Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
In this lecture we go further of Expectation I and do some example to deeper understand of it. Also we could get useful bound from expectation and Murphy's law. And last we learned the concept of variance and standard deviation.

For the English:
hand wavy, eventually, chances, deviation, variance, square

Lec 22. EXPECTATION II:

THM1: Given a prob- space S & events A1, A2 ... An ⊆ S, the expected number of these events to occur is ∑Pr(Ai)
PF: Let Ti(w) = 1 if w∈Ai || 0 otherwise (indicator)
Ti = 1 iff Ai occurs
Let T = T1 + T2 ... Tn // Ex(T) = ∑Ex(Ti) (linearity of expectation)
= ∑Pr(Ti = 1) == ∑Pr(Ai)
ex) flip n fair coins Ai = event I-th coin is H / T = # head
Ex(T) = Pr(A1) + Pr(A2) ... Pr(An) = n/2
1/2 1/2 1/2
Ex(T) = ∑i*Pr(T = I) mut indep = ∑i(n|i)2^-n = n/2
= ∑i(n|k) = n*2^n-1

THM2:
Pr(T) =< Ex(T) = useful bound if Ex(T) is small
PF: Ex(T) = ∑Pr(T >= I) => Ex(T) >= Pr(T >= 1)
COR: Pr(T >= 1) =< ∑Pr(Ai) // PF: THM1

- Meltdown
n = 1000 / Pr(Ai) = 1/100 Ex(T) = 10 // ∀i,j Pr(Ai|Aj) = 1 => Pr(T >= 1) = 1/100
case larger, possibility larger

THM3: Murphy's law
Given mut indep events A1 ~ An, then Pr(T = 0) =< e^-Ex(T)
PF: Pr(T = 0) Pr(!A1∩!A2...!An) = ∏Pr(!i) - product // <u>∀x 1-x =< e^-x</u> (fact)
=< ∏E^-Pr(Ai) = e^-∑Pr(Ai) = e^-Ex(T)
COR: If we expect 10 or more mut indep events to occur, the prob that no event occur is =< e^-10 < 1/22,000
coincidence = there are more possibility to not happen

- Card trick
deck order: 4 5 2 3 10 Q 3 7 6 8
start with 3 and interval is the vlaue of card
doesn't matter where we started we eventually arrive same card

secret is here : larger 1 proportion => get same lane together
mean move chances of longer game => Murphy's law
Ai = event that we colide on I-th jump

THM4: (product rule for exp)
for any independence rv's R1, R2 Ex(R1*R2) = Ex(R1)*Ex(R2)
ex) roll 2 dice (fair, indep, 6-sided)
R1 = value on 1^{st} die / R2 =2nd
Ex(R1*R2) = Ex(R1)*Ex(R2) = 7/2 * 7/2 = 12.25
Non ex) Ex(R1*R1) = Ex(R2)^2 => ∑i^2Pr(R1 = I) = 1/6(1+4+9+16+25+36) = 15 1/6 != 9/4
because it is not independence
COR4.1: If R1, R2 ~ Rn mut indep, then Ex(R1*R2~Rn) = Ex(R1)*Ex(R2)...Ex(Rn) - product
COR4.2: For any constants a, b and rv R => Ex(aR+b) = A*Ex(R) + b
COR?? Ex(1/R) = 1/Ex(R)
EX: R = { 1 w/prob 1/2 || -1 != '' 1/2} Ex(r) = 0, Ex(1/R) = 0

- Benchmark Problem
- Sadly, I attend the class but I felt it useless I should check the attribute of expected value

Pr(R = 1000) = 1/2
Pr(R = -1000) = 1/2
Ex(R) = 0

vs

Pr(S = 1) = 1/2
Pr(S = -1) = 1/2
Ex(S) = 0
DEF: the variance of a rv R is
Var(R) = Ex((R-Ex(R))^2) = variance = Ex of square of deviation

Var(R) = 1,000,000
Var(S) = 1