

Exercises 7.2

2. List all terms needed to generate the shape functions for a serendipity quadrilateral element with 12 nodes. Use computersoftware to compute all polynomials coefficients.

```
clear all
clc
```

```
syms xi eta
syms a1 a2 a3 a4 a5 a6 a7 a8 a9 a10 a11 a12
syms c1 c2 c3 c4 c5 c6 c7 c8 c9 c10 c11 c12
```

```
Cxy = a1 + a2*xi + a3*eta + a4*xi*eta + a5*xi^2 + ...
      a6*eta^2 + a7*xi^2*eta + a8*xi*eta^2 + a9*xi^3 + ...
      a10*eta^3 + a11*xi^3*eta + a12*xi*eta^3;
```

```
x = linspace (-1, 1, 4);

eqs =[subs(Cxy,[xi eta],[x(1) x(1)]) == c1, ...
      subs(Cxy,[xi eta],[x(4) x(1)]) == c2,...
      subs(Cxy,[xi eta],[x(4) x(4)]) == c3,...
      subs(Cxy,[xi eta],[x(1) x(4)]) == c4,...
      subs(Cxy,[xi eta],[x(2) x(1)]) == c5,...
      subs(Cxy,[xi eta],[x(3) x(1)]) == c6,...
      subs(Cxy,[xi eta],[x(4) x(2)]) == c7,...
      subs(Cxy,[xi eta],[x(4) x(3)]) == c8,...
      subs(Cxy,[xi eta],[x(3) x(4)]) == c9,...
      subs(Cxy,[xi eta],[x(2) x(4)]) == c10,...
      subs(Cxy,[xi eta],[x(1) x(3)]) == c11,...
      subs(Cxy,[xi eta],[x(1) x(2)]) == c12];

var = [a1, a2,a3, a4, a5, a6, a7, a8, a9, a10, a11, a12];
Cvar = [c1, c2, c3, c4, c5, c6, c7, c8, c9, c10, c11, c12];
```

```
A = solve(eqs,var);
a1 = A.a1;
a2 = A.a2;
a3 = A.a3;
a4 = A.a4;
a5 = A.a5;
a6 = A.a6;
a7 = A.a7;
a8 = A.a8;
a9 = A.a9;
a10 = A.a10;
a11 = A.a11;
a12 = A.a12;
```

```
Cxy = a1 + a2*xi + a3*eta + a4*xi*eta + a5*xi^2 + ...
      a6*eta^2 + a7*xi^2*eta + a8*xi*eta^2 + a9*xi^3 + ...
      a10*eta^3 + a11*xi^3*eta + a12*xi*eta^3;
```

```
[N,Ci] = coeffs(Cxy,Cvar);
N1 = simplify(N( 1))
```

$$N1 = \frac{(\eta - 1) (\xi - 1) (9 \eta^2 + 9 \xi^2 - 10)}{32}$$

$$N2 = \text{simplify}(N(2))$$

$$N2 = - \frac{(\eta - 1) (\xi + 1) (9 \eta^2 + 9 \xi^2 - 10)}{32}$$

$$N3 = \text{simplify}(N(3))$$

$$N3 = \frac{(\eta + 1) (\xi + 1) (9 \eta^2 + 9 \xi^2 - 10)}{32}$$

$$N4 = \text{simplify}(N(4))$$

$$N4 = - \frac{(\eta + 1) (\xi - 1) (9 \eta^2 + 9 \xi^2 - 10)}{32}$$

$$N5 = \text{simplify}(N(5))$$

$$N5 = \frac{9 (\eta - 1) (-3 \xi^3 + \xi^2 + 3 \xi - 1)}{32}$$

$$N6 = \text{simplify}(N(6))$$

$$N6 = - \frac{9 (\eta - 1) (-3 \xi^3 - \xi^2 + 3 \xi + 1)}{32}$$

$$N7 = \text{simplify}(N(7))$$

$$N7 = - \frac{9 (\xi + 1) (-3 \eta^3 + \eta^2 + 3 \eta - 1)}{32}$$

$$N8 = \text{simplify}(N(8))$$

$$N8 = \frac{9 (\xi + 1) (-3 \eta^3 - \eta^2 + 3 \eta + 1)}{32}$$

$$N9 = \text{simplify}(N(9))$$

$$N9 = \frac{9 (\eta + 1) (-3 \xi^3 - \xi^2 + 3 \xi + 1)}{32}$$

$$N10 = \text{simplify}(N(10))$$

$$N10 = - \frac{9 (\eta + 1) (-3 \xi^3 + \xi^2 + 3 \xi - 1)}{32}$$

$$N11 = \text{simplify}(N(11))$$

$$N11 =$$

$$-\frac{9(\xi-1)(-3\eta^3-\eta^2+3\eta+1)}{32}$$

N12 = simplify(N(12))

N12 =

$$\frac{9(\xi-1)(-3\eta^3+\eta^2+3\eta-1)}{32}$$