Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
Not alike normal probability, conditional probability means it have special given 'condition' and that change the ground of assumption and calculation. With special condition, we have to forsake intuition for assumption. If we only count the result of event, it looks like fair and flawless, but we multiple the probability for product rule, all the outcome is changed.

For the English:
alleging, tenure, discrimination, following, wing it, union, beauty here is, inherently, fifty fifty, does not hold

Lec 19. CONDITIONAL PROBABILITY:

- Condition
given some other event taken place before.
Pr(A|B) = prob of A given B
DEF: if Pr(B) != 0, Pr(A|B) = Pr(A∩B)/Pr(B)

- Product rule : Pr(A∩B) = Pr(B)*Pr(A|B)
- General Product rule :
Pr(A1∩A2...∩An) = Pr(A1)*Pr(A2|A1)*Pr(A3|A1∩A2)...Pr(An|A1∩A2...∩An-1)

EX: In a best 2 out of 3 series, the prob of winning 1st game 1/2.
the prob of winning a game following a win is 2/3 <-> lose 1/3

I draw tree node
{1/3 - WW, 1/18 - WLW, 1/9 - WLL, 1/9 - LWL, 1/18 - LWW, 1/3 - LL}
product rule gives : Pr(WW) = Pr(W1st)*Pr(W2nd|W1st) = 1/2 * 2/3 = 1/3
General product rule : Pr(WLW) = Pr(W1st)*Pr(L2nd|W1st)*<u>Pr(W3|W1L2) = Pr(W3|L2)</u>
A = event win series // B = event win 1st game
Pr(A|B) = Pr(A∩B)/Pr(B) = 7/18 / 1/2 = 7/9

- a postieri conditional probability
Pr(B|A) where B preceeds A in time / out of order
ex) when win a series then prob of winning first gmae (reversed condition)
not always same = meaningless -> if Pr(A) = Pr(B) or Pr(A∩B)

ex) suppose we have 2 coins
Fair coin : Pr(H) = Pr(T) = 1/2
Unfair coin : Pr(H) = 1, Pr(T) = 0

B = k straight heads
Pr(A|B) = Pr(A∩B) / Pr(b) = P^2-k + (1-p) = p/p+2^k*(1-p)

- Medical testing
10% population has disease / If you have disease, 10% chance test is negative = false negative vise versa = flase positive
we draw tree
a / a+b => .09/.09 + .27 = 1/4
Pr(test is correct) = 0.72 why the gap is too large?
Because disease is too rare == sample is too small

- Carnival Dice
Player picks a number N [1 ~ 6] and rolls 3 dice
player win iff N matches >= 1 die
claim : Pr(win) = 1/2
Pf : let Ai = event ith die I =< N
Pr(win) = Pr(A1UA2UA3) = Pr(A1) + Pr(A2) + Pr(A3) = 1/6 + 1/6 + 1/6 = 1/2
but it need Ai are disjointed -> does not hold
Pr(A1UA2UA3) = Pr(A1) + Pr(A2) + Pr(A3) - {Pr(A1∩A2) + Pr(A2∩A3) + Pr(A1∩A3)} + Pr(A1∩A2∩A3)
= 1/6 + 1/6 + 1/6 -(1/36 + 1/36 + 1/36) + 1/216
= 0.421...   => unfair

Pr(AUB|C) = Pr(A|C) + Pr(B|C) -Pr(A∩B|C)
Claim : If C∩D == 0, then Pr(A|CUD) = Pr(A|C) + Pr(A+D) => 1 = 1 + 1 => not true

- Discrimination
Events A = applicant is admitted / Fcs = app – si female & cs.. Fee, Mcs, Mee
Pr(A|Fcs) < Pr(A|Mcs) ->> 0/1 < 50/100
Pr(A|Fee) < Pr(A|Mee) ->> 70/100 < 1/1
However
Pr(A|FcsUFee) > Pr(A|McsUMee) ->> 70/101 > 51/101

and Airlines case too
샘플의 크기 차이는 클수록 오차는 잡아먹고 작을수록 작은 차이에 치명적이다
Mark Twain : 3 lies
lie, damn lie, statistics