# Signal and sistems
# Laboratory work - Inverted pendulum stabilization
# COUAP-4109

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October 23, 2019

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Figure 1: Laboratory module

# 1 Incroduction

The objective of this work is to stabilize the inverted pendulum in an upright position. For stabilization is used feedback controller based on the pole placement method. The pendulum consists of a driven arm. This arm rotates in the horizontal plane, and a pendulum attached to that arm is free to rotate in the vertical plane $[-\frac{\pi}{8}, +\frac{\pi}{8}]$. The arm has powered by the $\pm 10$V DC motor with constraints movement around.

The system is exceptionally non-linear due to the gravitational forces that affect him. To realize the stabilization of this system, we have the possibility to use two sensors that measure with a satisfactory defect. The first sensor measures the position of the rod angle relative to the vertical position, and the second sensor measures the position of the arm motor angle. Both sensors have powered a voltage of $\pm 1,33$V.

# 2 Model of a rotary pendulum

The non-linear equation are given by

$$\begin{aligned} &\ddot{\theta}_1 \left(J_{1zz} + m_1 l_1^2 + m_2 L_1^2 + (J_{2yy} + m_2 l_2^2)\sin^2(\theta_2) + J_{2xx}\cos^2(\theta_2)\right) \\ &\quad + \ddot{\theta}_2 m_2 L_1 l_2 \cos(\theta_2) - m_2 L_1 l_2 \sin(\theta_2)\dot{\theta}_2^2 \\ &\quad + \dot{\theta}_1\dot{\theta}_2 \sin(2\theta_2)(m_2 l_2^2 + J_{2yy} - J_{2xx}) + b_1\dot{\theta}_1 = \tau_1 \end{aligned} \tag{1}$$

and

$$\begin{aligned} &\ddot{\theta}_1 m_2 L_1 l_2 \cos(\theta_2) + \ddot{\theta}_2(m_2 l_2^2 + J_{2zz}) \\ &\quad + 1/2\dot{\theta}_1^2 \sin(2\theta_2)(-m_2 l_2^2 - J_{2yy} + J_{2xx}) + b_2\dot{\theta}_2 + g m_2 l_2 \sin(\theta_2) = \tau_2 \end{aligned} \tag{2}$$

These equations are required by Broyden's method, which is the method for developing non-linear mathematical models like in Figure 2 on page 3.

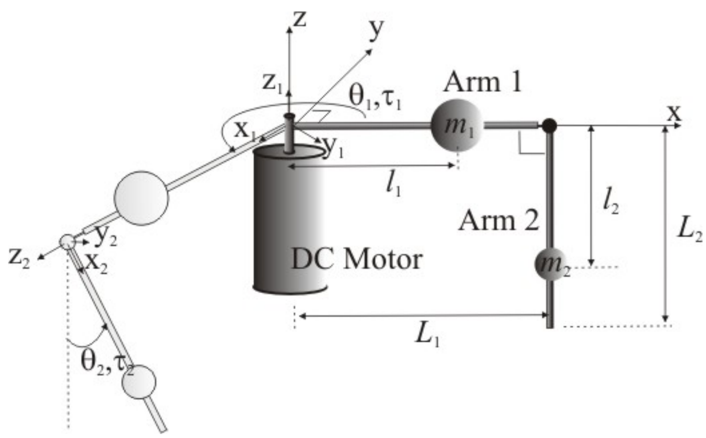


Figure 2: Model of Inverted Pendulum

# 3 Given Matlab/Simulink files and work to be done

**Modele_Pendule.m** State space model of opened loop inverted pendulum based on the data provided by the constructor.

**Observateur_Controleur_Dspace_IMC.slx** Observer-Controller discrete model used to generate the real time controller code.

**Contruct the inverted pendulum** open loop state space model in Simulink and analyse its **stability, reachability and observability**.

**Design a continuous** observer-controller capable to stabilize the inverted pendulum in its upright position. Realize the Simulink control scheme based on the figure given in the related topic and give it the name **Pendule_Observateur _Controleur_Continu.slx** and conduct the necessary tests.

**Design a discrete** observer-controller capable to stabilize the inverted pendulum in its upright position. Realize the Simulink control scheme based on the figure given in the related topic and give it the name **Pendule_Observateur _Controleur_Discret.slx** and conduct the necessary tests.

## 4 Specification

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## 5 Discussion of Experimental Uncertainty

The accepted value (periodic table) is $24.3\,\mathrm{g}\cdot\mathrm{mol}^{-1}$ Smith and Jones (2012). The percentage discrepancy between the accepted value and the result obtained here is 1.3%. Because only a single measurement was made, it is not possible to calculate an estimated standard deviation.

The most obvious source of experimental uncertainty is the limited precision of the balance. Other potential sources of experimental uncertainty are: the reaction might not be complete; if not enough time was allowed for total oxidation, less than complete oxidation of the magnesium might have, in part, reacted with nitrogen in the air (incorrect reaction); the magnesium oxide might have absorbed water from the air, and thus weigh "too much." Because the result obtained is close to the accepted value it is possible that some of these experimental uncertainties have fortuitously cancelled one another.

## 6 Answers to Definitions

a. The *atomic weight of an element* is the relative weight of one of its atoms compared to C-12 with a weight of 12.0000000…, hydrogen with a weight of 1.008, to oxygen with a weight of 16.00. Atomic weight is also the average weight of all the atoms of that element as they occur in nature.

b. The *units of atomic weight* are two-fold, with an identical numerical value. They are g/mole of atoms (or just g/mol) or amu/atom.

c. *Percentage discrepancy* between an accepted (literature) value and an experimental value is

$$\frac{\text{experimental result} - \text{accepted result}}{\text{accepted result}}$$

# References

Smith, J. M. and Jones, A. B. (2012). *Chemistry*. Publisher, 7th edition.