Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
we learned new concept of relation. It is sort of subset concept that shows element's relation by 4 properties(reflexive, symmetric, anti-sym, transitive). Throughout combination of properties, we deride 2 relations that Equivalence relations and Partial order. However parallel task scheduling is missing in lecture video

For the English:

Lec 11. RELATIONS, PARTIAL ORDERS, AND SCHEDULING:

- Relations (kind of directed graph)

a relation from a set A to a set B a subset R ⊆ A x B

ex) R = {(a, b): student a is taking class b} // (a, b) ∈: aRb, a~rb

Relation on A is a subset R ⊆ A x A

ex) A = Z(int) : xRy iff x =_ y (mod 5)

A = N(nature) : xRy iff x|y

A = N(") : xRy iff x =< y

set A together with R is a directed graph G = (V, E) with V = A, E = R

- July, Bill, Rob graph

- Properties

A relation R on A is

- reflexive : if xRx for all x ∈ A
- symmetric : if xRy => yRx for all x, y ∈ A
- antisymmetric : if xRy && yRx => x=y
- transitive : if xRy && yRz => xRz

ex)

	refl	symm	anti	trans
x=_y(mod 5)	yes	yes	no	yes
x|y	yes	no	yes	yes
x=<y	yes	no	yes	yes
- Equivalence relation = YYNY - Partial orders = YNYY				

- Equivalence relation

- reflexive, symm, trans

ex) equality (=) itself, x =_ y (mod n)

The equivalence class of x∈A is the set of all elements in A related to x by R : denoted [x]

[x] = {y : xRy}

ex) x =_ y (mod 5)
[7] = {.. , -3, 2, 7, 12, 17, ...}
[7] = [12] = [17] ...

A partition of A is a collection of disjoint, none empty sets
A1 ~ An ⊆ A, whose union is A
ex) {..., -5, 0, 5, 10 ...}, {... -4, 1, 6, 11 ...}, {...-2, 3, 8, 13 ...} ...

THM: the equivalence class of an equivalence relation on a set A form a partition of A
A relation is a (weak) Partial order if it is refl, anti, trans
A Partial order relation is denoted with ≦(one way) instead of R

- Hasse diagram
ex) wearing a clothes foot, underwear, shirt
DEF: a directed graph with vertex set A and edge set ≦ minus all self loops and edge implied by transitivity
THM: a PO set has no directed cycles other than self loops
PF: by contradiction, suppose ∃n >= 2, distinct elements a1 ~ an form a cycle
such that a1 ≦ a2 ≦ a3 ~~ ≦ an-1 ≦ an ≦a1
a1 ≦ a3 ~ by induction ~ a1 ≦ an ≦ a1 => contradiction / symmetric (self loop)
conclusion = after doing a self loop in PO set we get a directed acyclic graph (DAG)

a and b are incomparable if neither a≦b nor b≦a
are comparable if either a≦b or b≦a

- Total order
is partial order in which every pair of elements is comparable
ranking through straight line // PO -> TO we will learn this in next class

- Topological Sort
A total order consistent with a partial order is called a topological sort
a topological sort of aPO set (A, ≦) is a total order (A, ≦t) suchthat ≦ ⊆ ≦t
THM: every finite PO set has a Topological sort
X∈A is called minimal if !∃y∈A, y ! = x such that y≦x and vice versa
maximal x≦y
(Z, ≦) LEMMA: every finite PO set has a minimal element
DEF: a chain is a sequence of distinct elements // a1≦a2 ~ ≦at - length
PF: let a1 ≦ ~ ≦ an be a max length chain (assume it exists)
case1: a !∈ {a1 ~ an} : if a≦a1, then => contradiction (longer chain) - !a≦a1
case2: a ∈ {a1 ~ an} : if a≦a1 => there is cycle => contradiction = !a≦a1
=> a1 is minimal