Exercises 7.2

1. Solve the boundary value problem stated below using the three studied methods of weighted residuals and one basis function

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y = 2x, \qquad x \in (0,1)$$
$$y(0) = 0$$
$$y(1) = 0$$

Make a plot with the three approximate functions obtained and a second plot with the three corresponding residual functions.

```
clear all
clc
syms x
```

Weighted Residuals

```
syms c1
N1 = x*(x-1);
y  = @(x) c1*N1;
R  = @(x)diff((y(x)),2) + y(x) - 2*x;
w1 = @(x) 1;
A = int(R(x)*w1(x), 0, 1);
c1 = solve (A, c1);
y  = @(x) c1*N1;
R  = @(x)diff((y(x)),2) + y(x) - 2*x;
y1x = y(x)
```

```
y1x =
```

$$\frac{6\,x\,(x-1)}{11}$$

```
R1x = R(x)
```

```
R1x =
```

$$\frac{6\,x\,(x-1)}{11} - 2\,x + \frac{12}{11}$$

Least squares

```
syms c1
N1 = x*(x-1);
y  = @(x) c1*N1;
R  = @(x)diff((y(x)),2) + y(x) - 2*x;
w1 = diff(R(x),c1);
A = int(R(x)*w1, 0, 1);
c1 = solve (A, c1);
y  = @(x) c1*N1;
R  = @(x)diff((y(x)),2) + y(x) - 2*x;
y2x = y(x)
```

```
y2x =
```

$$\frac{55\,x\,(x-1)}{101}$$

```
R2x = R(x)
```

R2x =

$$\frac{55\,x\,(x-1)}{101}-2\,x+\frac{110}{101}$$

Galerkin

```
syms c1
N1 = x*(x-1);
y  = @(x) c1*N1;
R  = @(x)diff((y(x)),2) + y(x) - 2*x;
w1 = N1;
A = int(R(x)*w1, 0, 1);
c1 = solve (A, c1);
y  = @(x) c1*N1;
R  = @(x)diff((y(x)),2) + y(x) - 2*x;
y3x = y(x)
```

y3x =

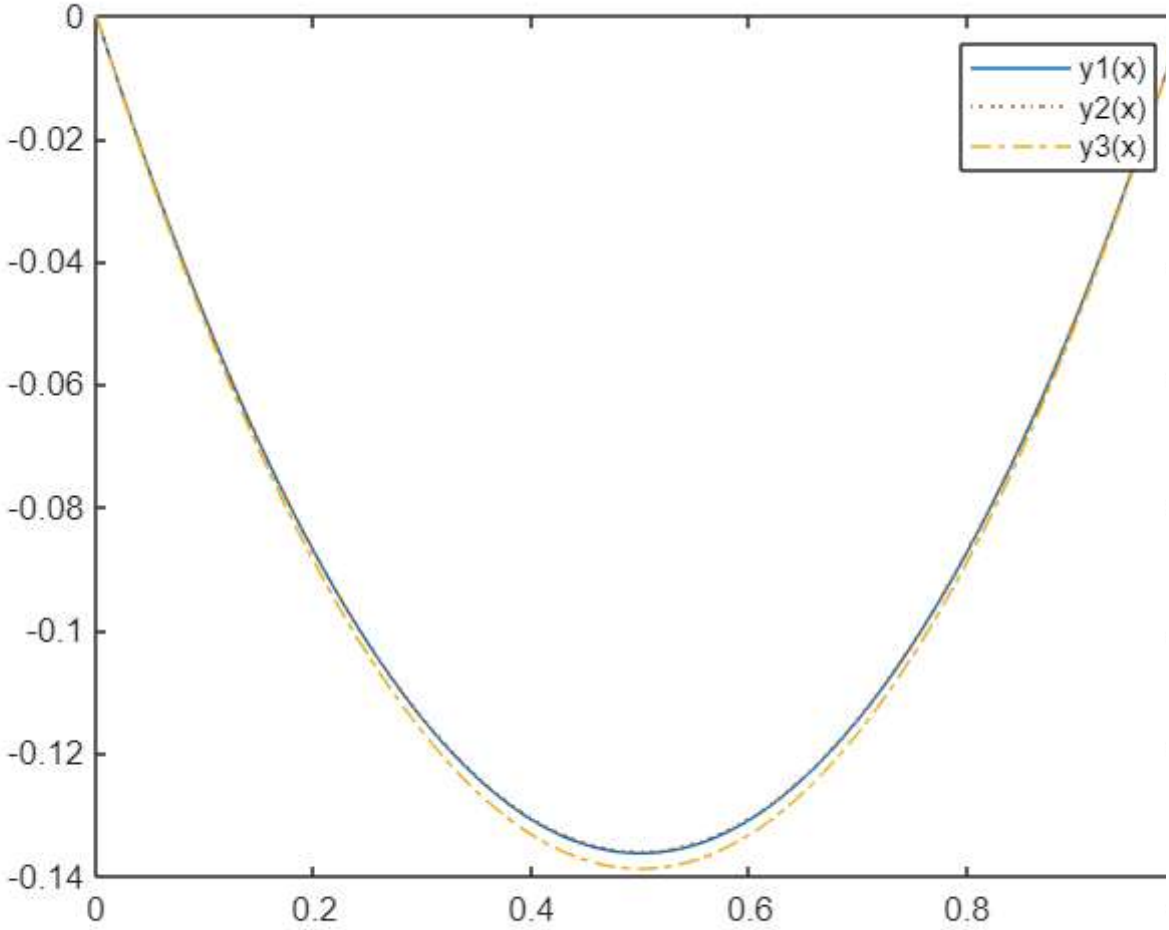
$$\frac{5\,x\,(x-1)}{9}$$

```
R3x = R(x)
```

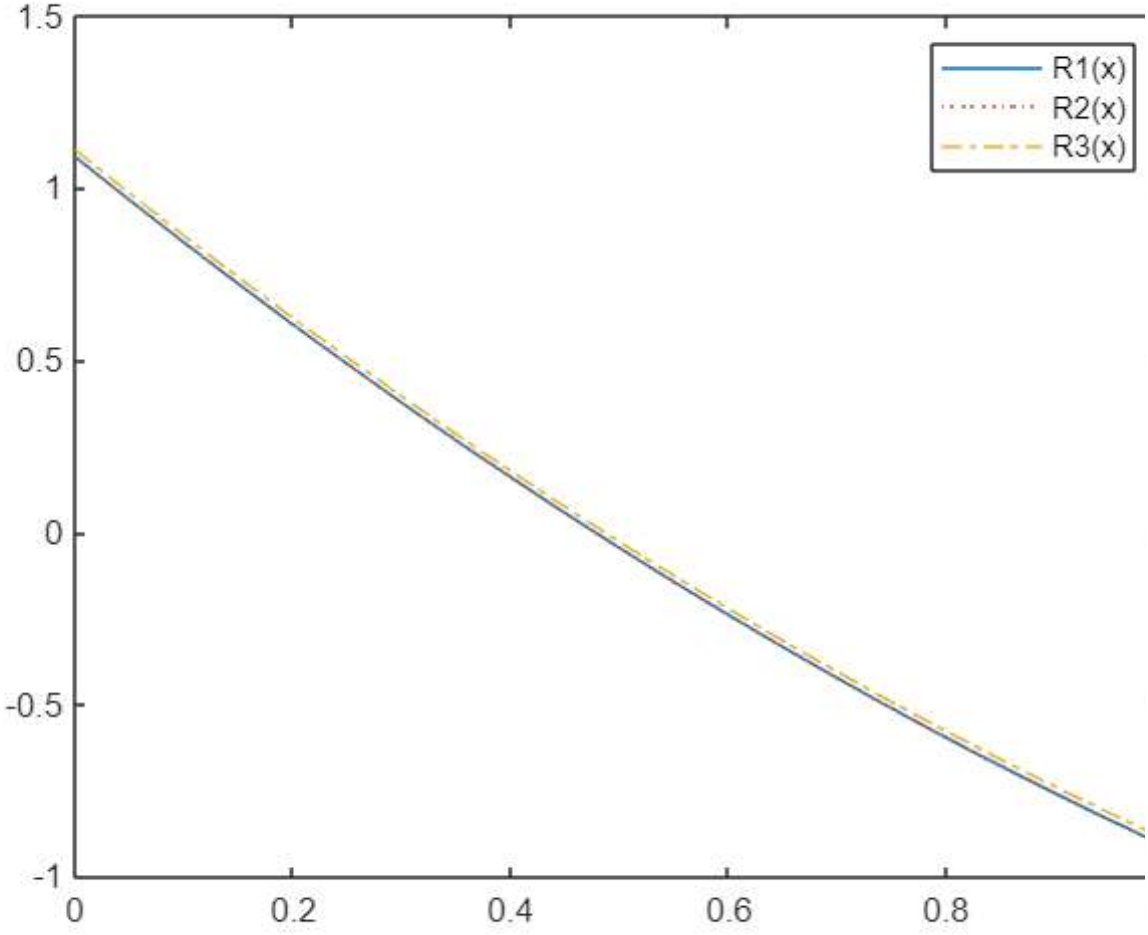
R3x =

$$\frac{5\,x\,(x-1)}{9}-2\,x+\frac{10}{9}$$

```
X = 0:0.01:1;
y1 = subs (y1x,x, X);
y2 = subs (y2x,x, X);
y3 = subs (y3x,x, X);
R1 = subs (R1x,x, X);
R2 = subs (R2x,x, X);
R3 = subs (R3x,x, X);
plot(X,y1,'-', X,y2,':', X,y3,'-.')
legend ('y1(x)','y2(x)','y3(x)')
```




```
plot(X,R1,'-', X,R2,':', X,R3,'-.')
legend ('R1(x)','R2(x)','R3(x)')
```


1.5
1
0.5
0
-0.5
-1
0
0.2
0.4
0.6
0.8
R1(x)
R2(x)
R3(x)