Mathematics of Computer Science. - M.I.T. opencourseware

Abstract:
Expectation is just a mean value, which is when the event occurs, people expected value of mean value. We can derive mean value from the all outcome's sum with probability weighted.

For the English:
corollary, attainable

Lec 22. EXPECTATION I:

DEF: the expected value (aka average or mena) of rv(random variable) R over a prob- space S is denoted by Ex(R) = ∑P(w)*Pr(w) == variable * weight
ex) roll fair 6-sided die, P = outcome
Ex(R) = 1*1/6 + 2*1/6 … 6*1/6 => 3.5 // doesn't have to attainable value of R
DEF: the median of R is x ∈ Range(R) s.t Pr(R<x) =< 1/2, Pr(R>x) < 1/2
in die => "4" is median

As always – game => Coin toss & draw tree node 4 times => 1/16
A B C (guessing) => Coin result
Ex(adam) = 0 => fair game
in lottery case => people don't pick number randomly // by the way lottery is terrible same state takes half

THM: Ex(R) = ∑x*Pr(P=x)
PF: Ex(P) = ∑R(w)Pr(w) = ∑∑R(w)Pr(w) = ∑∑x Pr(w) = ∑x∑Pr(w) Pr(P=x)
=> ∑x Pr(R=x)
COR: If R:S -> |N, then Ex(R) = ∑i Pr(R = I)
THM: If R:S -> |N, then Ex(R) = ∑Pr(R > I) = ∑Pr(R >= I) difference is started from 0 or 1
PF: ∑Pr(R>i) = Pr(R>0) = Pr(R = 1) + Pr(R = 2) … Pr(R = endless)
+ Pr(R>1) = Pr(R = 2) + …
…
= 1*Pr(R = 1) + 2*Pr(R = 2) …
= Ex(R) = ∑i (Pr(R = I)) a

EX: system fails with prob- P at each step (mut- indep-)
Ex(step) = ? // R = step when first failure occurs
Ex(R) = ∑Pr(R > I) -> Pr(no failure in first I steps) = Pr(ok in 1^{st} step)*(2nd)*…(ith step)
= (1-p)(1-p)…(1-p) = (1-p)^i = @^i wjere @ = 1-p
Ex(R) = ∑@^i = 1/@-1 = 1/p

Ex) Pr(Boy) - 1/2 quit when get girl birth / R = # boys / mut- indep-
Ex(r) = 1/p = # children = MTTF – 1 (girl one) = 1/1/2 = 1 (one boy before we get the girl)

Let D = delay of packet on channel
Let f(x) = Pr(d = x) be pdf for D -> graph be like fraction function graph
Pr(D >= I) = 1/i => THM Ex(D) = ∑Pr(D>=i) => ∑1/i => !! Endless !!
PF: 1/ 1/4 1/4 1/8 1/8 ... => 1/2^n
what about ∑1/i^2 => converge, let find the proof in Qoura

- Linearity of Expectation
THM: for any rv's R1 & R2 on a prob space S, <u>Ex(R1 + R2) = Ex(R1) + Ex(R2)</u>
COR: ∀k ∈ |N, & rv's R1, R2 ... Rk on S
Ex(R1 + R2 ... Rk) = ∑Ex(Ri)
No Independence needed!!
Ex) roll fair dice R1 = outcome on 1^{st} die, R2 = 2^{nd} // R = R1 + R2
Ex(R) = Ex(R1) + Rx(R2) = 3.5 + 3.5 = 7
only summation

ex) Hat check problem
- n men
- each man gets random hat
P = # man to get the right hat   Ex(r) = ∑k*Pr(R = k)
Pr(R = k) == 1/k!(n-k)   (k =< n-2) || 1/n!   (k = n-1, n)
=> use linearity of Expectation -> express R as Sum

easier solution
R = ∑Ri, Ri = ith man get his right hat back || 0, otherwise   => indicator
Ex(R) = Ex(R1) + ... Ex(Rn) = Pr(R1 = 1) + Pr(R2 = 1) ... Pr(Rn = 1)
= 1/n + 1/n ... + 1/n => 1
things happen simultaneously