Exercises 4.5

3. The nodal coordinates of a 4-node quadrilateral element are(x1,y1) =(1,1),(x2,y2) = (5,2),(x3,y3) = (6,5)and(x4,y4) = (2,4). Determine the element area using the Gauss quadrature with 1, 4, and 9 integration points.Compare the results.

```
clear all
clc
```

```
syms xi eta
```

```
C = [1 1; 5 2; 6 5; 2 4];
dn = quad4_deriv (xi, eta);
J = C' * dn;
eqs = det(J);
A = int(int(eqs, xi, -1, 1), eta, -1, 1);
valor_analitico = vpa(A)
```

```
valor_analitico = 11.0
```

```
quadrature_1_pts = [0.0  0.0  4.0];

quadrature_4_pts = [
    -0.577350269189626 -0.577350269189626  1.0
     0.577350269189626 -0.577350269189626  1.0
    -0.577350269189626  0.577350269189626  1.0
     0.577350269189626  0.577350269189626  1.0];

quadrature_9_pts = [
    -0.774596669241483 -0.774596669241483  0.3086419753086419
     0.0               -0.774596669241483  0.4938271604938271
     0.774596669241483 -0.774596669241483  0.3086419753086419
    -0.774596669241483  0.0                0.4938271604938271
     0.0                0.0                0.7901234567901234
     0.774596669241483  0.0                0.4938271604938271
    -0.774596669241483  0.774596669241483  0.3086419753086419
     0.0                0.774596669241483  0.4938271604938271
     0.774596669241483  0.774596669241483  0.3086419753086419];

N = (quadrature_9_pts);
L = 0.00;
[s, r, w] = quadrature(N);
for i = 1:size(N, 1)
    L = (subs(eqs, [xi eta], [s(i) r(i)])) * w(i) + L;
end
valor_numerico = vpa(L, 4)
```

```
valor_numerico = 11.0
```

```
function dn = quad4_deriv (xi, eta)
n = [1.0/4.0 * (1 - xi) * (1 - eta)
     1.0/4.0 * (1 + xi) * (1 - eta)
     1.0/4.0 * (1 + xi) * (1 + eta)
     1.0/4.0 * (1 - xi) * (1 + eta)];
dn = [diff(n, xi), diff(n, eta)];
```

```
end

function [xi, eta, w] = quadrature (quadrature)
Npst = size(quadrature, 1);
for i = 1:Npst
    xi(i, 1)  = quadrature (i, 1);
    eta(i, 1) = quadrature (i, 2);
    w(i, 1)   = quadrature (i, 3);
end
end
```