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1. Why the first three shape functions of a quadratic triangle element cannot be used to interpolate nodal values of a lineartriangle element?

Pois os coeficientes são quadráticos e não lineares como mostrado a seguir

$N_1(\xi,\eta) = 1 - \xi - \eta$ $N_2(\xi,\eta) = \xi$ $N_3(\xi,\eta) = \eta$	$N_1(\xi,\eta) = 1 - 3(\xi+\eta) + 2(\xi+\eta)^2$ $N_2(\xi,\eta) = 2\xi^2 - \xi$ $N_3(\xi,\eta) = 2\eta^2 - \eta$ $N_4(\xi,\eta) = 4\xi - 4\xi(\xi+\eta)$ $N_5(\xi,\eta) = 4\xi\eta$ $N_6(\xi,\eta) = 4\eta - 4\eta(\xi+\eta)$
Elemento linear com três nós	Elemento quadrático com seis nós. Os três primeiros termos são quadraticos, assim não podendo ser usasdo para elementos triangulares lineares com três nós.

2. List all terms needed to generate the shape functions for a serendipity quadrilateral element with 12 nodes. Use computersoftware to compute all polynomials coefficients.

```
n = 4;
xi = linspace (-1, 1, n);
eta = xi';
no = zeros(12:2);

no( 1, :) = [xi(1) eta(1)];
no( 2, :) = [xi(4) eta(1)];
no( 3, :) = [xi(4) eta(4)];
no( 4, :) = [xi(1) eta(4)];
no( 5, :) = [xi(2) eta(1)];
no( 6, :) = [xi(3) eta(1)];
no( 7, :) = [xi(4) eta(2)];
no( 8, :) = [xi(4) eta(3)];
no( 9, :) = [xi(3) eta(4)];
no(10, :) = [xi(2) eta(4)];
no(11, :) = [xi(1) eta(3)];
no(12, :) = [xi(1) eta(2)];
No = zeros (12:12);

for i = 1: 12
    No(i , :) = [1, no(i, 1), no(i, 2), no(i, 1)*no(i, 2), ...
        (no(i, 1))^2, (no(i, 2))^2, (no(i, 1))^2*(no(i, 2)), ...
        (no(i, 1))*(no(i, 2))^2, (no(i, 1))^3, (no(i, 2))^3, ...
        (no(i, 1))^3*(no(i, 2)), (no(i, 1))*(no(i, 2))^3];
end

C = inv(No);
```

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```
syms xi eta

for i = 1: 12
n = C(:, i) .* [1, xi, eta, xi*eta, xi^2, ...
    eta^2, xi^2*eta, xi*eta^2, xi^3, eta^3, ...
    xi^3*eta, xi*eta^3]';
N(i, 1) = sum(n);
end
simplify (N)
```

N1 =

$$\frac{(\eta-1)\ (\xi-1)\ (9\,\eta^2+9\,\xi^2-10)}{32}$$

N2 =

$$-\frac{(\eta-1)\ (\xi+1)\ (9\,\eta^2+9\,\xi^2-10)}{32}$$

N3 =

$$\frac{(\eta+1)\ (\xi+1)\ (9\,\eta^2+9\,\xi^2-10)}{32}$$

N4 =

$$-\frac{(\eta+1)\ (\xi-1)\ (9\,\eta^2+9\,\xi^2-10)}{32}$$

N5 =

$$\frac{9\ (\eta-1)\ (-3\,\xi^3+\xi^2+3\,\xi-1)}{32}$$

N6 =

$$-\frac{9\ (\eta-1)\ (-3\,\xi^3-\xi^2+3\,\xi+1)}{32}$$

N7 =

$$-\frac{9\ (\xi+1)\ (-3\,\eta^3+\eta^2+3\,\eta-1)}{32}$$

N8 =

$$\frac{9\ (\xi+1)\ (-3\,\eta^3-\eta^2+3\,\eta+1)}{32}$$

N9 =

$$\frac{9\ (\eta+1)\ (-3\,\xi^3-\xi^2+3\,\xi+1)}{32}$$

N10 =

$$-\frac{9\ (\eta+1)\ (-3\,\xi^3+\xi^2+3\,\xi-1)}{32}$$

N11 =

$$-\frac{9\ (\xi-1)\ (-3\,\eta^3-\eta^2+3\,\eta+1)}{32}$$

N12 =

$$\frac{9\ (\xi-1)\ (-3\,\eta^3+\eta^2+3\,\eta-1)}{32}$$

3. Find the shape functions of nodes 1, 4 and 10 of a 10-node 2D Lagrangian triangular element.

```
    1  3
    |  |\
    |  | \
    |  8  7
    |  |   \
   eta |    \
    |  9 10  6
    |  |      \
    |  |       \
    0  1--4--5--2
    |
    +--0----xi---1-->
```

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```
syms xi eta
syms a1 a2 a3 a4 a5 a6 a7 a8 a9 a10
syms c1 c2 c3 c4 c5 c6 c7 c8 c9 c10

Cxy = a1 + a2*xi + a3*eta + ...
    a4*xi^2 + a5*eta^2 + a6*xi*eta + ...
    a7*xi^3 + a8*eta^3 + a9*xi^2*eta + a10*xi*eta^2;

x = linspace (0, 1, 4);

eqs =[subs(Cxy,[xi eta],[x(1) x(1)]) == c1, ...
    subs(Cxy,[xi eta],[x(4) x(1)]) == c2,...
    subs(Cxy,[xi eta],[x(1) x(4)]) == c3,...
    subs(Cxy,[xi eta],[x(2) x(1)]) == c4,...
    subs(Cxy,[xi eta],[x(3) x(1)]) == c5,...
    subs(Cxy,[xi eta],[x(3) x(2)]) == c6,...
    subs(Cxy,[xi eta],[x(2) x(3)]) == c7,...
    subs(Cxy,[xi eta],[x(1) x(3)]) == c8,...
    subs(Cxy,[xi eta],[x(1) x(2)]) == c9,...
    subs(Cxy,[xi eta],[x(2) x(2)]) == c10];

var = [a1, a2,a3, a4, a5, a6, a7, a8, a9, a10];
Cvar = [c1, c2, c3, c4, c5, c6, c7, c8, c9, c10];

A = solve(eqs,var);
a1 = A.a1; a2 = A.a2; a3 = A.a3; a4 = A.a4; a5 = A.a5;
a6 = A.a6; a7 = A.a7; a8 = A.a8; a9 = A.a9; a10 = A.a10;

Cxy = a1 + a2*xi + a3*eta + ...
    a4*xi^2 + a5*eta^2 + a6*xi*eta + ...
    a7*xi^3 + a8*eta^3 + a9*xi^2*eta + a10*xi*eta^2;

[N,Ci] = coeffs(Cxy,Cvar);
N1  = simplify(N( 1)); N2  = simplify(N( 2)); N3  = simplify(N( 3));
N4  = simplify(N( 4)); N5  = simplify(N( 5)); N6  = simplify(N( 6));
N7  = simplify(N( 7)); N8  = simplify(N( 8)); N9  = simplify(N( 9));
N10 = simplify(N(10));
```

N1 =

$$-\frac{9\eta^3}{2}-\frac{27\eta^2\xi}{2}+9\eta^2-\frac{27\eta\xi^2}{2}+18\eta\xi-\frac{11\eta}{2}-\frac{9\xi^3}{2}+9\xi^2-\frac{11\xi}{2}+1$$

N4 =

$$\frac{9\xi\,(3\eta^2+6\eta\xi-5\eta+3\xi^2-5\xi+2)}{2}$$

N10 = $-27\,\eta\,\xi\,(\eta+\xi-1)$

4. Compute all polynomial coefficients of the shape functions of a 10-node tetrahedron element.

```
syms xi eta zeta
syms a1 a2 a3 a4 a5 a6 a7 a8 a9 a10
syms c1 c2 c3 c4 c5 c6 c7 c8 c9 c10

Cxyz = a1 + a2*xi + a3*eta + a4*zeta + ...
    a5*xi^2 + a6*eta^2 + a7*zeta^2 + ...
```

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```
    a8*xi*eta + a9*xi*zeta + a10*eta*zeta;

x = linspace (0, 1, 3);

eqs =[subs(Cxyz,[xi eta zeta],[x(1) x(1) x(1)]) == c1, ...
    subs(Cxyz,[xi eta zeta],[x(3) x(1) x(1)]) == c2,...
    subs(Cxyz,[xi eta zeta],[x(1) x(3) x(1)]) == c3,...
    subs(Cxyz,[xi eta zeta],[x(1) x(1) x(3)]) == c4,...
    subs(Cxyz,[xi eta zeta],[x(2) x(1) x(1)]) == c5,...
    subs(Cxyz,[xi eta zeta],[x(2) x(2) x(1)]) == c6,...
    subs(Cxyz,[xi eta zeta],[x(1) x(2) x(1)]) == c7,...
    subs(Cxyz,[xi eta zeta],[x(1) x(1) x(2)]) == c8,...
    subs(Cxyz,[xi eta zeta],[x(2) x(1) x(2)]) == c9,...
    subs(Cxyz,[xi eta zeta],[x(1) x(2) x(2)]) == c10];

var = [a1, a2,a3, a4, a5, a6, a7, a8, a9, a10];
Cvar = [c1, c2, c3, c4, c5, c6, c7, c8, c9, c10];

A = solve(eqs,var);
a1 = A.a1; a2 = A.a2; a3 = A.a3; a4 = A.a4; a5 = A.a5;
a6 = A.a6; a7 = A.a7; a8 = A.a8; a9 = A.a9; a10 = A.a10;

Cxyz = a1 + a2*xi + a3*eta + a4*zeta + ...
    a5*xi^2 + a6*eta^2 + a7*zeta^2 + ...
    a8*xi*eta + a9*xi*zeta + a10*eta*zeta;

[N,Ci] = coeffs(Cxyz,Cvar);
N1  = simplify(N( 1)); N2  = simplify(N( 2));
N3  = simplify(N( 3)); N4  = simplify(N( 4));
N5  = simplify(N( 5)); N6  = simplify(N( 6));
N7  = simplify(N( 7)); N8  = simplify(N( 8));
N9  = simplify(N( 9)); N10 = simplify(N(10));
```

N1 = $2\eta^2 + 4\eta\xi + 4\eta\zeta - 3\eta + 2\xi^2 + 4\xi\zeta - 3\xi + 2\zeta^2 - 3\zeta + 1$

N2 = $\xi\,(2\xi - 1)$

N3 = $\eta\,(2\eta - 1)$

N4 = $\zeta\,(2\zeta - 1)$

N5 = $-4\xi\,(\eta + \xi + \zeta - 1)$

N6 = $4\eta\xi$

N7 = $-4\eta\,(\eta + \xi + \zeta - 1)$

N8 = $-4\zeta\,(\eta + \xi + \zeta - 1)$

N9 = $4\xi\zeta$

N10 = $4\eta\zeta$

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5. (Opt.) Find all shape functions for the transition element shown below.

```
syms xi eta a b x

% ELEMENTO DE TRANSIÇÃO QUADRILATERAL COM 5 NÓS
NC =  1/4 * (1 + xi*a) * (1 + eta*b);    % i = 1...4 (linear)
NMP = 1/2 * (1 + xi*a) * (1 - eta^2);    % i = 6, 8  (quadratico)
NMI = 1/2 * (1 + xi^2) * (1 - eta*b);    % i = 5, 7  (quadratico)

N1 = subs (NC,  [a b], [-1 -1]);
N2 = subs (NC,  [a b], [ 1 -1]);
N3 = subs (NC,  [a b], [ 1  1]);
N4 = subs (NC,  [a b], [-1  1]);
N5 = subs (NMP, [a b], [ 1  0]);
```

N1 =

$$\left(\frac{\xi}{4}-\frac{1}{4}\right)(\eta-1)$$

N2 =

$$-\left(\frac{\xi}{4}+\frac{1}{4}\right)(\eta-1)$$

N3 =

$$\left(\frac{\xi}{4}+\frac{1}{4}\right)(\eta+1)$$

N4 =

$$-\left(\frac{\xi}{4}-\frac{1}{4}\right)(\eta+1)$$

N5 =

$$-(\eta^2-1)\left(\frac{\xi}{2}+\frac{1}{2}\right)$$

6. (Opt.) Using computer software plot the shape functions of nodes 1 and 5 for the element in the last exercise.

```
fsurf(N5,[-1 1 -1 1])
title 'N5'
xlabel ("ξ");
ylabel ("η");
```