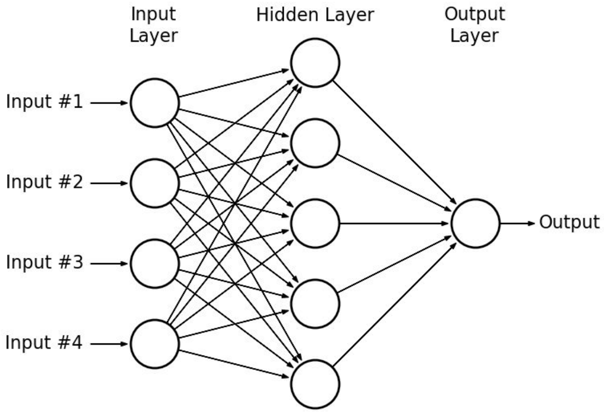
Week 7

Agenda

1. Neural Network discussion
2. Deep Learning notebook

For next week: Backpropagation discussion during next week's office hour

History



Timeline

- 40s-50s Idea emerges.
- 1962 Perceptron learning
- 1969 Minisky: XOR problem
- 1982 Multi-layer neural networks
- 1986 Backpropagation
- 1989 Universal Approximation Theorem
- 90s-00s SVMs gain favor
- 00s SGD popularized
- 2009-present Deep learning: return of neural nets

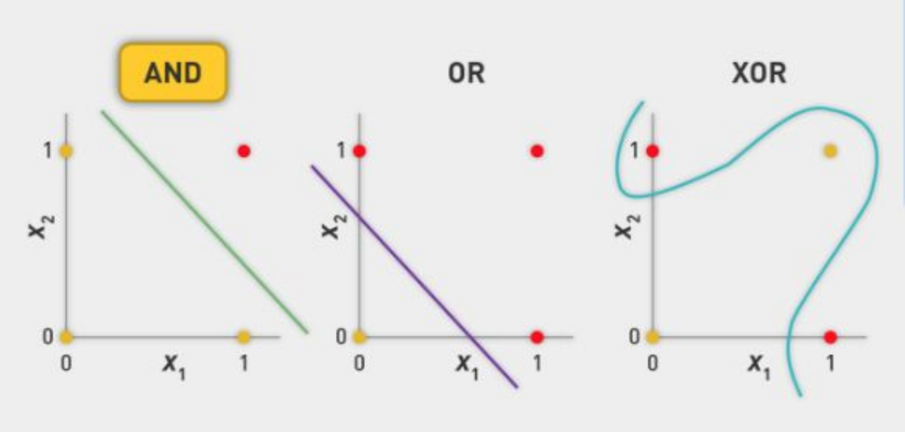
People to Know

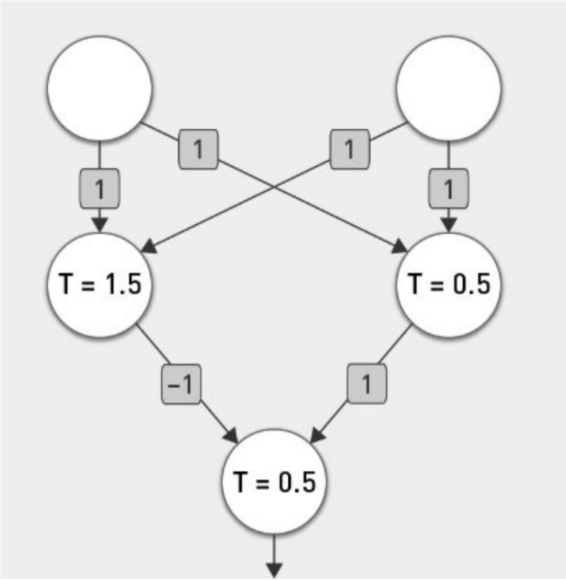
- Geoffrey Hinton. http://www.cs.toronto.edu/~hinton/ (https://www.coursera.org/learn/neural-networks)
- Yann LeCun. http://yann.lecun.com/
- Yoshua (and Samy) Bengio. http://www.iro.umontreal.ca/~bengioy/yoshua_en/
- Leon Bottou (SGD). http://leon.bottou.org/

Conferences

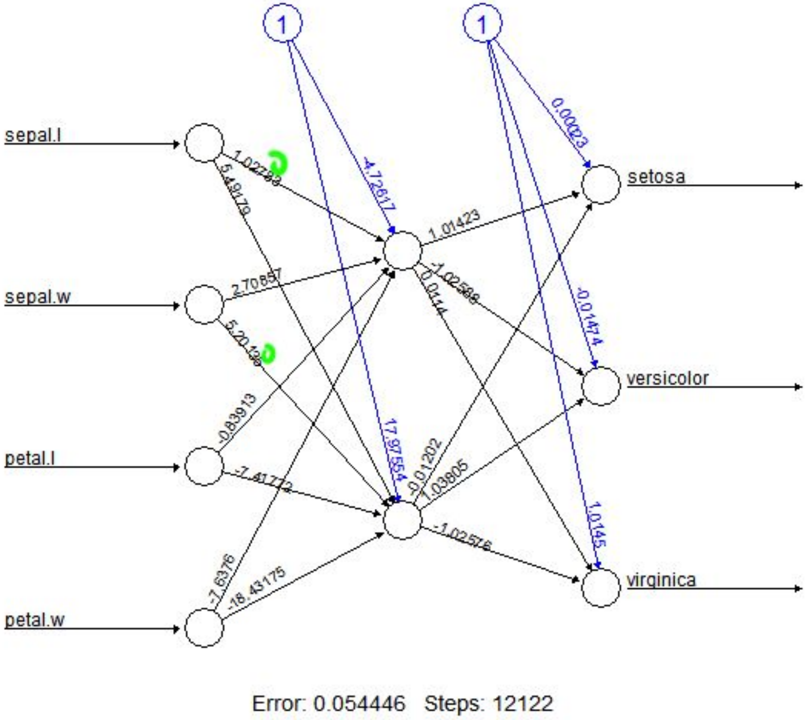
- NIPS, ICML
- APPLIED: KDD, SIGIR, AAAI

1. What's the limitation of a perceptron? What differs about Neural Nets that allow them to learn non-linear function?





Example Trained Neural Network



1. What do you remember about Iris dataset?
2. How many parameters in this model?
3. How is multi-class handled?
4. Sparse vs dense representation. Which one will we get? Why?
5. How many layers?
6. How can we think about this network as an ensemble/stacked model?
7. How can we think about this network as a series of matrix operations?

A challenge...

Describe logistic regression represented as a NN?

Accuracy on MNIST

Type	Classifier	Distortion	Preprocessing	Error rate (%)
Linear classifier	Pairwise linear classifier	None	Deskewing	7.6[9]
Non-Linear Classifier	40 PCA + quadratic classifier	None	None	3.3[9]
Neural network	2-layer 784-800-10	None	None	1.6[17]
Boosted Stumps	Product of stumps on Haar features	None	Haar features	0.87[15]
Neural network	2-layer 784-800-10	elastic distortions	None	0.7[17]
Support vector machine	Virtual SVM, deg-9 poly, 2-pixel jittered	None	Deskewing	0.56[16]
K-Nearest Neighbors	K-NN with non-linear deformation (P2DHMDM)	None	Shiftable edges	0.52[14]
Deep neural network	6-layer 784-2500-2000-1500-1000-500-10	elastic distortions	None	0.35[18]
Convolutional neural network	Committee of 35 conv. net, 1-20-P-40-P-150-10	elastic distortions	Width normalizations	0.23[8]

Universal Approximation Theorem

- Two-layer networks are universal function approximators
  - Let F be a continuous function on a bounded subset of D-dimensional space. Then there exists a two-layer neural network F' with a finite number of hidden units that approximate F arbitrarily well. Namely, for all x in the domain of F,

$$|F(x) - F'(x)| < \varepsilon$$

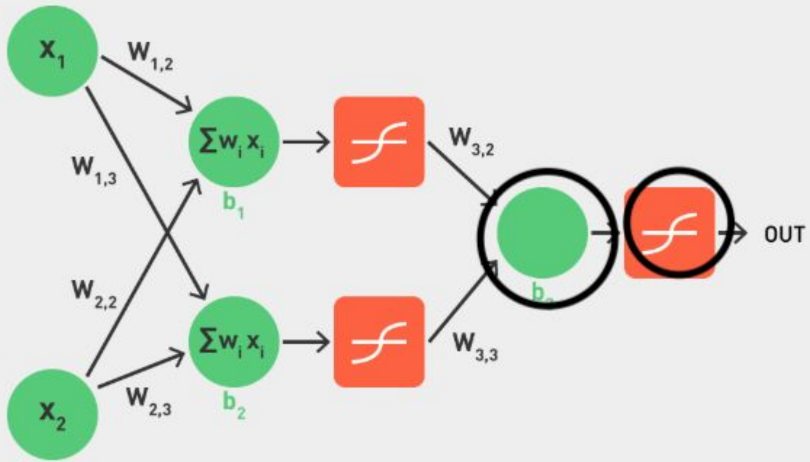
- Two-layer networks can approximate any function.
- Still may want more than two layers (fewer neurons, time to learn, time to compute, etc).

1. Why is this a theorem about representation rather than learning?

Training and Predicting

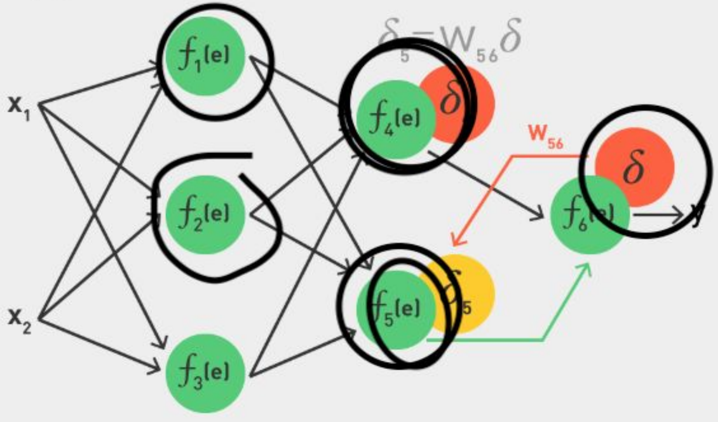
Intuition: Forward Propagation

- Given a training example (X_1, X_2) and output Y_i
- Propagate inputs/activations forward, applying sigmoid function on dot products



Intuition: Backward Propagation (cont.)

Propagate costs backward to earlier nodes:



- For each hidden unit h in k^{th} layer:

$$\delta_{hk} = Y_{hk}\left(1 - Y_{hk}\right) \sum_{j \in K} w_{hj} \delta_j$$

- Update each weight as $+\eta \delta_{hk} x_i$.
  - Daume ch. 8 for full algorithm

Backprop walkthrough

Stuff we know...

① Derivative of logistic cost

$y - \hat{y}$

② Derivative of l[illegible]t

$\hat{y}(1-\hat{y})$

③ Chain rule of g=f(h(

$\frac{dg}{dx} = \frac{dg}{dh}\frac{dh}{dx}$

④ Weight update rule

$w = w - \alpha \Delta w$

Hidden

$\frac{dE}{dw_1} = \frac{dE}{dz_1} \cdot \frac{dz_1}{da_1} \cdot \frac{da_1}{dw_1}$

Ø

$.3(1-.3)$

$\frac{dE}{dz_1} = \frac{dE}{d\hat{y}} \cdot \frac{d\hat{y}}{da_3} \cdot \frac{da_3}{dz_1}$

-.34

$\frac{dE}{dw_1} = -.34 \cdot .22 \cdot .5 \cdot .21 \cdot Ø$
$= Ø$

$\frac{dE}{dw_2} = Ø$

$\frac{dE}{dw_3} = -.34 \cdot .22 \cdot .5 \cdot .21$
$= -0.008$

$\frac{dE}{dw_4} = -.34 \cdot .22 \cdot .6 \cdot .24$
$= -0.01$

$w_3 = .3 - .2 \cdot (-0.008) =$

$w_4 = .4 - .2 \cdot (-0.010) =$

Output Layer

$\frac{dE}{dw_5} = \frac{dE}{d\hat{y}} \cdot \frac{d\hat{y}}{da_3} \cdot \frac{da_3}{dw_5}$

output of h's z_1

derivative of logit

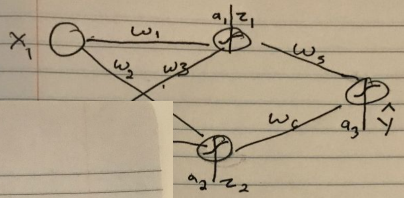
derivative of logistic cost

$\frac{dE}{dw_5} = [.66-1] \cdot [.66 \cdot (1-.66)] \cdot [.57]$
$= -.21$

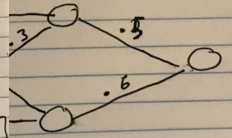
$\frac{dE}{dw_6} = [.66-1] \cdot [.66 \cdot (1-.66)] \cdot [.6]$
$= -.22$

$w_5 = .5 - .2 \cdot (-.21) = .54$

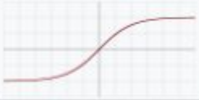
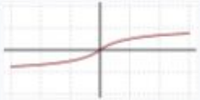
$w_6 = .6 - .2 \cdot (-.22) = .64$

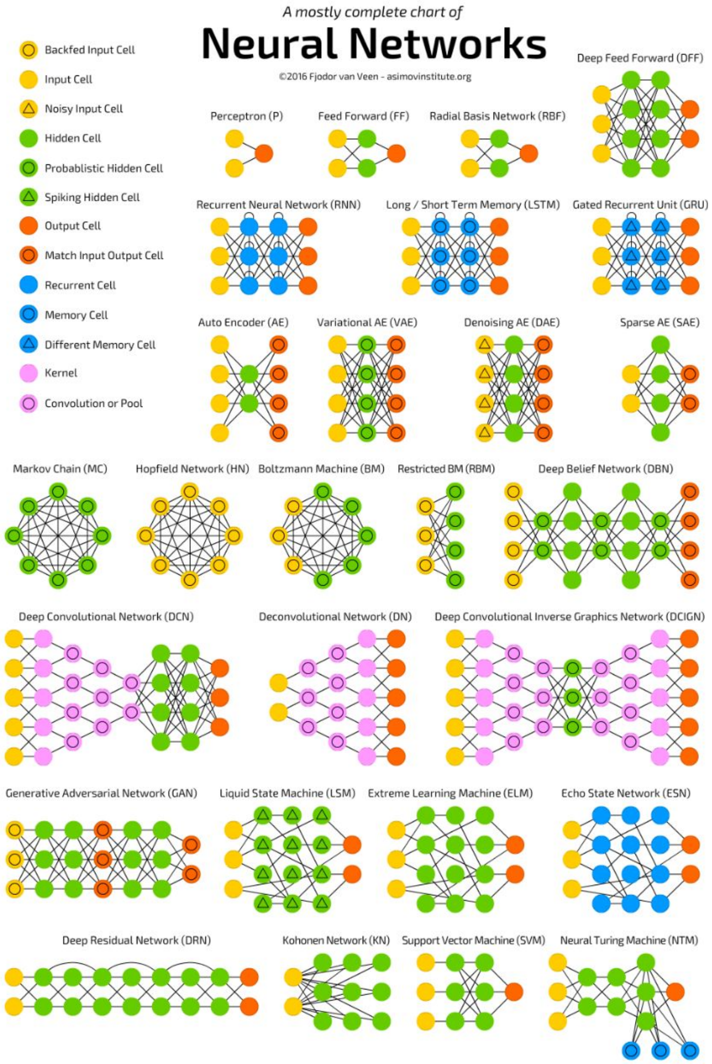


...int during training, the
...oks like this:



...encounter this example
..., x_2, y] = [0, 1, 1]

Name	Plot	Equation	Derivative (with respect to *x*)	Range
Identity		$f(x) = x$	$f'(x) = 1$	$(-\infty, \infty)$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$	$\{0, 1\}$
Logistic (a.k.a. Soft step)		$f(x) = \frac{1}{1 + e^{-x}}$	$f'(x) = f(x)(1 - f(x))$	$(0, 1)$
TanH		$f(x) = \tanh(x) = \frac{2}{1 + e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$	$(-1, 1)$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2 + 1}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Softsign [7][8]		$f(x) = \frac{x}{1 + \lvert x \rvert}$	$f'(x) = \frac{1}{(1 + \lvert x \rvert)^2}$	$(-1, 1)$
Rectified linear unit (ReLU)[9]		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$[0, \infty)$
Leaky rectified linear unit (Leaky ReLU)[10]		$f(x) = \begin{cases} 0.01x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0.01 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$(-\infty, \infty)$
Parameteric rectified linear unit (PReLU)[11]		$f(\alpha, x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(\alpha, x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$(-\infty, \infty)$


A mostly complete chart of
Neural Networks
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Backfed Input Cell
Input Cell
Noisy Input Cell
Hidden Cell
Probablistic Hidden Cell
Spiking Hidden Cell
Output Cell
Match Input Output Cell
Recurrent Cell
Memory Cell
Different Memory Cell
Kernel
Convolution or Pool
Perceptron (P)
Feed Forward (FF)
Radial Basis Network (RBF)
Deep Feed Forward (DFF)
Recurrent Neural Network (RNN)
Long / Short Term Memory (LSTM)
Gated Recurrent Unit (GRU)
Auto Encoder (AE)
Variational AE (VAE)
Denoising AE (DAE)
Sparse AE (SAE)
Markov Chain (MC)
Hopfield Network (HN)
Boltzmann Machine (BM)
Restricted BM (RBM)
Deep Belief Network (DBN)
Deep Convolutional Network (DCN)
Deconvolutional Network (DN)
Deep Convolutional Inverse Graphics Network (DCIGN)
Generative Adversarial Network (GAN)
Liquid State Machine (LSM)
Extreme Learning Machine (ELM)
Echo State Network (ESN)
Deep Residual Network (DRN)
Kohonen Network (KN)
Support Vector Machine (SVM)
Neural Turing Machine (NTM)

Final Thoughts?