Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
Independence means two or more events occurred irrelevantly. There are no related effect to both events and none of events effect to other. However that does not meant there are no intersect between of them. In the other words, it does not meant they are disjointed. With famous trial of ‘O.J Simpson’ is the well known example of conditional probability and independence.

For the English:
unusual, indictment, got booted, get by

Lec 20. INDEPENDENCE:

DEF: an event A is independence of an event B if Pr(A|B) = Pr(A) or if Pr(B) = 0
ex) flip 2 fair “Ind” coins
B = event 1^{st} coin is H => Pr(B) = 1/2 // A = ‘’ 2^{nd} ‘’ => Pr(A) = 1/2
then Pr(A|B) = 1/2 = Pr(A)

Disjoint != Tndependence => Pr(A|B) = 0 != Pr(A)
THM: product rule for indep events
If A is indep of B, then Pr(A∩B) = Pr(A)*Pr(B)
Pf: case 1 = Pr(B) = 0, then Pr(A|B) = 0 = Pr(A)*Pr(B)
case 2 = Pr(B) > 0, then Pr(A∩B) = Pr(A)*Pr(B) = Pr(A)

THM: Symmetry of Independence
If A is indep of B, then B is indep of A
ex) 2 fair indep coins => A = event coins match // B = ‘’ 1^{st} coin is H
Pr(A|B) = Pr(2nd coin is H) = 1/2 // Pr(A) = Pr(HH) + Pr(TT) = 1/4 + 1/4 = 1/2 -> indep
Pr(H) = p / Pr(T) = 1-p / Pr(A|B) = p / Pr(A) = p^2 + (1-p)^2
A&B are indep- iff Pr(B) = 0 (p = 0) or p = 1-2p+2p^2 => (1-2p)(1-p) = 0 => p = 1 or 1/2

- O.J.Simpson = Blood type and murdered by husband
1/10 => O type // 1/5 = rh factor // 1/4 = marker xyz => 1/200 match all three
DEF: events a1, a2 ... an are mutually indep if ∀i & ∀J ⊆ [1, n] - {i}
Pr(Ai|∩Aj) = Pr(Ai or Pr(∩Aj) = 0

Equivalnet DEF: Product rule
A1 ~ An are mutually indep- if ∀J ⊆ [1,N], Pr(∩Aj) = <u>∏Pr(Aj) product</u>
EX: n =3 a1 ~ a3 are mutually indep- if Pr(A1∩A2) = Pr(A1)Pr(A2) vice versa in 3 cases
Pr(A1∩A2∩A3) = Pr(A1)Pr(A2)Pr(A3)
ex) flip 3 fair mut- indep- coins A1 = event coin 1 matches coin 2 / A2 = ‘’ 2A3 / A3 = ‘’ 1A3
A1, A2, A3 are mut- indep- ??

Pr(Ai) = Pr(HH) + Pr(TT) = 1/4 + 1/4 = 1/2
Pr(Ai∩Aj) = Pr(HHH) + Pr(TTT) = 1/8 + 1/8 = 1/4 – Pr(Ai)Pr(Aj)
Pr(A∩A∩A) = 1/4 = Pr(A)()()? = 1/8 => not independent
DEF: events A1, A2 ~ An are pairwise indep-, if ∀I, J, Ai & Aj are indep-
Pairwise !=> but <= mutual Pr(M∩M∩M) =< Pr(M∩M) = Pr(M)Pr(M) = 1/50

- Killing by anyone VS murdered by husband
Pr(K|B) = 1/2000 => Pr(K|B∩M) > 1/2 // Pr(K|M)

- Birthday pardox
N birthdays (ex N = 365) // M people (ex M =100)
what is the prob- 2 or more people have same birthday?
by the proof of pigeonhole principle we easily assume that if we have 366 people (only one more people than year's day), we got more than 1/2 probability, however only more than 23 ppopulation we could get 1/2 probability and 57 people, is 99%. how could it possible? it is derived by hash

!P(n) = 1*(1 – 1/365)*(1 – 2/365) ...(1 – n-1/365) = 365!/365^n*(365-n)!
P(n) = 1 – 365!/365^n*(365-n)!

n	$p(n)$
1	0.0%
5	2.7%
10	11.7%
20	41.1%
23	50.7%
30	70.6%
40	89.1%
50	97.0%
60	99.4%
70	99.9%
100	99.99997%
200	99.9999999999999999999999999998%
300	$(100 - (6\times10^{-80}))\%$
350	$(100 - (3\times10^{-129}))\%$
365	$(100 - (1.45\times10^{-155}))\%$
366	100%