Exercises 5.2

3. Find an expression for the matrix B of a three-node bar element located in 2D space as a function of the Jacobian and the local coordinate ξ. Consider ε = ∂u/∂s wheres is the curviline arco ordinate along the bar path.

```
clear all
clc
```

```
syms x1 x2 x3
syms y1 y2 y3
syms xi
```

```
C = [x1 y1; x2 y2; x3 y3];
```

```
B = compute_B(C, xi)
```

B =

$$\begin{pmatrix} \frac{\left(\xi-\frac{1}{2}\right)\sigma_3}{\sigma_1} & 0 & \frac{\left(\xi+\frac{1}{2}\right)\sigma_3}{\sigma_1} & 0 & -\frac{2\,\xi\,\sigma_3}{\sigma_1} & 0 \\ 0 & \frac{\left(\xi-\frac{1}{2}\right)\sigma_2}{\sigma_1} & 0 & \frac{\left(\xi+\frac{1}{2}\right)\sigma_2}{\sigma_1} & 0 & -\frac{2\,\xi\,\sigma_2}{\sigma_1} \end{pmatrix}$$

where

$$\sigma_1 = \left|\overline{x_1}\left(\xi-\frac{1}{2}\right)+\overline{x_2}\left(\xi+\frac{1}{2}\right)-2\,\xi\,\overline{x_3}\right|^2+\left|\overline{y_1}\left(\xi-\frac{1}{2}\right)+\overline{y_2}\left(\xi+\frac{1}{2}\right)-2\,\xi\,\overline{y_3}\right|^2$$

$$\sigma_2 = y_1\left(\overline{\xi}-\frac{1}{2}\right)+y_2\left(\overline{\xi}+\frac{1}{2}\right)-2\,y_3\,\overline{\xi}$$

$$\sigma_3 = x_1\left(\overline{\xi}-\frac{1}{2}\right)+x_2\left(\overline{\xi}+\frac{1}{2}\right)-2\,x_3\,\overline{\xi}$$

```
function B = compute_B(C, xi)
nnodes = size(C, 1);
ndof = 2;
dN   = lin3_derivs(xi);
J    = (C'*dN);
dNdX = dN * (J/(norm(J)*norm(J)))';
    for i = 1: nnodes
    c = (i-1) * ndof;
    B(1, c+1) = dNdX(i,1);
    B(2, c+2) = dNdX(i,2);
    end
end

function dN = lin3_derivs (xi)
n = [ xi*xi/2 - xi/2
      xi*xi/2 + xi/2
```

```
            1 - xi*xi];
dN = [diff(n, xi)];
end
```