Exercises 5.4

1. Using computer software, compute the stiffness matrix for the trian-gular element at right. Use E= 20GPa and v= 0.25.

```
clear all
clc
```

```
E  = 20;
nu = 0.25;
q  = 4;
C  = [1 1; 2 1; 1 2];
```

```
D = E/(1-nu^2)*[1 nu 0;
                nu 1 0;
                0 0 (1-nu)/2];
```

```
K = compute_k(C, D, q)
```

K =

$$\begin{pmatrix} \frac{44}{3} & \frac{20}{3} & -\frac{32}{3} & -4 & -4 & -\frac{8}{3} \\ \frac{20}{3} & \frac{44}{3} & -\frac{8}{3} & -4 & -4 & -\frac{32}{3} \\ -\frac{32}{3} & -\frac{8}{3} & \frac{32}{3} & 0 & 0 & \frac{8}{3} \\ -4 & -4 & 0 & 4 & 4 & 0 \\ -4 & -4 & 0 & 4 & 4 & 0 \\ -\frac{8}{3} & -\frac{32}{3} & \frac{8}{3} & 0 & 0 & \frac{32}{3} \end{pmatrix}$$

```
function Q = quadrature(q)
quadrature_1_pts = [1/3  1/3  1/2];

quadrature_3_pts = [1/6  1/6  1/6
                    2/3  1/6  1/6
                    1/6  2/3  1/6];

quadrature_4_pts = [1/3  1/3  -9/32
                    3/5  1/5  25/96
                    1/5  3/5  25/96
                    1/5  1/5  25/96];
if q == 1
    Q = quadrature_1_pts;
elseif q == 3
    Q = quadrature_3_pts;
elseif q == 4
    Q = quadrature_4_pts;
else
    Q = 0;
end
end

function K = compute_k(C, D, q)
```

```
q = quadrature(q);
Npst = size(q, 1);
nnodes = size(C, 1);
ndof = 2;
K = zeros(nnodes*ndof, nnodes*ndof);
for i = 1:Npst
    xi  = q (i, 1);
    eta = q (i, 2);
    w   = q (i, 3);
    B   = compute_B(C, xi, eta);
    dN  = tri3_derivs(xi, eta);
    J   = C'*dN;
    K = K + B'*D*B*det(J)*w;
end
end

function B = compute_B(C, xi, eta)
nnodes = size(C, 1);
ndof = 2;
dN   = tri3_derivs(xi, eta);
J    = C'*dN;
dNdX = dN/J;
    for i = 1: nnodes
    c = (i-1) * ndof;
    B(1, c+1) = dNdX(i,1);
    B(2, c+2) = dNdX(i,2);
    B(3, c+1) = dNdX(i,2);
    B(3, c+2) = dNdX(i,1);
    end
end

function dN = tri3_derivs (r, s)
syms xi eta
N = [ 1 - xi - eta
          xi
               eta];
dN = subs([diff(N, xi) diff(N, eta)], [xi eta], [r s]);
end
```