Exercises 7.1

1. Find an approximate solution for the BVP using weighted integrals with an approximate solution of the form u(x) =c1x2+c2x+c3 and a weight function w(x) = 1.

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + u = 1 \qquad \text{for } x \in (0,1)$$
$$u(0) = 1$$
$$u(1) = 0$$

```
clear all
clc
```

```
syms c1 c2 c3
syms t1 t2
syms x
u  = @(x) c1*x*x+c2*x+c3;
w1 = @(x) 1;
w2 = @(x) x;
R  = @(x) diff(u(x),2) + u(x);
u0 = u(0) == 1
```

u0 = $c_3 = 1$

```
u1 = u(1) == 0
```

u1 = $c_1 + c_2 + c_3 = 0$

```
A1  = int(R(x)*w1(x), 0, 1) == 1
```

A1 =

$$\frac{7\,c_1}{3} + \frac{c_2}{2} + c_3 = 1$$

```
A2  = int(R(x)*w2(x), 0, 1) == 1
```

A2 =

$$\frac{5\,c_1}{4} + \frac{c_2}{3} + \frac{c_3}{2} = 1$$

```
eqs = [subs(A1, c3, 1), ...
       subs(A2, c3, 1)];
A = solve (eqs, [c1 c2]);
C1 = A.c1
```

C1 =

$$-\frac{18}{11}$$

```
C2 = A.c2
```

C2 =

$$\frac{84}{11}$$

```
C3 = 1
```

```
C3 = 1
```

```
u  = @(x) C1*x*x+C2*x+C3;
R  = @(x) diff(u(x),2) + u(x);
ux = u(x)
```

ux =

$$-\frac{18\,x^2}{11}+\frac{84\,x}{11}+1$$

```
Rx = R(x)
```

Rx =

$$-\frac{18\,x^2}{11}+\frac{84\,x}{11}-\frac{25}{11}$$

```
% clear all
% f = @(c1,c2) 7*c1/3 + c2/2 + 1
% f(-18/11,84/11)
```