Algorithm-1

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	N+1
3	1	$\sum_{l=1}^{n} n-l+2$
4	1	$\sum_{l=1}^{n} n-l+1$
5	1	$\sum_{u=1}^{n}\sum_{i=1}^{u} i+1$
6	6	$\sum_{u=1}^{n}\sum_{i=1}^{u} i$
7	4	$\sum_{l=1}^{n} n-l+1$
8	1	1

Multiply col.1 with col.2, add across rows and simplify

$T_1(n) = 1 + n + 1 + \frac{n^2+3n}{2} + \frac{n^2+n}{2} + \frac{n(n+1)(n+5)}{6} + \frac{6n(n+1)(n+2)}{6} + 4\frac{n^2+n}{2} + 1$

$= 3 + n + \frac{6n^2+8n}{2} + \frac{n(n+1)(7n+17)}{6} = 3 + n + 3n^2 + 4n + \frac{7n^3+24n^2+17n}{6}$

$= \frac{7n^3}{6} + 7n^2 + \frac{47n}{6} + 3$

$= O(n^3)$

Algorithm-2

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	n+1
3	1	n
4	1	$\sum_{l=1}^{n} n-l+2$
5	6	$\sum_{l=1}^{n} n-l+1$
6	4	$\sum_{l=1}^{n} n-l+1$
7	1	1

Multiply col.1 with col.2, add across rows and simplify

$T_2(n) = 1 + n + 1 + n + 1 + \frac{n^2+3n}{2} + 6\frac{n^2+n}{2} + 4\frac{n^2+n}{2}$

$= 3 + 2n + \frac{11n^2+13n}{2} = \frac{(11n+6)(n+1)}{2}$

$= \frac{11n^2}{2} + \frac{17n}{2} + 3$

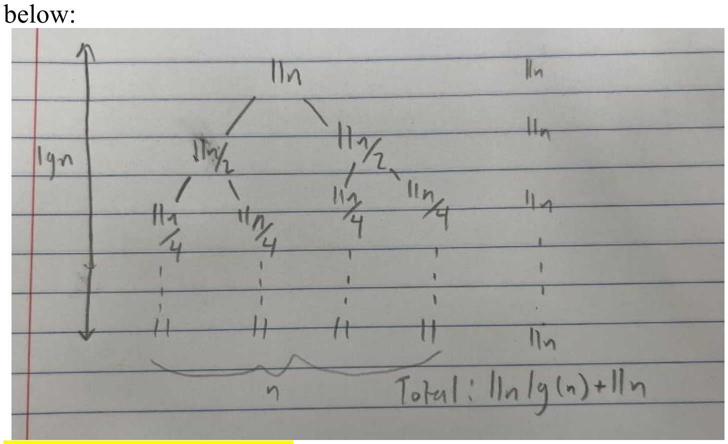
$= O(n^2)$

Algorithm-3

Step	Cost of each execution	Total # of times executed in any single recursive call
1	3	1

2	8	1
Steps executed when the input is a base case: 1, 2		
First recurrence relation: T(n=1 or n=0) = 4		
3	5	1
4	2	1
5	1	$\frac{n}{2} + 1$
6	6	$\frac{n}{2}$
7	4	$\frac{n}{2}$
8	2	1
9	1	$\frac{n}{2} + 1$
10	6	$\frac{n}{2}$
11	4	$\frac{n}{2}$
12	4	1
13	1	(cost excluding the recursive call) 1
14	1	(cost excluding the recursive call) 1
15	5	1
Steps executed when input is NOT a base case: 3 - 15		
Second recurrence relation: T(n>1) = 11n+33		
Simplified second recurrence relation (ignore the constant term): T(n>1) = 11n		

Solve the two recurrence relations using any method (recommended method is the Recursion Tree). Show your work below:



$T_3(n) = 11nlog(n) + 11n$
$= O(nlog(n))$

Algorithm-4

Step	Cost of each execution	Total # of times executed
1	1	1
2	1	1
3	1	n+1
4	7	n
5	4	n
6	2	1

Multiply col.1 with col.2, add across rows and simplify

$T_4(n) = 1 + 1 + n + 1 + 7n + 7 + 4n + 2$
$= 12n + 12$
$= O(n)$