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Routing fleets with multiple driving ranges: Is it possible to use greener fleet configurations?

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ABSTRACT

This paper discusses the vehicle routing problem with multiple driving ranges (VRPMDR), an extension of the classical routing problem where the total distance each vehicle can travel is limited and is not necessarily the same for all vehicles – heterogeneous fleet with respect to maximum route lengths. The VRPMDR finds applications in routing electric and hybrid-electric vehicles, which can only cover limited distances depending on the running time of their batteries. Also, these vehicles require from long charging times, which in practice makes it difficult to consider en route recharging. The paper formally introduces the problem, describes an integer programming formulation and a multi-round heuristic algorithm that iteratively constructs a solution for the problem. Using a set of benchmarks adapted from the literature, the algorithm is then employed to analyze how distance-based costs are increased when considering 'greener' fleet configurations – i.e., when using electric vehicles with different degrees of autonomy.

1. Introduction

It has been echoed in the literature that future transportation systems will include heterogeneous fleets consisting of common internal combustion engine (ICE) vehicles, as well as those using "green" technologies such as plug-in hybrid electric vehicles (PHEV) and electric vehicles (EV) (e.g., [1]). Several factors necessitate resorting to such new technologies, such as (a) companies to focus on reducing their carbon footprint of their transportation activities, either as part of their corporate social responsibility strategies or other external requirements, (b) variability of cost of and long-term cost risk associated with dependence on oil-based energy sources, (c) availability of government subsidies to reduce acquisition cost as well as the potential for increased regulation on corporate fleets, and (d) advances in alternative energy technologies (such as EVs and PHEVs), which have potential for a more environmentally sustainable solutions at a cost that is starting to be competitive from an economic point of view [2,3].

In providing a useful service, one can assume that ICEs and PHEVs have unlimited driving-range capabilities since they can refuel at any service station along their route. The driving range for electric vehicles, however, is constrained by the amount of electricity stored in their batteries since these vehicles cannot quickly recharge en-route. In effect, EVs issues due to long charging times and short driving ranges are recognized by several authors. For instance, as stated in Ferreira et al. [4]: "Being the next big step in automobile industry, electric vehicles continue to have limited autonomy which associated with the long charging times, limited charging stations and undeveloped smart grid infrastructures demands a hard planning of the daily use of the vehicle". Similar arguments can be found in Achtnicht et al. [5], Wirasingha et al. [6], or Chan et al. [7]. Also, as reported by the ISOE institute: "As long as there is no global infrastructure for replacing batteries or directly inducting power from outside the car while driving, the reduced range will remain the main issue concerning electric mobility. According to experts this is not likely to change considerably in the medium term." (http://www.isoe.de/english/projects/futurefleet.htm). Electric vehicles with different battery sizes give rise to problems where each vehicle having its own driving ranges which needs to be accounted for in route-planning. With EVs becoming more prevalent, an efficient routing of heterogeneous fleets with multiple driving-range vehicles is emerging as a new issue in the transportation industry.

The vehicle routing problem with multiple (heterogeneous) driving ranges (VRPMD) is an extension of the well-known

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capacitated vehicle routing problem (CVRP) where each vehicle has its own restriction on the total distance it can cover. To the best of our knowledge, the VRPMDR has not yet been addressed in the literature, neither from a theoretical nor a practical perspective. The main goals and contributions of this article are the following ones: (i) to formally define the VRPMDR and discuss the difficulty of obtaining exact solutions for it; (ii) to introduce a heuristic method for solving the VRPMDR based on a multi-round approach; and (iii) to employ this method to analyze how distance-based costs vary when "greener" fleet configurations – those in which ICE vehicles are partially replaced by EVs – are considered.

In order to solve the aforementioned VRPMD, we were interested in taking benefit of the existence of well-tested and efficient algorithms for solving simpler versions of the VRP. For that reason, we designed a strategy based on decomposing the VRP with multiple (heterogeneous) driving ranges into a series of CVRPs with route-length restrictions (CVRP*). Each of the latter can be solved by adapting any of the metaheuristics already published for the CVRP. In particular, the one employed in this paper to solve each of the CVRP* in the series is an adaptation of the algorithm by Juan et al. [8]. For that purpose, our algorithm uses a multi-round process, and at each round a new homogeneous problem is solved. An overview of the logic behind this multi-round approach is advanced next (it will be explained later in more detail): Consider a heterogeneous fleet composed of several types of vehicles, each type characterized by a different degree of driving autonomy. At each round, the algorithm considers a different type of vehicle. Then, assuming an unlimited number of vehicles of that type, it solves a CVRP*. At this moment, those routes which are feasible – i.e., those which can be covered by available vehicles – are saved as routes of a partial solution and a new round is started. Once all vehicle types have been iterated over, the final solution is constructed by aggregating the feasible routes previously saved at each round. The algorithm in Juan et al. [8] that we used in our experimental section to solve each CVRP* employs a reduced number of parameters. It combines a multi-start biased randomization of a classical VRP heuristic with a memory-based local search, and is able to quickly generate alternative solutions with similar costs to classical CVRP instances. Thus, the approach described in this paper is practical as it is simple to implement with only a few parameters, and provides a set of alternative routing solutions for a fleet manager to consider.

The rest of the paper is structured as follows. Section 2 provides a review of literature relevant to the VRPMDR. Section 3 formally defines the VRPMDR and presents an integer linear programming formulation of the problem. A more detailed description of the proposed multi-round methodology is described in Section 4. Data generation and the details of computational experiments are given in Section 5, which also includes a discussion of the results. Finally, Section 6 presents general conclusions and suggestions for future research directions.

2. Related work

Numerous variants of vehicle routing problems (VRPs) have been studied for decades, and different approaches have been explored. These approaches range from the use of pure optimization methods – such as mixed-integer programming to solve small-size to medium-size problems with relatively simple constraints – to the use of heuristics and metaheuristics – which provide near-optimal solutions for medium and large-size problems with more complex constraints [9]. VRPs are still attracting researchers' attention due to the potential applications in practice and to the stimulus for developing new algorithms, optimization methods, and metaheuristics for solving combinatorial problems [10]. Moreover, as some researchers have already pointed out, there is a need for more simple and flexible methods to solve VRPs when considering the numerous side constraints that arise in practice [11]. This is especially the case whenever technology breakthroughs, like the emergence and inclusion of EVs and PHEVs in heterogeneous fleets, introduce new challenges. The VRP variant discussed in this paper is related to the heterogeneous VRP and also to the VRP with maximum route length – or distance constrained VRP. Therefore, a brief literature review on both topics is included next.

One of the most recent surveys on VRPs with heterogeneous fleets was authored by Baldacci et al. [12]. In their chapter, the authors review several heuristic-based approaches for solving VRPs with either fixed- or variable-size heterogeneous fleets. Still, they do not make any explicit reference to heterogeneous fleets involving vehicles with different driving range capabilities. Hoff et al. [13] offer another excellent and updated literature review on articles combining fleet composition and VRPs. In their extensive survey, the authors explicitly mention "driving time restrictions" and "driving speed" as important industrial aspects that should be considered in modern VRP research. However, no reference to "driving range restrictions" or "driving distance restrictions" was found in their survey. While time and distance are definitely related, distance is the dominant factor in energy usage. Additionally, they distinguish several categories of heterogeneous fleets according to: (a) vehicle physical dimensions, which determine vehicle capacity and also might restrict vehicle access to some narrow roads; (b) vehicle compatibility constraints, which determine the type of products a vehicle can load as well as the customers a vehicle can serve; and (c) varying costs of vehicles with respect to factors such as size and age. Still, there is not any explicit reference to heterogeneous fleets including multiple driving range vehicles or electric vehicles with limited driving autonomy, despite the fact that range is an important limitation for vehicles running on alternative fuel technologies [14]. This indicates a gap in the VRP literature on heterogeneous fleets composed of different types of vehicles according to their driving ranges.

A related problem to the one studied in this paper is the distance constrained VRP (DVRP), which has received some attention from the literature [15–18]. The definition of the DVRP is such that each vehicle has a common limit on the total length of the route it can traverse. The problem studied here can be seen as an extension of the DVRP in that the vehicle fleet is homogeneous with respect to capacity *but* heterogeneous on the driving ranges – i.e., several route-length constraints have to be considered in the problem, one for each type of vehicle. The DVRP has appeared in the context of Alternative Fuel Powered vehicles in, e.g., Erdoğan and Miller-Hooks [19], but with only a single (common) driving range limitation on tour durations.

One important component of the heuristic introduced in this paper is an algorithm that combines biased randomization of a classic heuristic with a memory-based local search. The biased randomization process used here is similar to the heuristic-biased stochastic sampling (HBSS) proposed by Bresina [20]. Both approaches use a skewed distribution function combined with a sampling methodology. In fact, our approach can be seen as a natural extension of the HBSS methodology, since we incorporate a single-parameter statistical distribution and a memory-based local search. Additionally, while the HBSS methodology was originally proposed for solving flow-shop problems, our approach solves VRPs by introducing an efficient biased randomization of the classical Clarke & Wright savings (CWS) heuristic [35]. Despite its age, the CWS heuristic is still the basis for most modern metaheuristics for solving different VRPs. The CWS heuristic is an iterative procedure that starts out by considering an initial dummy solution in which each customer is served by a dedicated vehicle. Next, the heuristic initiates an iterative process for merging some of the routes in the initial solution. Merging routes can improve the expensive initial solution so that a unique vehicle serves the nodes of the merged route. The merging criterion is based upon the concept of edge savings. Roughly speaking, given a pair of nodes to be served, a savings value can be assigned to the edge connecting these two nodes. This savings value is given by the reduction in the total cost function due to serving both nodes with the same vehicle instead of using a dedicated vehicle to serve each node – as proposed in the initial dummy solution. This way, the algorithm constructs a list of savings, one for each possible edge connecting two demanding nodes. At each iteration of the merging process, the edge with the largest possible savings is selected from the list as long as the following conditions are satisfied: (a) the nodes defining the edge are adjacent to the depot; and (b) the two corresponding routes can be feasibly merged. The CWS algorithm usually provides relatively good solutions for CVRP instances, especially for small and medium-size problems. Thus, the CWS is frequently used to generate the initial solution in most current metaheuristics. For a comprehensive discussion on the various CWS variants and uses, the reader is referred to Rand [21].

A multi-start randomized version of the CWS has been described in Juan et al. [8] to generate state-of-the-art solutions for most classical CVRP benchmarks. As part of the general multi-round methodology presented in this paper, the randomized algorithm used here goes one step further by incorporating route length constraints during the solution generation process. Still, it does not use any splitting techniques as described in Juan et al. [22] since, for the VRPMDR, they would add complexity to the algorithm without providing significant cost improvements. In fact, as the experimental section will reveal, the simplest version of the randomized algorithm is able to offer, in a reasonable time period, alternative fleet configurations for most tested CVRP benchmarks. This way, multiple driving range vehicles can be used without incurring additional (distance-based) costs or creating infeasible solutions. Moreover, when integrated inside the multi-round approach, and assuming some feasibility conditions hold, our method can offer routing solutions that fit fleet configurations specified by the decision maker – as far as they are still feasible, since using vehicles with distance constraints might cause that some nodes go out of the delivery range.

Finally, we note that a significant number of algorithms have been published to solve the vehicle routing problem (VRP) and some of its variations, including, but not limited to, those based on genetic algorithms [23,24], particle swarm optimization [25] and variable neighborhood search [26]. The particular variation of the VRP tackled in this paper, to our knowledge, has not been addressed by any of these algorithms. The VRPMD is an important problem in the area of green logistics, and there is a need to be able to solve this problem efficiently and effectively.

3. Formal definition of the problem

The VRPMDR is a natural extension of the classical capacitated vehicle routing problem (CVRP) defined on a complete graph $G = (N, A)$, where $N = \{0, 1, \ldots, n\}$ is the set of nodes including a depot (node 0) and n customers denoted by subset $N_0 = N \setminus \{0\}$, where each customer $i \in N_0$ has known demand q_i for a given commodity. The set $A = \{(i, j)\colon i, j \in N, i \neq j\}$ includes arcs (i.e., roads) between all pairs of nodes where $c_{\{ij\}}$ denotes the cost of travel on each arc $(i, j) \in A$ and d_{ij} is the shortest (Euclidean) distance between nodes i and j. Demands must be served using a set of vehicles $K = K_1 \cup K_2 \cup \ldots \cup K_r$ where each K_l, $l \in L = \{1, \ldots, r\}$ denotes a category of vehicle with a maximum load capacity Q ($Q > \max_{i=2,\ldots,n}\{q_i\}$), where Q and q_i are given in the same unit of measure. The parameter T_l shows the maximum distance each vehicle in set K_l, $l \in L = \{1, \ldots, r\}$ can cover without returning to the depot for refueling/recharging. We assume, without loss of generality, that $T_l < T_m$ for any $1 \le l < m \le r$. In other words, the vehicle types in set K appear an increasing order of their driving ranges.

The VRPMDR consists of finding a minimal-cost solution, i.e., at most $|K|$ routes that minimize the total cost of travel such that each customer is visited once by any vehicle and vehicle load and driving range constraints are respected. As stated earlier, the problem is inspired by the fact that vehicles using different energy sources or even different battery types might have different driving range constraints. For the latter category of vehicles, their driving range is typically much lower than those running on fuel-combustion engines. In defining the problem, priority is therefore given to using as much electric vehicles as possible, i.e., we use of vehicles in the order of priority $l = 1, \ldots, r$.

3.1. An integer programming formulation

In what follows, we describe a mathematical formulation of the problem described above. The model uses binary variables x_{ijk} equal to 1 if vehicle $k \in K$ travels on arc $(i, j) \in A$, and 0 otherwise. Continuous variables u_i are defined for each customer $i \in N_0$, showing the cumulative amount of load carried when leaving node i. Similarly, continuous variables v_{ik} show the cumulative amount of distance traveled by vehicle $k \in K$ when leaving node $i \in N_0$.

$$\text{Minimize} \quad \sum_{k \in K} \sum_{(i,j) \in N} c_{ij} x_{ijk} \tag{1}$$

subject to

$$\sum_{k \in K} \sum_{j \in N} x_{ijk} = 1 \quad i \in N_0 \tag{2}$$

$$\sum_{k \in K} \sum_{j \in N} x_{jik} = 1 \quad i \in N_0 \tag{3}$$

$$\sum_{j \in N} x_{ijk} - \sum_{j \in N} x_{jik} = 0 \quad i \in N_0, \; k \in K \tag{4}$$

$$\sum_{j \in N} x_{0jk} \le 1 \quad k \in K \tag{5}$$

$$u_i - u_j + Q \sum_{k \in K} x_{ijk} + (Q - q_i - q_j) \sum_{k \in K} x_{jik} \le Q - q_j \quad (i, j) \in A \tag{6}$$

$$u_i \le Q - (Q - q_i) \sum_{k \in K} x_{0ik} \quad i \in N_0 \tag{7}$$

$$u_i - q_i \sum_{k \in K} x_{0ik} \ge 0 \quad i \in N_0 \tag{8}$$

$$\begin{aligned} v_{ik} - v_{jk} + (T_l - d_{0j} - d_{i0} + d_{ij}) x_{ijk} + (T_l - d_{0j} - d_{i0} - d_{ji}) x_{jik} \\ \le T_l - d_{0j} - d_{i0} \quad (i, j) \in A, \; k \in K_l, \quad l \in L \end{aligned} \tag{9}$$

$$v_{ik} = T_l - d_{i0} - (T_l - d_{0i} - d_{i0}) x_{0ik} \quad i \in N_0, \; k \in K_l, \; l \in L \tag{10}$$

$$v_{ik} - d_{0i} x_{0ik} \ge 0 \quad i \in N_0, \; k \in K \tag{11}$$

$$\sum_{k \in K_1} \sum_{j \in N} x_{0jk} > \sum_{k \in K_2} \sum_{j \in N} x_{0jk} > \ldots > \sum_{k \in K_r} \sum_{j \in N} x_{0jk} \tag{12}$$

$$x_{ijk} \in \{0, 1\} \quad (i, j) \in A, \; k \in K \tag{13}$$

$$u_i, \; v_{ik} \ge 0 \quad i \in N_0, \; k \in K. \tag{14}$$

The formulation (1)–(14) presented above minimizes the total (distance-based) cost of travel. Constraints (2) and (3) are used to ensure that each customer is visited exactly once, whereas constraint (4) is used to preserve the flow of vehicles from and to a given customer node. Constraints (5) are used to limit the number of vehicles used by the available fleet size. Capacity limitations on each tour are modeled using constraints (6)–(8), which also serve to prohibit the formation of subtours, which are tours forming amongst customer nodes only and are disconnected from the depot. A proof of this can be found in Kara et al. [27]. These constraints also ensure that vehicle capacity constraints give precise meanings to variables u_i, as shown in the proposition below.

Proposition 1. Constraints (6)–(8) set variables u_i, for each customer $i \in N$, to denote the load on the vehicle leaving node $i \in N$ and ensure that the total demand on each tour does not exceed the vehicle capacity Q.

Proof: Consider a node $i \in N_0$ that is first visited on a given tour for vehicle $k \in K$, i.e., $x_{0ik} = 1$. By virtue of the assignment constraints (5) the condition $\sum_{k \in K} x_{0ik} = 1$ holds. Constraints (7) and (8) then jointly imply $u_i = q_i$ for this particular node $i \in N_0$. Constraints (6) written for any arc $(i, j) \in A$ such that $x_{ijk} = 1$ for a given $k \in K$ enforce the condition that $u_j = u_i + q_j$, which "link" the u_i variables over all nodes on a given tour, which implies the last node on a tour will denote the total amount of load carried by the vehicle on this particular tour. Constraints (7) guarantee that $u_i \leq Q$ for any $i \in N_0$, also valid for the last node on a tour, hence the capacity constraints are respected. □

Similarly, constraints (9)–(11) model route length restrictions. More specifically, these constraints ensure that a tour's length which is composed of the total travel time between nodes cannot exceed T_l for each class $l \in L$. Constraints (6)–(11) are based on and variations of Miller et al. [28] subtour elimination constraints for the CVRP and adapted to our problem with varying driving range limitations. Further details on the basic version of these constraints for the "homogeneous" driving-range version can be found in Kulkarni and Bhave [29]; Achuthan and Caccetta [30]; Desrochers and Laporte [31]; Naddef [17]; Kara et al. [27]. For our problem, these constraints are used to give precise definitions to variables v_{ik} as shown in Proposition 2 below.

Proposition 2. Constraints (9)–(11) initialize variables v_{ik}, for each customer $i \in N$ and vehicle $k \in K_l$, $l \in L$ visiting this customer, to denote the total distance traversed by the vehicle until node $i \in N$. These constraints ensure that the total distance on the tour of this vehicle $k \in K$ does not exceed the driving range T_l.

Proof: For a customer $i \in N_0$ visited first on the tour of a vehicle $k \in K_l$, $l \in L$, one has $x_{0jk} = 1$, for which case constraints (10) and (11) jointly imply $v_{ik} = d_{0i}$. "Linking" variables v_{ik} on a given tour, similar to Proposition 1, is done through constraints (9). For any node apart from the first node on a tour, but including the last node on the tour, constraints (10) imply $v_{ik} + d_{i0} \leq T_l$, which allow the vehicle to return to the depot by traversing distance d_{i0} in addition to what it already has traversed, i.e., v_{ik}, but still be within the driving range T_l. □

Constraints (12) state the priorities amongst the different classes of vehicles, i.e., priority increases with decreasing driving ranges. Finally, constraints (13) and (14) represent the integrality and nonnegativity conditions on the set of variables.

We note that for a given $(i, j) \in A$ such that $T_l - d_{0j} - d_{i0} < 0$, one can enforce the condition

$$x_{0j} + x_{ij} + x_{i0} \leq 2, \tag{15}$$

as it is clear that any solution with $x_{0j} = x_{ij} = x_{i0} = 1$ will violate the distance restrictions. In this case, constraints (9) can be replaced by the inequality (15) for such arcs. A similar argument can be stated for any node $i \in N_0$ for which $T_l - d_{0i} - d_{i0} < 0$, implying that a solution with $x_{0i} = x_{i0}$ will violate the driving range restrictions. In this case, one can replace these constraints by inequality $x_{0i} + x_{i0} \leq 1$ for any such node.

There are several challenges in solving the formulation presented above. The first of these is the nonlinearity of constraints (12). One way of overcoming this difficulty is to convert these constraints into an objective function of the following type, minimize

$$\omega_1 \sum_{k \in K_1} \sum_{j \in N} x_{0jk} + \omega_2 \sum_{k \in K_2} \sum_{j \in N} x_{0jk} + \ldots + \omega_r \sum_{k \in K_r} \sum_{j \in N} x_{0jk}, \tag{16}$$

where $\omega_1, \ldots, \omega_r$ are nonnegative weights such that $\omega_1 > \omega_2 > \ldots > \omega_r$. Removing constraints (12) from the formulation and appending (16) to the formulation above as a second objective function would result in a multi-objective integer programming formulation [32]. One can then further combine objectives (1) and (16) to convert the formulation into a single-objective formulation. One way to choose the weights $\omega_1, \ldots, \omega_r$ is to use the "costs" of the different types of vehicles. However, vehicle costs depend on various factors (e.g., depreciation, running costs, or battery costs) and are difficult to calculate, particularly for vehicles using new technologies. Our approach in this work has been to relax somewhat the importance of distance-based costs in our model and, instead, to consider also the number of vehicles of each type as one of the primary determinants to evaluate a solution. In other words, we are interested in computing distance-based costs for alternative fleet configurations – some of them "greener" than others – and then analyze how these costs vary as more ICE vehicles are replaced by EVs with restricted driving ranges. In our opinion, this approach is interesting for decision makers since it allows them to answer two critical questions: (a) "can I use more EVs in my fleet configuration?", and (b) "how much will my distance-based costs be increased by using more EVs"?

Even if the problem has been modeled above as a single objective integer program, the challenge of solving it still remains as the resulting formulation is variation of the VRP, an NP-Hard problem, with $O(n^2k)$ binary variables. In fact, the aforementioned model – expressions (1)–(14) excluding constraints (12) – has been found to be difficult to even produce a feasible solution in our preliminary experiments on one of the smallest benchmark VRPs, namely A-n32-k5 (http://www.branchandcut.org) using CPLEX 12.4 within reasonable computing time. Given that the VRPMDR is a problem at an operational level of decision making and requires to be solved repeatedly (e.g., daily), a fast heuristic algorithm is required for its solution. We describe one such algorithm in Section 4.

4. The multi-round heuristic algorithm

The approach proposed in this paper is a multi-round or successive-approximations heuristic, with the number of necessary rounds limited by the number of distinct driving-range classifications for the fleet. For example, when considering a fleet configuration composed of ICEs, PHEVs, and two types of EVs (i.e., EVs using batteries with different capacity), a global feasible solution will typically require three rounds: a first one associated with the 'unlimited' range ICEs and PHEVs that can refuel easily, a second one associated with 'medium' range EVs with larger batteries, and a third one associated with 'short' range EVs with smaller batteries. At each round of the problem-solving process, a different CVRP with route length restrictions (CVRP*) is solved. A CVRP* is a CVRP with an additional constraint limiting the maximum distance for any route. The maximum route distance considered for each round is given by the maximum driving range for all unused vehicles. When the problem specification includes some unlimited driving-range vehicles, no restriction will be assumed on the route length for the first round, i.e., a classical CVRP is solved in this first round. In the remaining rounds, when no more unlimited driving range vehicles are available, a CVRP* must be solved considering the maximum route distance for that round. From all the routes generated by solving the CVRP* in that round, those that are feasible – i.e., those that can be covered by unused vehicles – are saved as a partial solution, while the remaining routes are discarded and dissolved so that their associated nodes and vehicles can be dealt with in the next round. Once the multi-round process has finished and all nodes' demands have been satisfied in a feasible way, the final global solution for the VRPMDR is obtained as the union of the partial and disjoint sub-solutions obtained during each round.

A more structured description of the multi-round process is given below as a six-step algorithm and summarized in a flowchart (Fig. 1).

1. ***Define the CVRP*:*** Define a new CVRP* – original CVRP in the first round – with the non-served nodes and unused vehicles. The maximum route distance allowed in this problem is given by the maximum of the driving range distances for all unused vehicles – which will be unlimited in the first round if ICE vehicles are available.
2. ***Solve the CVRP*:*** Assume all vehicles have the same driving range capacity and then solve the associated CVRP*, (i.e., consider a single maximum route distance) using any efficient algorithm, e.g., Juan et al. [8].
3. ***Assign routes to vehicles:*** Sort the routes obtained in the previous step by total distance (from longest to shortest routes); sort the unused vehicles according to their driving range capacity (from longest to shortest range); then, assign the routes to vehicles in the order specified until the feasibility of the assignment does not hold, i.e., until the next vehicle driving range is lower than the corresponding route distance.
4. ***Extract assignments and reset data:*** Save in memory the routes (including nodes) assigned to vehicles; delete these nodes and vehicles from the problem to be solved; then dissolve the unassigned routes and reset the data of non-served nodes and unused vehicles for the next round.
5. ***Repeat steps*** **1–4** until all nodes have been served.
6. ***Create the final solution:*** The final solution for the VRPMDR is the union of all previously saved (partial and disjoint) sub-solutions.

Finally, a simple but a step-by-step illustrative example of how this multi-round algorithm runs on a small-scale instance of 17 customers is shown in Fig. 2. The figure show two rounds of the algorithm where two ICE and three EV vehicles are available, where iterations corresponding to the steps above are shown in the numbers inside the hexagons. In the first round, a CVRP* on the instance is solved which results in using two ICE vehicles to serve eight customers. The two routes form a partial solution and are subsequently taken out from the solution. The second round then allocates three EVs to the remaining nine customers, which results in another feasible solution. The combination of two feasible solutions in the first and second round results in a complete solution for the overall problem instance.

5. Data generation and numerical experiments

The algorithm described in this paper has been implemented as a Java application. For the heuristic randomization process we employed the linear feedback shift register (LFSR) generator, which is a fast and well tested pseudo-random generator (RNG) [33]. Using the LFSR113 as our base RNG with an approximate period length of 2^{113}, the biased randomization process has been performed by employing a truncated Geometric distribution with a single parameter, β. More specifically, at each step of the edge-selection process a new pseudo-random number υ is generated using the LFSR113, and then the position ρ of the next-edge in the sorted savings list is selected according to the following formula,

$$\rho = \lfloor \lambda \rfloor \bmod \eta$$

where η is the number of edges in the savings list and λ is a random variate from a Geometric distribution with parameter $0 < \beta < 1$, i.e.,:

$$\lambda = \frac{\log \upsilon}{\log(1-\beta)}$$

Since β is the only user-defined parameter that our approach employs, the algorithm does not require complex fine-tuning of a parameter, which is often time-consuming. Roughly speaking, higher values of β imply more bias. For instance, using a value of β close enough to 1 will always select the next position in the sorted list and, therefore, it will provide the same results that the classical (deterministic) heuristic. On the other extreme, using a value of β close enough to 0 will provide uniform random selection of the edges in the list, i.e., it will not consider the sorting (savings) criteria at all. Of course, better results will be obtained by using intermediate values for β. In our experiments, we have used random values for β in the interval (0.10, 0.20), which according to some previous studies seem to provide reasonable results for most benchmarks [8].

A total of 20 classical CVRP benchmark instances were randomly selected from the website http://www.branchandcut.org, an online repository which contains detailed information on a large number of benchmark CVRP instances. Only those instances meeting the following criteria were considered: (a) instances for which an optimal or pseudo-optimal solution was given, which included instances from sets A, B, E, F, M, and P; (b) instances offering complete information, e.g. specific routes, for the optimal or pseudo-optimal solution; and (c) mid-size instances between 22 and 135 nodes. We consider this size as quite representative of most small and medium transportation enterprises. The selected benchmark files are shown in Table 1. These benchmarks differ in the number of nodes, the vehicle capacity, the location of the depot with respect to the clients (in a corner or in the center) and their topology (scattered or clustered nodes). Finally, three classifications of vehicles were considered in our tests: (a) ICE and PHEV vehicles, which are vehicles with non-restricted driving range; (b) medium range EVs, which are able to complete tours up to 200 distance units (du.); and (c) short range EVs, which are able to traverse tours up to 100 du. The difference in driving range between EVs would likely be due to different battery capacities. The goals of these tests were: (a) to estimate the distance-based costs associated with alternative fleet configurations – some of them using EVs; and (b) to analyze how these costs increase as we substitute ICE vehicles by other vehicles with limited driving ranges.

A standard personal computer Intel® Xeon® CPU E5504 @ 2.00 GHz and 4 GB RAM running Windows 7 Pro operating system was used to perform all tests. In order to test the efficiency of our approach, the algorithm was allowed a maximum computational time of 300 s for each instance. The results of these tests are summarized in Table 1, which contains the following columns for each instance: (a) name of the instance; (b) number of nodes (customers) in the instance; (c) the vehicle capacity Q; (d) distance-based costs obtained using the best-known solution (BKS), these costs have been obtained using the solutions available at http://www.branchandcut.org; (e) different feasible fleet configurations found by our algorithm; these configurations are shown by S/M/L, where L is the number of long-range (more than 200 du.) vehicles, M is the number of medium-range (up to 200 du.)

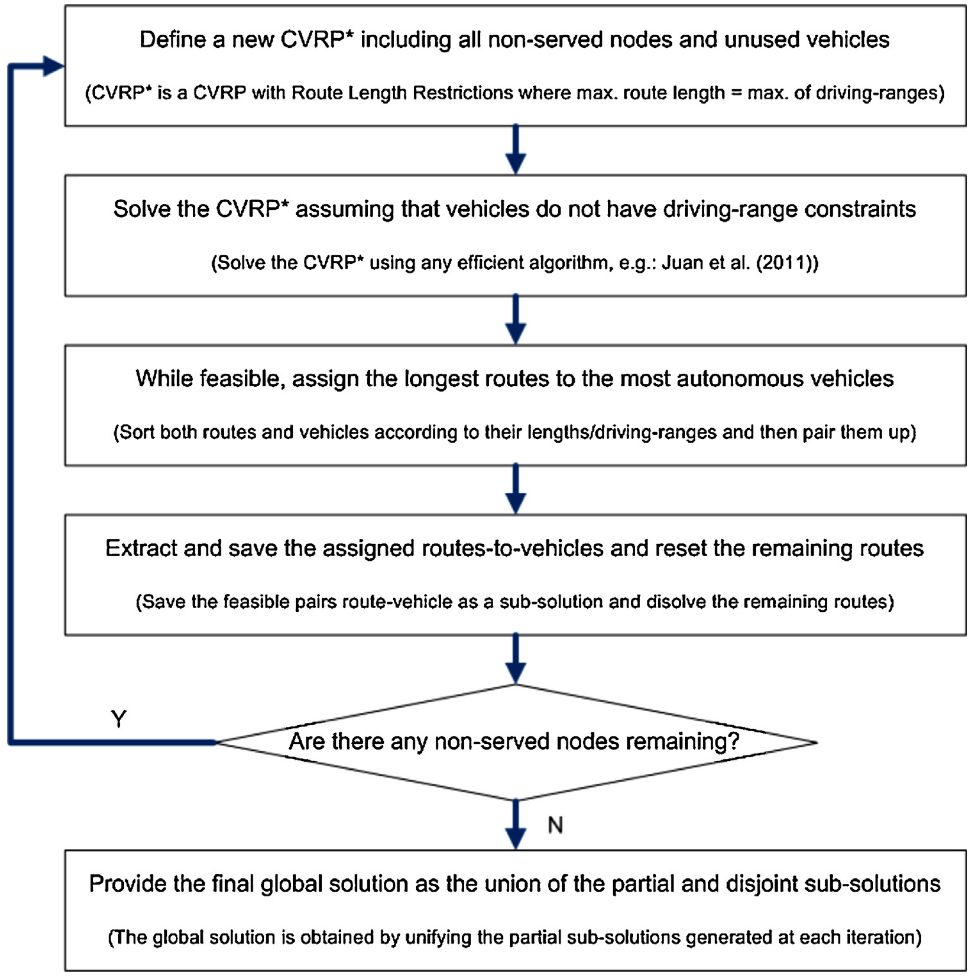

Fig. 1. Flow-chart of the proposed algorithm.

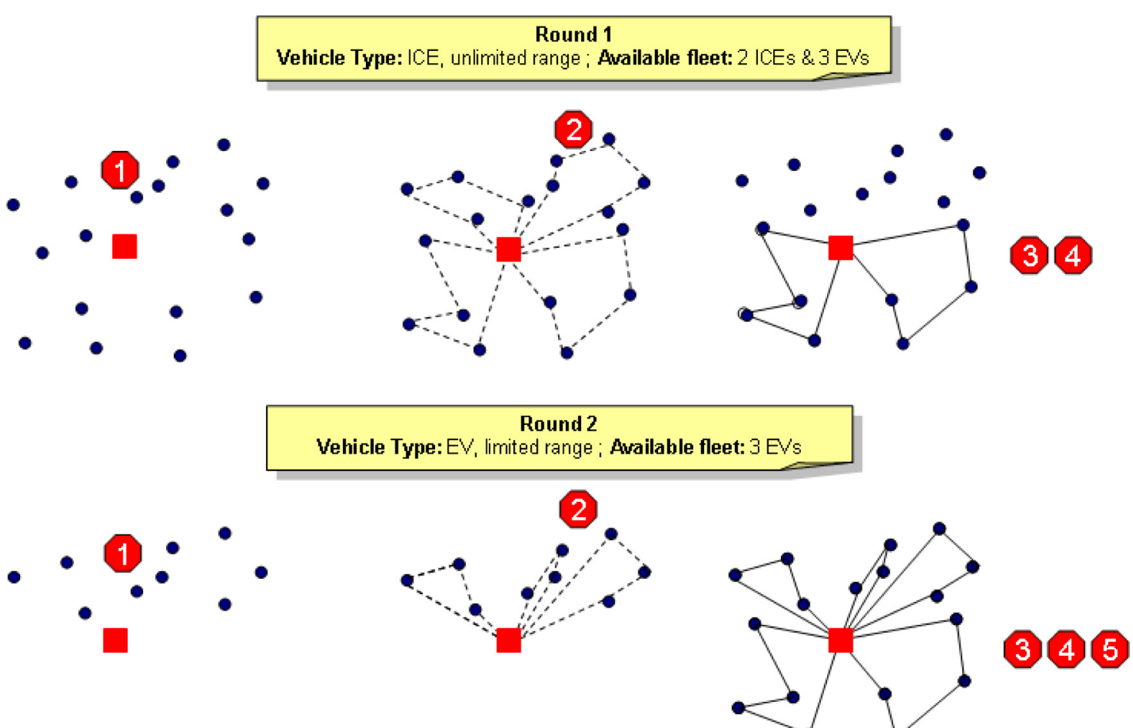

Fig. 2. An illustration of the multi-round heuristic on a small-scale instance.

Table 1
Experimental results for 20 mid-size, randomly selected, classical CVRP instances.

(a) Name	(b) # Nodes	(c) Capacity	(d) BKS Cost	(e) Fleet configuration S/M/L	(f) OS cost	(g) Gap (d)–(f)
A-n32-k5	32	100	787.81	2/1/2	787.08	−0.09%
				1/3/1	829.41	5.28%
A-n38-k5	38	100	734.18	0/5/0	733.95	−0.03%
				1/3/1	734.18	0.00%
				1/4/0	735.05	0.12%
				3/3/0	763.13	3.94%
A-n65-k9	65	100	1181.69	1/8/0	1183.31	0.14%
				2/7/0	1191.27	0.81%
				3/6/1	1238.33	4.79%
				4/6/0	1253.81	6.10%
				5/5/0	1276.21	8.00%
				3/5/1	1297.31	9.78%
A-n80-k10	80	100	1766.50	2/5/3	1776.19	0.55%
				1/7/2	1785.05	1.05%
B-n50-k7	50	100	744.78	2/5/0	744.23	−0.07%
				3/4/0	744.67	−0.01%
				4/3/0	751.24	0.87%
B-n52-k7	52	100	750.08	4/2/1	752.63	0.34%
				3/4/0	756.71	0.88%
B-n57-k9	57	100	1603.63	0/4/5	1602.29	−0.08%
				0/5/4	1603.37	−0.02%
				0/6/3	1631.66	1.75%
				1/3/5	1642.53	2.43%
				1/4/4	1646.65	2.68%
B-n78-k10	78	100	1229.27	4/5/1	1236.33	0.57%
				3/7/0	1251.83	1.83%
				4/4/2	1252.76	1.91%
				4/6/0	1253.10	1.94%
				6/5/0	1292.60	5.15%
E-n22-k4	22	6000	375.28	2/2/0	375.28	0.00%
				3/1/0	383.52	2.19%
E-n30-k3	30	4500	535.80	1/3/0	505.01	−5.75%
E-n51-k5	51	160	524.94	3/2/0	524.63	−0.06%
E-n76-k10	76	220	837.36	7/3/0	845.80	1.01%
				8/2/0	856.70	2.31%
				11/0/0	854.42	2.04%
E-n76-k14	76	100	1026.71	13/2/0	1031.94	0.51%
				14/1/0	1041.58	1.45%
				13/1/0	1043.29	1.62%
				15/0/0	1045.77	1.86%
F-n135-k7	135	2210	1170.65	3/1/3	1175.73	0.43%
				3/2/2	1190.07	1.66%
M-n101-k10	101	200	819.81	8/2/0	821.11	0.16%
M-n121-k7	121	200	1045.16	2/3/2	1047.96	0.27%
				1/7/0	1274.60	21.95%
P-n50-k10	50	100	699.56	10/0/0	700.66	0.16%
P-n55-k15	55	70	991.48	16/0/0	952.02	−3.98%
P-n70-k10	70	135	830.02	8/2/0	834.38	0.53%
				10/0/0	841.56	1.39%
P-n76-k5	76	280	635.04	1/4/0	638.44	0.54%
				2/3/0	647.51	1.96%
				4/2/0	696.63	9.70%

vehicles, and S shows the number of short-range (up to 100 du.) vehicles; more feasible configurations have been found for each instance, but only those which are non-dominated, either in cost or in 'green level' have been considered in this table; notice that we are considering that one configuration is 'greener' than another if it substitutes vehicles of type L by vehicles of type M or S (with S preferred over M), or vehicles of type M by vehicles of type S (without increasing the number of vehicles of type L); (f) distance-based costs associated to each of the feasible and non-dominated configurations provided by our algorithm in the aforementioned computing-time period; and (g) percentage gap in distance-based costs between the BKS and each of the alternative solutions provided by our algorithm.

We first evaluated the robustness of the algorithm by solving the test problems without driving range constraints and comparing with the best-known solution from the reference site. As shown in column (f), the cost of the best solution provided by our algorithm – in a reasonable computing time – is always quite similar to the one provided by the BKS. In some cases our costs are even better than the BKS costs due to the fact that we are considering real costs while most BKS in the reference site are obtained using integer-rounded costs. There is even one case, the P-n55-k15, in which our best solution is able to clearly outperform the BKS by using one more vehicle.

We then created problems using the same base data but with pre-defined fleets having three classifications of vehicle driving ranges that represent one set of ICEs/PHEVs and two sets of EVs, as described above. These alternative problems allow evaluating the ability to meet customer needs using a higher number of EVs while keeping distance-based costs comparable. Note that in 15 of the 20 tested instances our approach was able to generate alternative solutions that use 'greener' fleet configurations by employing fewer long- or medium-range vehicles. In most cases, this is attained without significantly increasing distance-based costs. Regarding

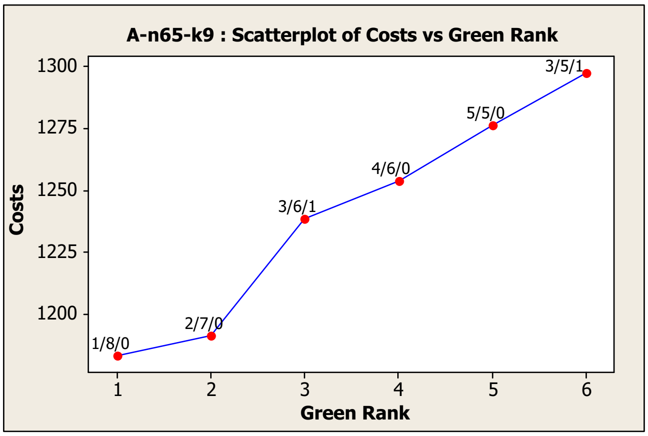

Fig. 3. Comparison of different fleet configurations for the instance A-n65-k9.

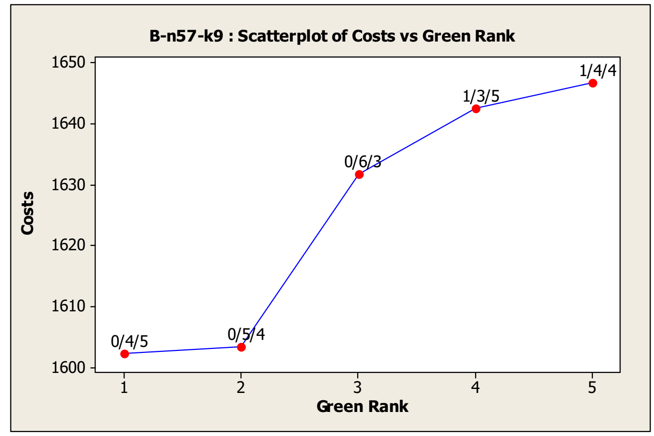

Fig. 4. Comparison of different fleet configurations for the instance B-n57-k9.

the remaining 5 instances, two of them (P-n50-k10 and P-n55-k15) were already using only S-type vehicles and, therefore, it was not possible to obtain alternative 'greener' solutions for them.

Using the results in Table 1, Figs. 3 and 4 illustrate several alternative fleet configurations for the instances A-n65-k9 and B-n57-k9, respectively. Both figures show: (a) that multiple fleet configurations, some of them 'greener' than others, can be chosen to solve the problem; and (b) how distance-based costs vary as the decision maker selects 'greener' fleet configurations. Notice that these figures resemble to Pareto Frontiers in the sense that they provide a series of (best-found) non-dominant solutions. Thus, while the B-n57-k9 instance can be solved by using 4 vehicles of type M (medium-range EVs) and 5 vehicles of type L (ICEs), it can also be solved using other fleet configurations which happen to be 'greener', e.g.: 1 vehicle of type S (short-range EVs), 4 vehicles of type M, and 4 vehicles of type L. A similar observation can be done by analyzing Fig. 4. Of course, this information can be very valuable to the decision maker while considering not only distance-based costs but also other factors such as: environmental costs, fixed and variable costs associated to each type of vehicle, etc.

Fig. 5 shows a comparative plot between the BKS and our first alternative (and 'greener') solution for the instance A-n80-k10. While both solutions show similar distance-based costs and contain the same number of routes, our alternative solution uses only two long-range vehicles while the BKS requires three of these vehicles. Similarly, Fig. 6 shows a comparative plot between the BKS and our alternative solution for the instance B-n57-k9. Distance-based costs are basically equivalent for both solutions, but our alternative solution is able to serve the same customers using fewer long-range vehicles. Again, a similar situation can be observed in Fig. 7, where our alternative solution for the M-n121-k7 is able to reduce from three to two the number of long-range vehicles required to satisfy customers' demands without increasing too much the distance-based costs.

The results shown above that our approach is able to provide alternative solutions that are similar (in terms of distance-based

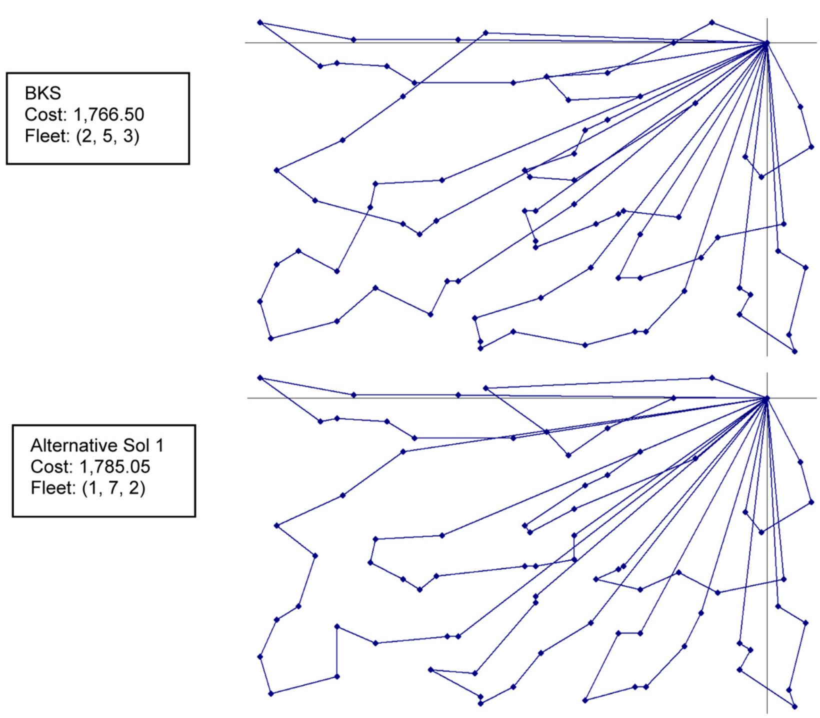

Fig. 5. A visual comparison of two alternative solutions for instance A-n80-k10.

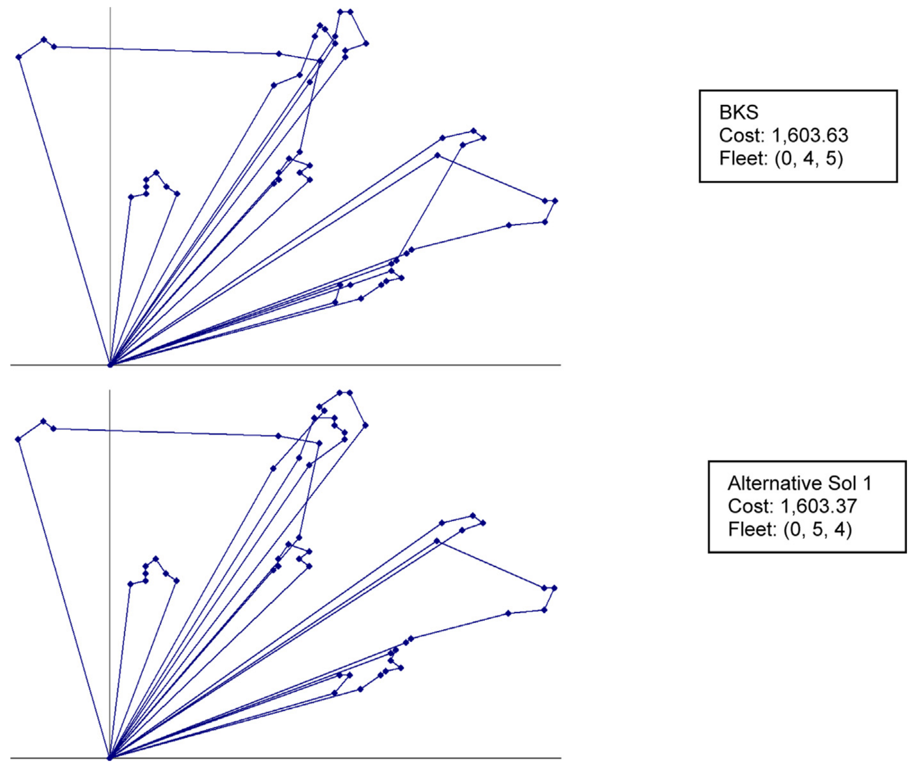

Fig. 6. A visual comparison of two alternative solutions for instance B-n57-k9.

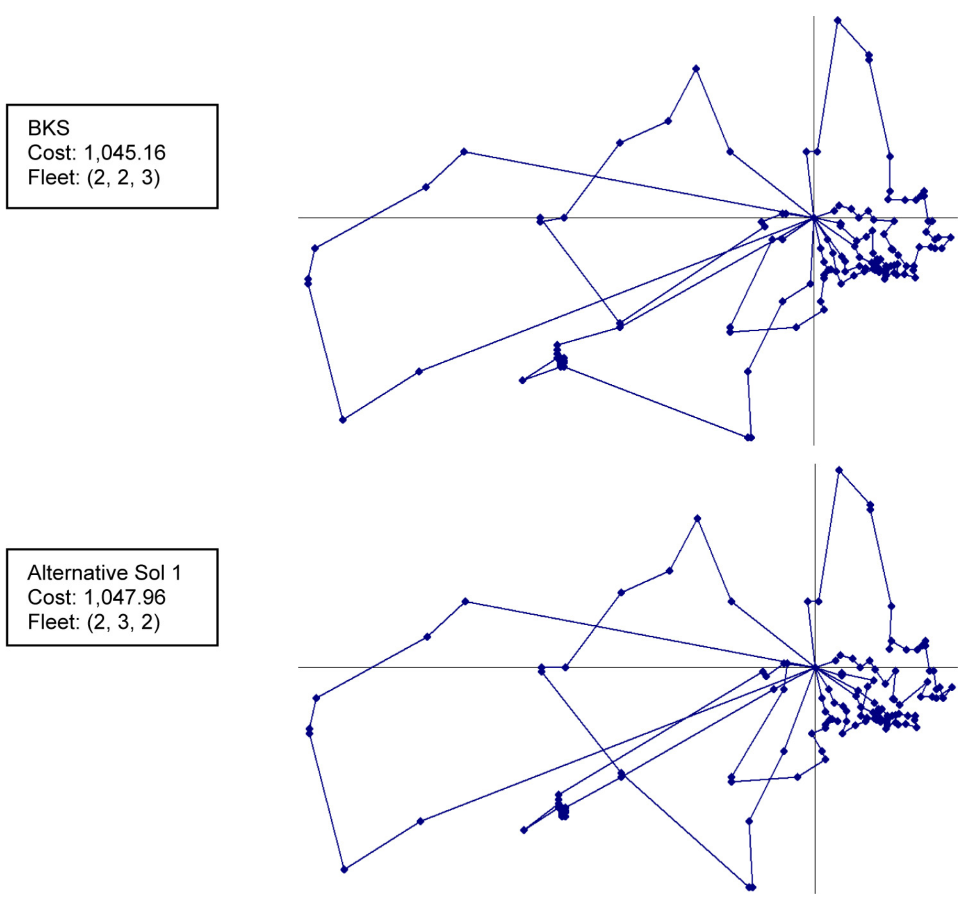

Fig. 7. A visual comparison of two alternative solutions for instance M-n121-k7.

costs) to state-of-the-art solutions but, at the same time, offer an improved – i.e. 'greener' – fleet configuration. We note that the use of EVs sometimes result in longer total distances (hence distance-based costs), but this additional cost may well be more than offset by the lower operating costs of the EVs. The actual operating costs of EVs are beyond the scope of this paper and this is a difficult problem to analyze in itself; for a new model and methodology to evaluate competitiveness of EVs and the factors affecting it, the reader is referred to Figliozzi and Davis [34]. However, the results provided in this paper allow for an assessment of the trade-off between the additional costs incurred and the lower operational costs.

6. Conclusions and future work

This paper introduces the vehicle routing problem with multiple driving ranges (VRPMDR), which is motivated by the practical challenge driven by the increasing number of EVs – with different degrees of autonomy – in corporate fleets. To the best of our knowledge, it is the first time that multiple values for the maximum route length have been simultaneously considered in the vehicle routing literature. A further contribution has been made by describing an integer programming formulation of the problem as well as a multi-round heuristic algorithm that can be used to generate solutions for the VRPMDR with pre-specified fleet configurations. The multi-round approach relies on a biased randomized algorithm that can also be used on its own to generate alternative fleet configurations whenever feasibility of the pre-specified fleet configuration is not guaranteed. Numerical experiments demonstrate that this approach provides attractive solutions for all tested benchmarks. Most of the alternative solutions obtained offer competitive distance-based costs while requiring fewer long- or medium-range vehicles than the best-known solution. Consequently, the paper also contributes to support the hypothesis that hybrid and electric vehicles can be used in routing problems without necessarily incurring significantly higher distance-based costs.

The VRPMDR offers numerous research lines to explore, both from a theoretical and from a practical point of view. First, scenario analysis could be used to identify an effective fleet composition. Exploring how distance-based costs vary as the fleet composition changes, one could determine the best number of EVs to add before incurring prohibitive costs or infeasible service situations. Second, including fixed investment and infrastructure costs, such as those mentioned in De Los Rios et al. [3], would offer a more formal, economically driven analysis of the tradeoff between fleet configuration and distance-based cost. Furthermore, models that monetize the carbon footprint could be incorporated to evaluate solutions across an economic-environmental continuum. Third, the impact of topography could be explored. It could be assumed that EVs work better in dense clusters of customers close to the depot. But the characteristics of customer density and their distances from the depot and the impact of these parameters on the ability to deploy EVs is an avenue yet to be explored. Fourth, the general structure of the multi-round procedure could incorporate new algorithms (both of exact and approximate nature) that exploit the problem structure to provide better solutions.

Acknowledgements

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