

Notes for the course  
- – REAL ANALYSIS

Josenaldo Júnior

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Part I

*Real Analysis I*, by Elon L.  
Lima

## Chapter 6

# Some Topological Notions

**Definition** (Interior Point). The point  $a$  is *interior* to the set  $X \subseteq \mathbb{R}$  when there is some  $\epsilon > 0$  st  $(a - \epsilon, a + \epsilon) \subseteq X$ . The set of interior points of  $X$ , called the *interior* of  $X$ , is represented as  $\text{int } X$ .

**Definition** (Neighborhood). If  $a \in \text{int } X$ , then  $X$  is a *neighborhood* of  $a$ .

**Definition** (Open Set).  $X \subseteq \mathbb{R}$  is open iff  $\text{int } X = X$ .

**Fact** (Limit of a Sequence in Terms of Open Sets).  $a = \lim x_n$  iff for every open set  $A$ ,  $x_n$  is eventually contained in  $A$ .

**Theorem 1.** *Arbitrary unions of opens are open; finite intersections of opens are open.*

**Definition** (Adherent Point).  $a$  is *adherent* to the set  $X \subseteq \mathbb{R}$  when  $a$  is the limit of a sequence of points of  $X$ .

**Definition** (Closure). The *closure* of  $X \subseteq \mathbb{R}$  is the set of points that adhere to  $X$ . It is denoted by  $\overline{X}$ .

**Definition** (Closed Set). A set  $X$  is *closed* when  $\overline{X} = X$ .

**Definition** (Dense Set). Let  $X \subseteq Y$ .  $X$  is *dense* on  $Y$  if  $Y \subseteq \overline{X}$ .

**Theorem 2.**  *$a$  adheres to  $X$  iff every neighborhood of  $a$  intersects  $X$ .*

**Corollary.** *Closures are closed.*

**Theorem 3.** *A set is closed iff its complement in  $\mathbb{R}$  is open.*

**Theorem 4.** *Arbitrary intersections of closed sets are closed; finite unions of closed sets are closed.*

**Definition** (Cision). The sets  $A$  and  $B$  are a *cision* of  $X = A \cup B$  if  $A \cap \overline{B} = \overline{A} \cap B = \emptyset$ .

**Theorem 5.** *Intervals of the real line only admit the trivial division.*

**Corollary.** *The only clopen subsets of  $\mathbb{R}$  are  $\emptyset$  and  $\mathbb{R}$ .*

**Definition** (Accumulation Point).  $a$  is an *accumulation point* of  $X$  if every neighborhood of  $a$  intersects  $X \setminus \{a\}$ . The set of accumulation points of  $X$  is denoted as  $X'$ .

**Definition** (Isolated Point).  $a \in X$  is a point *isolated* from  $X$  if  $a$  is not an accumulation point of  $X$ .

**Definition** (Discrete Set). A set is *discrete* if all of its elements are isolated points.

**Theorem 6.** *Given  $X \subseteq \mathbb{R}$  and  $a \in \mathbb{R}$ , the following are equivalent:*

1.  $a \in X'$
2.  $a \in \overline{X \setminus \{a\}}$
3. *Every interval centered in  $a$  has infinite elements of  $X$ .*

**Theorem 7.** *Every bounded infinite set has an accumulation point.*