Exercises 7.1

2. Find an approximate solution for the BVP using weighted integrals with an approximate solution of the form u(x) =c1x3+c2x2+c3x+c4 and weight function sw1(x) = 1 and w2(x) =x.

$$\frac{d}{dx}\left(x\frac{du}{dx}\right) + u = 0 \qquad \text{for } x \in (0, 1)$$

$$u(0) = 1$$

$$\left(x\frac{du}{dx}\right)\bigg|_{x=1} = 0$$

```
clear all
clc
```

```
syms c1 c2 c3 c4
syms x
u   = @(x) c1*x^3+c2*x^2+c3*x+c4;
w1 = @(x) 1;
w2 = @(x) x;
R   = @(x) diff(diff(u(x))*x) + u(x);
u0 = u(0) == 1
```

u0 = $c_4 = 1$

```
ux = diff(u(x))*x == 0;
ux1 = subs (ux, x, 1)
```

ux1 = $3\,c_1 + 2\,c_2 + c_3 = 0$

```
A1   = int(R(x)*w1(x), 0, 1) == 0
```

A1 =
$$\frac{13\,c_1}{4} + \frac{7\,c_2}{3} + \frac{3\,c_3}{2} + c_4 = 0$$

```
A2   = int(R(x)*w2(x), 0, 1) == 0
```

A2 =
$$\frac{49\,c_1}{20} + \frac{19\,c_2}{12} + \frac{5\,c_3}{6} + \frac{c_4}{2} = 0$$

```
eqs = [subs(A1, c4, 1), ...
        subs(A2, c4, 1)];
A = solve ([eqs ux1], [c1 c2 c3]);
C1 = A.c1
```

C1 =
$$-\frac{60}{17}$$

```
C2 = A.c2
```

C2 =

$$\frac{138}{17}$$

```
C3 = A.c3
```

C3 =

$$-\frac{96}{17}$$

```
C4 = 1
```

C4 = 1

```
u  = @(x) C1*x^3+C2*x^2+C3*x+C4;
R  = @(x) diff(diff(u(x))*x) + u(x);
ux = u(x)
```

ux =

$$-\frac{60\,x^3}{17}+\frac{138\,x^2}{17}-\frac{96\,x}{17}+1$$

```
Rx = R(x)
```

Rx =

$$\frac{180\,x}{17}-x\left(\frac{360\,x}{17}-\frac{276}{17}\right)-\frac{42\,x^2}{17}-\frac{60\,x^3}{17}-\frac{79}{17}$$

```
% clear all
% f = @(c1,c2,c3,c4) (13*c1)/4 + (7*c2)/3 + (3*c3)/2 + c4
% f(-60/17,138/17,-96/17,1)
```