PEC 1. Problems in 1D

Finite Differences and Finite Elements

URV-UOC. 2020-21

1) Modify the program L2Projector1D in order to compute the approximations L^2 in the case of the following functions.

a) $f(x) = 1 + 2x$

b) $g(x) = x^3(x-1)(1-2x)$

c) $h(x) = \arctan((x-0.5)/\epsilon)$, **with** $\epsilon = 0.1$ **and** $\epsilon = 0.01$

Use uniform partitions in the interval $I = [-1,1]$ **with** $n = 25$ **and** $n = 100$ **subintervals. Give also an estimation of the error produced in these approximations.**

2) a) Write down the equations needed to resolve numerically, using the Finite Elements Method (FEM), the 1D elliptic general problem:

$$
\begin{aligned}
-(au')' + bu' + cu = f, \quad x \in I = [0,L] \\
au'(0) = \kappa_0(u(0) - g_{0D}) + g_{0N} \\
-au'(L) = \kappa_L(u(L) - g_{LD}) + g_{LN}
\end{aligned}
$$

being $a = a(x)$, $b = b(x)$, $c = c(x)$ **and** $f = f(x)$ **real functions with sufficienty high degrees in their derivatives.**

b) Modify accordingly the programs of the book to solve numerically the problem:

$$
\begin{aligned}
u'' + u = x^2,\ x \in I = [0,1] \\
u(0) = 0 \\
u(1) = 0
\end{aligned}
$$

Use the midpoint or trapezoidal approximation for the numerical integrations. Compare the numerical solution with the exact solution given by:

$$
u(x) = \frac{\sin x + 2\sin(1-x)}{\sin 1} + x^2 - 2
$$

c) Solve now numerically the boudary value problem

$$
\begin{aligned}
u'' + u' - 2u = x,\ x \in I = [0,1] \\
u(0) = 0 \\
u'(1) = 1
\end{aligned}
$$