Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
In order to solve matching problems, the lecture shows Mating Algorithm that is stable, fairness, terminated in finite time, and everyone get their proper mate.
We leared the idea of rogue couple and diverse case of graph that holds preference, perfection and weight.

For the English:
compatible, asymmetric, instability, optimistic <-> pessimistic (optimal <-> pessimal), vice versa, et cetera, without loss of generality, takes place, suitor, wooer, write off, crossout, get the boot or boots, condition is invoked, vacuously true, favorable

Lec 7. MATCHING PROBLEMS:

DEF: given graph G = (V, E), a matching is a subgraph of G where every node had degree 1 — ex) {x1, x6 - x2, x5} is matching of size 2 // 3 is best (check the original graph)

DEF: a matching is perfect if it has size |V|/2 (which is pairing)

DEF: the weigh of matching M is the sum of the weights on the edges in M
DEF: min-weight matching for G is a perfect matching for G with the min weight

Preferences
DEF: given a matching x and y form a rogue couple if they prefer each other to their mates = there is possibility of adultery(affair)
DEF: a matching is stable if there are no rogue couple
GOAL: find a perfect matching that is stable => not guarantee happy but stable(even)

Unisex scenario(bad case)
PF: by contradiction. Assume for some stable matching M
then mergatoid matched with someone in M
WLOG(by symmetry) -> assume mergatoid matched to Alex
Alex & Robin form rogue couple for M => not stable

Stable marriage problem
- N boys & N girls // each boy has his own ranked preference list of all the girl and vice versa

GOAL: find perfect matching without(w/o) rogue couple
greedy algorithm - does not work

Serenades - Mating Algorithm
NEED TO SHOW(TMA)
1. TMA terminates (quickly)
2. Everyone get married
3. No rogue couple (stable)
4. Fairness

THM1: TMA terminates in =< (less than) n^2 + 1 days
PF: by contradiction. suppose TMA does not terminates in n^2 + 1 days
CLAIM: if we don't terminate on a day, then some boy crosses a girl off his list that night => crossout more than n^2+1 girls => N =< n^2+1 ?? is contradiction

Let P (invariant): "if a girl G ever rejected a boy B, that means(then) G has a suitor who she prefers to B
LEMMA1: P is an invariant for TMA
PF: by induction . on #(n) days
Base case: day 0 -> 'vacuously true' // because yet No B got rejected
Inductive step: assume P holds at the end of the day D
-case1: G rejects B on day d+1: G got other suitor => there was battle = P is true
-case2: G rejected B before d+1: G had better suitor on day D/ G has same or better suitor on d+1 => P true on d+1

THM2: Everyone is married in TMA
PF: by contradiction, assume some boy is not married at the end => B rejected by every girl => every girl has better suitor(L1) => every girl is married and vice versa => contradiction

THM3: TMA produces a stable matching
PF: Let Bob & Gail be any pair(WLOG) that are not married
-case1: Gail rejected Bob => Gail has another auitor = better than Bob(L1) = not rogue
-case2: Gail did not rejected Bob => Bob never serenaded Gail => Bob married better spouser = not rogue
=> TMA is stable

-which is better: Proposer(boys got power-favorable) or Accepter
Fairness
Let S = set of all stable matchings (TMA produce stable matching/at least one)
For each person P, we define the realm of possibility for P to be set of mates $\{Q | \exists m \in S, \{P,Q\} \in m\}$

DEF: a person's optimal(pessimal) mate is his/her favorite(least) from the realm of possibility

THM4: TMA marries every boy with his optimal mate (THM5: vice versa in her / pessimal)
PF: by contradiction, suppose that there exist ∃stable matching m, ∃girl G whi faires worse than in TMA = rogue couple in m => m is not stable(contradiction)

=> TMA is balance and nice distributed way