Practice 2

Numerical Interpolation and Derivation

Starting date: 21/10/2019. Delivery date: 8/11/2019.

- This activity is a theoretical study numerical interpolation and derivation. It can be done with the use of MATLAB/Octave.
- You can deliver the resolution in any language (Catalan, Spanish, English), using any text processor or even an scanned document if this writing is legible. In any case, the filename must be stated as Surname1-Name-PEC1T19.pdf If you use a MATLAB/Octave file, you must also deliver it.
- Your document must clearly show the resolution strategy used, that is, you do not need to write down all the calculations but there should be the sufficient elements to complete the resolution.
- It will be necessary to justify all the answers properly.

Exercise 1 (4 points)

Given the following table:

x_i	$f(x_i)$
-1.0	0.038462
-0.8	0.058824
-0.6	0.100000
-0.4	0.200000
-0.2	0.500000
0.0	1.000000
0.2	0.500000
0.4	0.200000
0.6	0.100000
0.8	0.058824
1.0	0.038462

a) Write a MATLAB/Octave program to find an estimation for the values of the function at the following locations.

$$x = -0.95$$
$$x = 0.11$$
$$x = 0.75$$

Use a classical third degree polynomial (splines) and also a polynomial with the highest degree possible. Comment the results.

b) Compare the resulting values of the interpolation with the exact values of the function, defined as:

$$f(x) = \frac{1}{1 + 25x^2}$$

c) Now we are going to create a table with 21 equispaced nodes in the interval $[-1, 1]$ ($x_i = -1.0, -0.9, -0.8, ..., 1.0$) and we will calculate their images using the proposed function in b) ($f(x_i) = f(-1.0), f(-0.9), f(-0.8), ..., f(1.0)$). Calculate a classical third degree polynomial and also a polynomial with the highest degree possible. Represent in a single plot the two calculated polynomials. Use this computed polynomial to estimate (again):

$$x = -0.95$$
$$x = 0.11$$
$$x = 0.75$$

Comment the results.

d) Make a plot that shows the original function and the interpolating polynomial and other plot with the original function and the spline interpolation.

Exercise 2 (3 points)

a) Find the interpolating polynomials of degrees 2, 4, 10 and 20 for the function:

$$f(x) = arctan(x)$$

using evenly spaced nodes, within the interval $[-5, 5]$. Plot both, the function $f(x)$ and the interpolating polynomial and compare the results.

b) Repeat the computations but using this time the properly selected Chebyshev nodes. Write the Chebyshev nodes used in each grade, and plot all the polynomials obtained in the same picture. What can be said of the interpolaing polynomial in these cases?

Exercise 3 (3 points)

Consider the function

$$f(x) = e^{-3x}cos(2x)$$

which has the derivative

$$f'(x) = -e^{-3x}(3cos(2x) + 2sin(2x))$$

Write a MATLAB/Octave program to compare the accuracy of the non-centered derivative formula:

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

$$f'(x) \simeq \frac{-25f(x) + 48f(x+h) - 36f(x+2h) + 16f(x+3h) - 3f(x+4h)}{12h}$$

at the point $x = 0$, using values $h = 4^{-n}$ for $1 \leq n \leq 15$. Make a table that, for each value of h, shows the values of the two approximations of the numerical derivative and the exact value of $f'(0)$. Comment the results.