Exercises 5.5

1. The nodal coordinates of a three-node bar element are (x1,y1) = (−1,−1),(x2,y2) = (1,1) and (x3,y3) = (0,1). At the endof a finite element analysis, the axial stresses calculated at the three integration points were:σ1= 5kPa,σ2= 10kPa andσ3= 15kPa. Compute the equivalent nodal loads.

```
clear all
clc
```

```
C = [-1 -1; 1 1; 0 1];
q = 3;
sig = [5; 10; 15];
nnodes = size(C, 1);
for i = 1: nnodes
    q = quadrature(nnodes);
    xi = q(i, 1);
    [dN, n] = lin3_derivs(xi);
    N(i,:) = n';
end
q = size(q,1);
sig_nodal = N\sig;
F = compute_F(C, sig, q);
vpa(F, 3)
```

```
ans =
```

$$\begin{pmatrix} -2.43 \\ -7.54 \\ 12.2 \\ -2.68 \\ -9.74 \\ 10.2 \end{pmatrix}$$

Exercises 5.5

2. Recover the nodal stresses for the last exercise.

```
vpa(sig_nodal, 3)
```

```
ans =
```

$$\begin{pmatrix} 3.55 \\ 16.5 \\ 10.0 \end{pmatrix}$$

```
function F = compute_F(C, sig, q)
Q = quadrature(q);
nodeselement= size(C, 1);
ndof        = 2;
numelement  = 1;
F           = zeros(nodeselement*ndof,1);
C           = C;
for i = 1: q
    xi      = Q (i, 1);
    w       = Q (i, 2);
    B = compute_B(C, q, ndof, xi);
```

```
    [dN, N] = lin3_derivs(xi);
    J = C'*dN;
    F = F + B'*sig(i)*norm(J)*w;
end
end

function B = compute_B(C, q, ndof, xi)
q           = quadrature(q);
Npst        = size(q, 1);
nnodes      = size(C, 1);
    [dN, N] = lin_shape_form(nnodes, xi);
    J       = C'*dN;
    dNdX    = dN*pinv(J);
    for j = 1: nnodes
    c = (j-1) * ndof;
    B(1, c+1) = dNdX(j,1);
    B(1, c+2) = dNdX(j,2);
    end
end

function Q = quadrature(q)
quadrature_1_pts = [0.0  2.0];
quadrature_2_pts = [
    -0.577350269189626 1.0
     0.577350269189626 1.0];
quadrature_3_pts = [
    -0.774596669241483 5.0/9.0
     0.0               8.0/9.0
     0.774596669241483 5.0/9.0];

if q == 1
    Q = quadrature_1_pts;
elseif q == 2
    Q = quadrature_2_pts;
elseif q == 3
    Q = quadrature_3_pts;
else
    Q = 0;
end
end
```