Public key cryptology, RSA, ElGamal, Elliptic Curve

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Key Terms

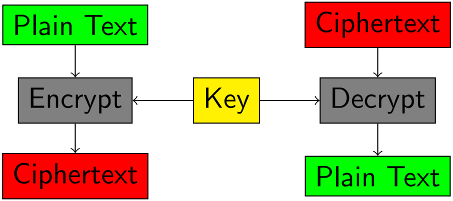
Plain text: typically a simple text such as this line
Cipher text: a message after it has been encrypted
Prime Number: a whole number that can only be divided evenly by one and itself also it is greater than one

Types of Encryption

Symmetric Encryption
Asymmetric Encryption
Hashing
Hybrid Encryption

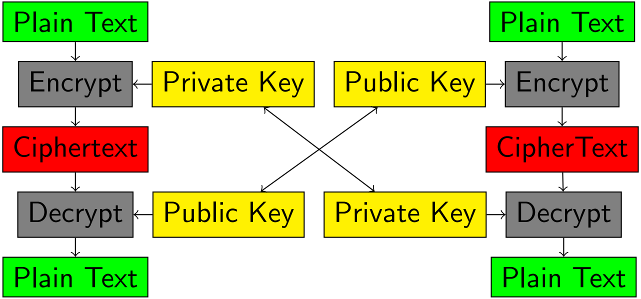
Symmetric Encryption

Encryption and Decryption use the same key



Asymmmetric Encryption

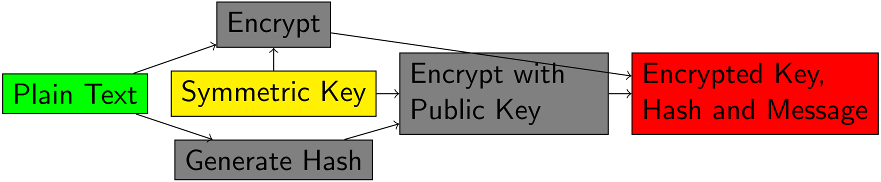
Public key and private key pair
Public key is used to encrypt a message
Private key is used to decrypt a message
Creating the key tends to be computationally expensive



Hashing

We may or may not want to talk about this?

Uses ideas from symmetric and asymmetric encryption methods
An asymmetric cryptosystem is used for key encapsulation and an symmetric system is used for data encapsulation



Padding Schemes

RSA Cryptosystem

First designed in 1973 and declassified in 1997.

Named after its founders Ron Rivest, Adi Shamir and Leonar Adleman

Uses large prime numbers to create a private and public key

Security arises from the presumed difficulty of factoring large prime numbers

RSA Generating Public and Private Keys

Generate two large prime numbers and use to calculate n
Compute Euler's Totient Function
Create Public Key and Private Key

Generating Large Prime Numbers

Generate two large prime numbers.
Typically uses AKS testing and/or the Miller-Rabin test for prime numbers
These two values we will call p and q
p = 991
q = 821
We next calculate n = p * q
So we have n = 991 * 821 = 813611

Public Information	Secret Information
n = 813611	q = 821
	p = 991

Euler's Totient

We then can determine Euler's totient value by using the following equation.

$\phi(n) = \phi(p)\phi(q) = (p-1)(q-1) = n - (p+q-1)$

And for $\phi(n) = \phi(813611) = 811800$

Public Information	Secret Information
n = 813611	q = 821
	p = 991
	$\phi(n) = 811800$

Create Public and Private Keys

To create a public key we pick an arbitrary number e between $1 < e < \phi(n)$ for example we will use e = 7423

To create a private key we need to find the modular multiplicative inverse of e.

This is commonly done using the Extended Euclidean Algorithm.

$d \equiv e^{-1} \pmod{\phi(n)}$

This makes our value of d = 788287

Public Information	Secret Information
n = 813611	q = 821
e = 7423	p = 991
	$\phi(n) = 811800$
	d = 788287

Working Example

Using these values we can create a cipher text c and decrypt it using the following equations

$c \equiv m^e \,(mod\,(n))$ and $m \equiv c^d \,(mod\,(n))$

Finishing up our example we will encrypt "Hi" using our new values

plain text m = 72105, e = 7423, d = 788287, n = 813611

Public Encrypt	Private Encrypt
$c \equiv m^e \,(mod\,(n))$	$c \equiv m^d \,(mod\,(n))$
$c \equiv 72105^{7423} mod(813611)$	$c \equiv 72105^{788287} mod(813611)$
$c = 707473$	$c = 616895$

Private Decrypt	Public Decrypt
$m \equiv c^d \,(mod\,(n))$	$m \equiv c^e \,(mod\,(n))$
$m \equiv 707473^{788287} mod(813611)$	$m \equiv 616895^{7423} mod(813611)$
$m = 72105$	$m = 72105$

ElGamal Cryptosystem

Elliptic Curve

Conclusion

References

http://caislab.kaist.ac.kr/lecture/2010/spring/cs548/basic/B02.pdf
http://doctrina.org/Why-RSA-Works-Three-Fundamental-Questions-Answered.html
http://doctrina.org/How-RSA-Works-With-Examples.html