum f(i)$ when $f(x)$ is a <u>positive increasing function</u> and vice versa

check the original graph that show integral (mensuration[quadrature] by parts) : picture proof

$f(n) + \int f(x)dx >= \sum f(i) >= f(1) + \int f(x)dx$ (lower bound and upper bound)
ex) $f(i) = \sqrt{i}$
$\int \sqrt{x}dx = 2/3*(n^{3/2} - 1)$ if n = 100
$2/3*n^{3/2} + 1/3 =< \sum f(i) =< 2/3*n^{3/2} + \sqrt{n} - 2/3$ => 667 ~ 676 -> $\sqrt{100}-1$
if n grows -> gap grows bigger but in '$O(\sqrt{n})$'

$\sum\sqrt{i} = 2/3*n^{3/2} + d(n)$ - (delta) where $1/3 =< d(n) =< \sqrt{n} - 2/3$
tilde notation [~] => $\sum\sqrt{i} \sim 2/3*n^{3/2}$
DEF: $g(x) \sim h(x)$ means $\lim g(x)/h(x) = 1$