Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
N X N network means the network connects n times input with n times output. There are 4 type of network system. Binary tree, 2D array, butterfly and last Benes. For analyzing stability and speed of network, there are criteria have to be checked. Diameter, switch size, # of switches, congestion are them.

For the English:

Lec 9. COMMUNICATION NETWORKS:

- Binary tree (complete)
- 0 = switch : direct packet through network
- □ = terminal : computer/source & destination of data

- Latency is the time required for a packet to travel from an input to output
- Diameter of a network is the length of the shortest path between the input & output that are furthest apart
- Permutation is a function Π: 0 – n-1 s/t no two member are mapped to the same value $\Pi(i) = \Pi(j)$ iff I = j // permutation routing problem for Π for each I direct the packet at In_i to $Out_{\Pi(i)}$: Path taken is denoted by $P_{i,\ \Pi(i)}$

ex) $\Pi(i)$ = N-1-i -> congestion = 4 // $\Pi(i)$ = i -> congestion = 1
=> congestion of the paths $P_{0,\Pi(0)}$ ~ $P_{n-1,\Pi(n-1)}$ is equal to rhe largest number of paths that pass through single switch
worst senario -> max congestion = max min congestion $P_{0,\Pi(0)}$ ~ $P_{n-1,\Pi(p-1)}$

- 2D Array

congestion THM: the congestion of an N-input array is 2
PF: let Π be a pem
$P_{i,\Pi(i)}$ = I row and $\Pi(i)$ column // path from In_i = rightward to column $\Pi(i)$ and downward to $Out_{\Pi(i)}$

PF: switch in row I and column $\Pi(i)$ transmits =< 2 packets if $\Pi(0) = 0$, $\Pi(n-1) = n-1$ => cong 2

- Butterfly

switch is unique identified it's row & col(min) (b1 ~ blogN, I (level))
this parted to (b1 ~ be+1, ... , blogN, l+1) and (b1 ~ bl+1, blogN, l+1) // (x, ~ xlogN, 0) -> (yq, x2 ~ xlogN, 1) -> (y1, y2, x3 ~ '', 2) -> (y ~ ylogN, logN) - final level
-check the diagram

- Benes

fold-duplicate Butterfly structure – recursive
symmetric -> subgraph also Benes (in middle top, bottom) => congestion is "1"

why? => each switch's input is 2 from top and bottom. however Benes structure contain same Benes structure subgraph in middle top and mid-bottom so there is no congestion, they are not mapped same switch

THM: the congestion of th N-input Benes is 1 who N =2^a for some a >= 1
PF: by induction on a P(a) = "the thm is true for a"
Base case: N = 2^1 In0 -> 0 - 0 - ㅁ out0 $\pi(0) = 0$ | $\pi(0) =1$
In1 -> 0 - 0 - ㅁ out1 $\pi(1) = 1$ | $\pi(1) =0$
Inductive step : assume P(a) is true
ex) $\pi(0) = 1$ 4 =3
1 =5 5 = 6 <u>constraint graph</u>
2= 4 6 = 0 If 2 packets must pass through different subnetwork, then
3 = 7 7 = 2 there is an edge between them

ex) he packet destined for Out0 (n(6) = 0) and the packet for Out4 (n(2) = 4) cannot pass through the same subnetwork

key insight : A 2-coloring of the constraint graph
재귀함수 느낌 (by induction) + permutation