Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:

The counting rule II is deepening of counting rule. We advanced set's sum rule and product rule to bookkeeper rule and binomial theorem. This lecture is centralized to count the number of cases without redundancy so we can find the value of cases.

For the English:

union, intersection, consecutive, coefficient

Lec 17. COUNTING RULES II:

- Inclusion & Exclusion

|MUE| = |M| + |E| - |M∩E|

|MUEUS| = |M| + |E| + |S| - {|M∩E| + |M∩S| + |E∩S|} + |M∩E∩S|

Generalize : ∑(-1)^k+1∑|∩Ai| (S=<{1~n} i∈S) s/t |S| = k

ex) How many permutation of {0, 1 ~ 9} have (consecutive) 42(M), 04(E), 60(S)

e.g (7, 2, 5, 6, 0, 4, 3, 5, 1, 9)

size of p60

Trick : count both as 1 = 9! = |p64| = |p42|

|p60| -biject-> {60, 1, 2 - 9}

p42∩p60 -> perm {42, 60, 1, 3, 5 ~}

|''| = 8!

p60∩p04 -> perm {604, ~}

|''| = 8! = |p04∩p42| = 8!

p60∩p04∩p42 = {6042, ~} = 7!

- Bookkeeper rule

Distinct copies of a letters l1, l2, ~lk # of sequences with n1 copies of l1, n2 copies l2, ~ nk copies lk => (n1 + n2 ... nk)!/(n1!*n2!...nk!) = (n1 + ...nk | n1, n2 ... nk

k=2 -> binomial coefficient

(n, k => (n, k,n-k => "combination = nCk or C(n, k)

ex) # bit sequences of length 16 and with 4 ones (16, 4 = 16!/4!*(16-4)!

- Subset rule

# of k element subsets of an n element set (n, k

THM(Binomial THM) ∀n (a+b)^n = ∑(n, k)a^n-k b^k

n=2 a^2 + ab + ba b^2 -> a^2 2ab + b^2

n=3 a^3 + aab + aba + baa + abb + bab + bba + b^3 => a^3 + 3a^2b + 3b^2a + b^3

3a 2a 1b 2b 1a 3 3 3 3

terms with k*a, (n-k)*b = # length n-sequences with k a's, (n-k) b's = (n, k

ex) n=3 (3:0, 2:1, 1:2, 0:3

- Poker Hands

Deck = 52 card, Card = suit {S, H, C, D}, Values = 2 ~ 10, j, q, k, a (13)
"Hand" = subset of deck of 5 cards
hands = (52, 5 => 2,598,960

4 Card : {8S, 9D, 8D, 8H, 8C}
representation : 1. value of the 4 cards -> 13
2. '' extra card = 12,   3. suit '' = 4
포카드 : {val (8), val2 (9), val3 (d)} = {13, 12, 4}
= # sequence = 13*12*4 = 624(gen prod rule)

Full House : {2C, 2S, 2D, JC, JD} => { 2(C,S,D) - J(C,D) }
representation : 1. value of triple -> 13
2. suit of triple -> 4
3. -> 12   4. -> 4C2 = 6
13*4*12*6 = 3744

- Guide Lines

1. for f:A -> B, check # elements of A mapped to each element of B
2. try solving peoblem in a different way (extra check)

1) value of 2 pairs(13, 2 // 2) suit of smaller pair -> (4, 2 // 3) '' larger '' -> (4,2
4) value of extra value 11   // 5) suit '' -> (4, 1 => TWO pair

- Hands with every suit

ex) {7D, KC, 3D, AH, 2S}
1) value of each suit in the order(D, C, H, S) - 13, 12, 11, 10
2) suit extra card -> 4
3) value '' -> 12 (D만 중복) -> 1/2 division rule

- Combinatorial Proofs

n-shirts, keep K, trash n-k   == (n, k = (n, n-k
ex) choose a team of k elements (student) out of n students
teams with Bob = (n-1, k-1 // # teams without bob = (n-1, k
=> (n-1, k-1 + (n-1, k == (n, k
1. define S   // 2. show |S| = n one way by counting
3. show |S| = m   ''   // 4. conclude n = m

THM: ∑(n, r *(2n, n-r == (3n, n
PF: S = student of n=balls chosen from a basket of n-red balls and 2n green balls
|S| = (3n, n => # n-element subsets from 3n-element set
subset with r red balls (n, r *(2n, n-r // |S| = ∑(n, r *(2n, n-r == (3n, n