SDRT 1: Information Content

Julian J. Schlöder

Formal Pragmatics, Lecture 3, Jan 15th

Why discourse relations?

- DRT does not (always) make the right predictions for anaphora.

(1) a. John dropped off his car for repairs.
    b. He got a rental.
    c. But it had a broken fuel pump.

- DRT: flat structure.
  - → *his car* available for *it*.
- Discourse Relations: complex structure.
  - → Narration(a,b) blocks this binding.

Complex Structure

– Rhetorical structure can be complex, and smaller structures can be embedded in larger ones.

(2) a. A: What would you like to order?
b. B: Can I have a pizza without cheese?
c. A: Yes.
d. B: I'll have the Margherita without cheese.

-Q-Elab
-QAP
-QAP

Complex Structure

– Rhetorical structure can be complex, and smaller structures can be embedded in larger ones.

(2) a. A: What would you like to order?
b. B: Can I have a pizza without cheese?
c. A: Yes.
d. B: I'll have the Margherita without cheese.

-Q-Elab
-QAP
-QAP

– Structures themselves can be part of rhetorical relations.

(3) a. A: What did you have for dinner?
b. B: Oh, I made a huge meal.
c. B: Soup, steak, potatoes, salad, icecream and cheese!

-Elab
-QAP

SDRT

- SDRT is an integrated theory of discourse relations.
  - → What discourse relations mean.
  - → How they are inferred.

- Four main component logics:
- Logic of Information Content for the truth-conditional semantics of a discourse.
- Logic of Cognitive States for the pragmatic attitudes of speakers in dialogue.
- Glue Logic to construct these logical forms.
- (Underspecified Logical Form)

(4) Many puzzles preoccupy every logician.

Subordinating and Coordinating Relations

- For some discourse relations, if *Rel*(a,b), then anaphora from after b can access referents in a.
  - → Call these subordinating.
- E.g. *Elaboration*
- For some discourse relations, if *Rel*(a,b), then anaphora from after b cannot access referents in a.
  - → Call these coordinating.
- E.g. *Narration*
- (Some exceptions: Contrast and Parallel are coordinating because not *all* anaphora can go to the left, but *some* can.)
  - → These two get special semantics.

Parallel and Contrast

- Parallel (cuewords *too*, *also*, …) and Contrast (cueword *but*) have a syntactic requirements: similar structure.
- Parallel: similar structure, similar content.
- Contrast: similar structure, dissimilar content.

Parallel and Contrast

- Parallel (cuewords *too*, *also*, ...) and Contrast (cueword *but*) have a syntactic requirements: similar structure.
- Parallel: similar structure, similar content.
- Contrast: similar structure, dissimilar content.
- Anaphora follows the structure.

(5) a. Thatcher respects Reagan, but Blair admires him. {✓He | ✗She} ..
#b. Thatcher respects Reagan, but Blair admires her.

(6) a. Thatcher admires Reagan. Blair likes him, too. {✓He | ✗She} ...
#b. Thatcher admires Reagan. Blair likes her, too.

(7) a. Thatcher admires Reagan, but he hates her. {✓He | ✓She} ...
b. Thatcher admires Reagan. He admires her too. {✓He | ✓She} ...

The Right Frontier

- Let's graph coordinating relations with horizontal arrows and subordinating relations with vertical arrows.
- Then accessible anaphora are on the right frontier of the resulting graph.

Right Frontier (extended example)

(8) π_1 : John had a great day.
π_2 : In particular, he had a great lunch .
π_3 : He particularly liked the cheese.

π_1 : John had a great day.

Elaboration

π_2 : *j* had a great lunch.

Elaboration

π_3 : *j* liked *c*.

Right Frontier (extended example)

(8) π_1 : John had a great day.
π_2 : In particular, he had a great lunch .
π_3 : He particularly liked the cheese.

π : It [the cheese] was a Dutch Gouda.

π_1 : John had a great day.

Elaboration

π_2 : *j* had a great lunch.

Elaboration

π_3 : *j* liked *c*.

π : ? was a Dutch Gouda.

Right Frontier (extended example)

(8) π_1 : John had a great day.
π_2 : In particular, he had a great lunch .
π_3 : He particularly liked the cheese.

π : It [the cheese] was a Dutch Gouda.

π_1 : John had a great day.

Elaboration

π_2 : *j* had a great lunch.

Elaboration

π_3 : *j* liked *c*.

Elaboration

π : *c* was a Dutch Gouda.

Right Frontier (extended example)

(8) π_1 : John had a great day.
π_2 : In particular, he had a great lunch .
π_3 : He particularly liked the cheese.
π_4 : Afterwards, he visited his girlfriend.

π_1 : John had a great day.

Elaboration

π_2 : *j* had a great lunch.

π_4 : ? visited his girlfriend.

Elaboration

π_3 : *j* liked *c*.

Right Frontier (extended example)

(8) π_1 : John had a great day.
π_2 : In particular, he had a great lunch .
π_3 : He particularly liked the cheese.
π_4 : Afterwards, he visited his girlfriend.

π_1 : John had a great day.

Elaboration

Narration

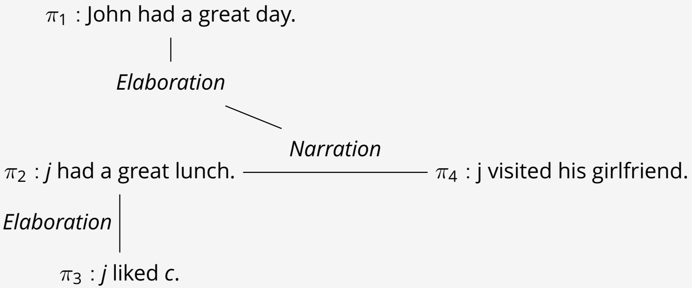
π_2 : *j* had a great lunch. —— π_4 : j visited his girlfriend.

Elaboration

π_3 : *j* liked *c*.

Right Frontier (extended example)

(8) π_1 : John had a great day.
π_2 : In particular, he had a great lunch .
π_3 : He particularly liked the cheese.
π_4 : Afterwards, he visited his girlfriend.
π : #It [#the cheese] was a Dutch Gouda.



π : ? was a Dutch Gouda.

Logical Form of Discourse 1

- We want to write down logical forms for such trees in a way that we can truth conditionally evaluate them.

- (vocab-1) the vocabulary of dynamic predicate logic.
  - → But with typed variables for entities ($x, y, z, \ldots$), events ($e_1, e_2, \ldots$) and times ($n, t_1, t_2, \ldots$)
  - → Throw in a Groenendijk-Stokhof question operator ?, if you like.

- (vocab-2) an unbounded set of labels (α, β, λ, π...).

- (vocab-3) a finite inventory of discourse relations (Narration, Elaboration etc.) and an unbounded set of variable symbols ranging over that inventory (R_1, R_2, ...).

Logical Form of Discourse 2

- The Language of Information Content (LIC):
  - $\rightarrow$ All well-formed DPL formulae are LIC formulae
  - $\rightarrow$ If *R* is a discourse relation symbol and α, β are labels, then *R*(α, β) is a LIC formula.
  - $\rightarrow$ if φ, ψ are LIC wff, then $(\varphi \wedge \psi)$ and $\neg\varphi$ are LIC wff.

Logical Form of Discourse 3

- A Segmented DRS is a triple $(\Pi, \mathcal{F}, L)$ such that:
- Π is a set of labels.
- $\mathcal{F} : \Pi \rightarrow$ LIC is a function mapping labels to LIC wffs
- $L \in \Pi$ (the "last" added label).
- Frequently, I write $\pi : K$ for $\mathcal{F}(\pi) = K$.

Logical Form of Discourse (Example)

(9) π_1 : John had a great day.
π_2 : In particular, he had a great lunch .
π_3 : He particularly liked the cheese.
π_4 : Afterwards, he visited his girlfriend.

- $\Pi = \{\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$.
- $L = \pi_4$.
- $\mathcal{F}(\pi_1) = K_1$, $\mathcal{F}(\pi_2) = K_2$, $\mathcal{F}(\pi_3) = K_3$, $\mathcal{F}(\pi_4) = K_4$ (per DPL)
- $\mathcal{F}(\pi_5) = \mathit{Elaboration}(\pi_2, \pi_3) \wedge \mathit{Narration}(\pi_2, \pi_4)$
- $\mathcal{F}(\pi_0) = \mathit{Elaboration}(\pi_1, \pi_5)$

Logical Form of Discourse 4

- Note that $\mathcal{F}$ induces an order on Π.
- Say that $\alpha < \beta$ iff α occurs in $\mathcal{F}(\beta)$.
- Let $\prec$ denote the reflexive transitive closure of $<$.
  - → Call this "outscoping".
- A SDRS is coherent if:
  - → There is a unique outscoping-maximal label in Π ("root").
  - → $\prec$ is anti-symmetric (in particular, then, it has no circles)

Accessibility

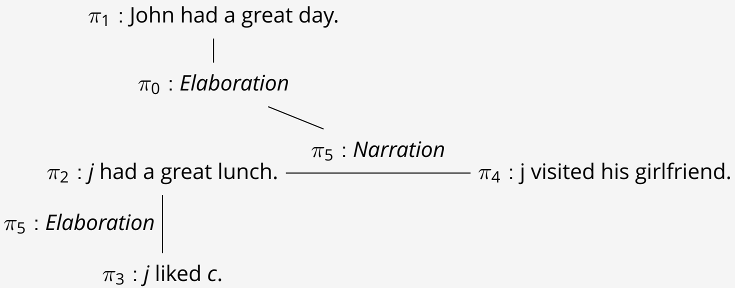
- Let $(\Pi, \mathcal{F}, L)$ be a coherent SDRS.
- Accessibility is defined as follows (recursively):
  - $\rightarrow$ *L* is accessible.
  - $\rightarrow$ If α is accessible and $\alpha \prec \beta$, then β is accessible.
  - $\rightarrow$ If (i) α is accessible, and
    (ii) $R(\beta, \alpha)$ occurs in some $\mathcal{F}(\gamma)$, and
    (iii) *R* is subordinating

  then β is accessible.

(10) a. π_1 : John dropped off his car for repairs.
b. π_2 : He got a rental.
c. π_3 : But it had a broken fuel pump.

- After we get π_0 : *Narration*(π_1, π_2), the only segment on the right frontier is π_2.
  - $\rightarrow$ So *it* = *the rental*, no matter what.

- $\Pi = \{\pi_0, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5\}$.
- $L = \pi_4$.
- $\mathcal{F}(\pi_1) = K_1$, $\mathcal{F}(\pi_2) = K_2$, $\mathcal{F}(\pi_3) = K_3$, $\mathcal{F}(\pi_4) = K_4$ (per DPL)
- $\mathcal{F}(\pi_5) = \mathit{Elaboration}(\pi_2, \pi_3) \wedge \mathit{Narration}(\pi_2, \pi_4)$
- $\mathcal{F}(\pi_0) = \mathit{Elaboration}(\pi_1, \pi_5)$



Truth Conditions

- For DPL formulae φ we know what $f[\![\varphi]\!]g$ means (for two contexts f, g) [at the ILLC we also know for questions].
- But what is $[\![R]\!]$ for a discourse relation R?
- Most discourse relations are veridical, i.e. entail their segments.
- If R is veridical, define:

$$f[\![R(\alpha, \beta)]\!]g \text{ iff } f[\![\mathcal{F}(\alpha) \wedge \mathcal{F}(\beta) \wedge \Phi_R]\!]g.$$

where Φ_R is a meaning postulate associated with R.

Some Postulates

- Some veridical relations are: Narration, Elaboration, Explanation, Contrast, Background.

- Narration(α, β): $\mathcal{F}(\alpha)$ is before $\mathcal{F}(\beta)$.
- Elaboration(α, β): $\mathcal{F}(\beta)$ defeasibly entails $\mathcal{F}(\alpha)$ and $\mathcal{F}(\alpha)$ and $\mathcal{F}(\beta)$ temporally overlap.
- Explanation(α, β): $\mathcal{F}(\beta)$ is before $\mathcal{F}(\alpha)$ and $\mathcal{F}(\beta)$ explains $\mathcal{F}(\alpha)$ (i.e. enthymeme).
- Contrast(α, β): $\mathcal{F}(\alpha)$ defeasibly entails $\neg\mathcal{F}(\beta)$.
- Background(α, β): $\mathcal{F}(\alpha)$ and $\mathcal{F}(\beta)$ spatiotemporally overlap.

Non-Veridical Relatoins

– Alternation (corresponds to dynamic $\vee$).

(11) a. A: John is at home.
b. B: Or at the office.

– Consequence (corresponds to dynamic $\Rightarrow$).

(12) a. If a farmer owns a donkey, he beats it.

– Correction & Counterevidence (only right-veridical).

(13) a. A: John is at home.
b. B: No, he's at the office.
b.′ B: His car isn't in the driveway.

Dialogue doesn't quite work out

(14) π_1 : A: Max owns several classic cars.
π_2 : B: No he doesn't.
π_3 : A: He owns two 1967 spiders

-Correction
-Counterevidence
-Elaboration

- $\pi_0 = \textit{Correction}(\pi_1, \pi_2) \wedge \textit{Counterevidence}(\pi_2, \pi_3) \wedge \textit{Elaboration}(\pi_1, \pi_3)$.

$$f[\![\mathcal{F}(\pi_0)]\!]g \text{ iff } f[\![\mathcal{F}(\pi_2)]\!] \circ [\![\Phi_{Corr}]\!] \circ [\![\mathcal{F}(\pi_3)]\!] \circ [\![\Phi_{CE}]\!] \circ [\![\mathcal{F}(\pi_1)]\!] \circ [\![\mathcal{F}(\pi_3)]\!] \circ [\![\Phi_{Elab}]\!]g$$

Today's Half-Baked Thing

(15) a. A: ⟨unclear⟩
b. B: What?
c. A: I said that it's <u>very loud in here.</u>
d. B: Yeah, you're right!

-Clarification Request
-QAP
-Clarifiy
-Accept

Today's Half-Baked Thing

(15) a. A: ⟨unclear⟩]-Clarification Request
b. B: What?
c. A: I said that it's <u>very loud in here.</u>]-QAP]-Clarifiy
d. B: Yeah, you're right!]-Accept

– Needed:
  - → Formal representation of "⟨unclear⟩"
  - → Semantics for CR and Clarify.
  - → Mechanism to explain how Accept can accept the clarified content.

Nonmonotonic Inference

The need for defeasibility

- New information can change the interpretation of a dialogue at any time.

(16) α : May fell.
β : John kicked him.
$\rightsquigarrow \pi_0$: *Explanation*(α, β)

(17) α : May fell.
β : John kicked him.
γ : But this is not why he fell.
δ : John kicked him after he fell.
$\rightsquigarrow \pi_0$: *Narration*(α, π_1)
π_1 : *Contrast*$(\beta, \gamma) \wedge$ *Elaboration*(β, δ)

Brief Excursion: Generics

(18) Birds (can) fly.

(19) Mammals give live birth.

Brief Excursion: Generics

(18) Birds (can) fly.

(19) Mammals give live birth.

- Habituals:

(20) John smokes.

(21) Lisa rides her bike to work.

Brief Excursion: Generics

(18) Birds (can) fly.

(19) Mammals give live birth.

- Habituals:

(20) John smokes.

(21) Lisa rides her bike to work.

- Mixed:

(22) Actors smoke.

Why We Care

- They Tolerate Exceptions

(23) Birds fly. (unless they are penguins)

(24) Mammals give live birth. (unless they are platypus)

(25) John smokes. (except when he’s trying to quit)

(26) Lisa rides her bike to work (unless it rains).

(27) Actors smoke. (not all; not always)

Why We Care

- They Tolerate Exceptions

(23) Birds fly. (unless they are penguins)

(24) Mammals give live birth. (unless they are platypus)

(25) John smokes. (except when he's trying to quit)

(26) Lisa rides her bike to work (unless it rains).

(27) Actors smoke. (not all; not always)

- The exceptions do not seem to impeach on our intuitions that the statements are, somehow, true.

- We feel entitled to use these sentences in inference.

(28) Birds fly.
Tux is a bird.
____
Tux flies.

(29) Robert uses the whiteboard to teach.
Robert is teaching.
____
Robert is using the whiteboard.

Reasoning with Exceptions

- We feel entitled to use these sentences in inference.

(28) Birds fly.
Tux is a bird.
―――――――――
Tux flies.

(29) Robert uses the whiteboard to teach.
Robert is teaching.
―――――――――
Robert is using the whiteboard.

- But we also feel that the inferences can be blocked without contradiction.

(30) Birds fly.
Tux is a bird.
Tux is a penguin.
―――――――――
~~Tux flies.~~
~~Contradiction.~~

They are *intensional*, too!

(31) Mail from Antarctica goes to Jenny.
(we've never gotten any mail from Antarctica)

(32) This button makes decaf coffee.
(nobody ever pressed that button).

(33) Around here, we help each other out in emergencies.
(there has never been an emergency)

- The generic statement need not occur even once.

They are *intensional*, too!

(31) Mail from Antarctica goes to Jenny.
(we've never gotten any mail from Antarctica)

(32) This button makes decaf coffee.
(nobody ever pressed that button).

(33) Around here, we help each other out in emergencies.
(there has never been an emergency)

- The generic statement need not occur even once.

⇒ Extensional analysis is generally incorrect.

The Epistemic Argument (Pelletier & Asher 1997)

- Exception-tolerant statements form a large part of our knowledge.
  - → And that knowledge is true, inferentially tractable, *good*.
  - → “intellectually satisfying and practically useful” (P&A 97)

The Epistemic Argument (Pelletier & Asher 1997)

- Exception-tolerant statements form a large part of our knowledge.
  - $\rightarrow$ And that knowledge is true, inferentially tractable, *good*.
  - $\rightarrow$ "intellectually satisfying and practically useful" (P&A 97)
- Commonsense Knowledge.
- $\approx$ the knowledge of regularity while being simultaneously aware that regularities can be broken.

The Epistemic Argument (Pelletier & Asher 1997)

- Exception-tolerant statements form a large part of our knowledge.
  - $\rightarrow$ And that knowledge is true, inferentially tractable, *good*.
  - $\rightarrow$ "intellectually satisfying and practically useful" (P&A 97)
- Commonsense Knowledge.
- $\approx$ the knowledge of regularity while being simultaneously aware that regularities can be broken.
- That is, we want:

(a) Truth-conditional semantics.

(b) Inference.

Nonmonotonic Inference

Commonsense Entailment

Commonsense Entailment

Towards Commonsense Knowledge

- Let's understand "Birds fly" as something like a generalised quantifier.

- Recall generalised quantifiers:
  "All boys kissed John." $\approx$ $\mathit{generalisedQ}(B)x.K(x,j)$

- "Birds fly" $\approx$ $\mathit{genericQ}(B)x.Fx$.

Towards Commonsense Knowledge

- Let's understand "Birds fly" as something like a generalised quantifier.
- Recall generalised quantifiers:
  "All boys kissed John." $\approx$ $generalisedQ(B)x.K(x,j)$
- "Birds fly" $\approx$ $genericQ(B)x.Fx$.
- Note that we can already do some basic reasoning with this representation.

(34)

Birds fly	$genericQ(B)x.Fx$.
All flyers have wings.	$generalisedQ(F)x.Wx$.
Birds have wings.	$genericQ(B)x.Wx$.

Default Logic

– Default Logics are logics of nonmonotonic inference.
  → Monotonicity: If $\Gamma \vdash \psi$ then $\Gamma, \varphi \vdash \psi$.

Default Logic

- Default Logics are logics of nonmonotonic inference.
  - → Monotonicity: If $\Gamma \vdash \psi$ then $\Gamma, \varphi \vdash \psi$.
- That is, Right Weakening fails.

$$(\mathrm{WR})\frac{\Gamma \quad \vdash \Delta}{\Gamma, A \vdash \Delta} \qquad\qquad (\mathrm{WR})\frac{\Gamma \quad \vdash B}{\Gamma, A \vdash B}$$

Default Logic

- Default Logics are logics of nonmonotonic inference.
  - → Monotonicity: If $\Gamma \vdash \psi$ then $\Gamma, \varphi \vdash \psi$.
- That is, Right Weakening fails.

$$\text{(WR)}\frac{\Gamma \quad \vdash \Delta}{\Gamma, A \vdash \Delta} \qquad \text{(WR)}\frac{\Gamma \quad \vdash B}{\Gamma, A \vdash B}$$

- The idea is that added information can cancel inferences.
  - → "A entails B unless we are in a state where it just happens to be the case that A doesn't entail B."

Default Logic

- Default Logics are logics of nonmonotonic inference.
  - $\rightarrow$ Monotonicity: If $\Gamma \vdash \psi$ then $\Gamma, \varphi \vdash \psi$.
- That is, Right Weakening fails.

$$(\mathrm{WR})\frac{\Gamma \quad \vdash \Delta}{\Gamma, A \vdash \Delta} \qquad\qquad (\mathrm{WR})\frac{\Gamma \quad \vdash B}{\Gamma, A \vdash B}$$

- The idea is that added information can cancel inferences.
  - $\rightarrow$ "A entails B unless we are in a state where it just happens to be the case that A doesn't entail B."
- Goal: define an all else being equal-conditional $>$ and with a default entailment relation $\mid\!\sim$ (ceteris paribus inference).
- Defeasible Modus Ponens:

  $A, A > B \mid\!\sim B.$

  $A, A > B, \neg B \mid\!\not\sim B.$

Generic $\approx$ Default

- $generalisedQ(P)x.Qx \; = \; \forall x.Px \rightarrow Qx$

Generic $\approx$ Default

- $generalisedQ(P)x.Qx \; = \; \forall x.Px \rightarrow Qx$
- How about this, then?
- $genericQ(P)x.Q(x) \; = \; \forall x.Px > Qx$. (P&A 1997)

Generic $\approx$ Default

- $generalisedQ(P)x.Qx \;=\; \forall x.Px \rightarrow Qx$

- How about this, then?
- $genericQ(P)x.Q(x) \;=\; \forall x.Px > Qx$. (P&A 1997)

- This means that “birds fly” expands to “under sufficiently normal circumstances and all else being equal, if *x* is a bird, then *x* flies.”

Generic $\approx$ Default

- $\mathit{generalisedQ}(P)x.Qx \;=\; \forall x.Px \rightarrow Qx$
- How about this, then?
- $\mathit{genericQ}(P)x.Q(x) \;=\; \forall x.Px > Qx$. (P&A 1997)
- This means that "birds fly" expands to "under sufficiently normal circumstances and all else being equal, if *x* is a bird, then *x* flies."
- Now we need to give semantic meaning to the "sufficiently normal circumstances."
- For simplicity, let's not worry about quantification from now on and zoom in on the propositional semantics of $>$.

Simple Transformations

(35) Birds fly.
Airplanes fly.
———————————
Birds and Airplanes fly.

- $B(x) > F(x), A(x) > F(x) \mid\!\sim (B(x) \vee A(x)) > F(x)$.
  (Disjunction of the Antecedent).

Simple Transformations

(35) Birds fly.
Airplanes fly.

Birds and Airplanes fly.

- $B(x) > F(x), A(x) > F(x) \mathrel{|\!\sim} (B(x) \vee A(x)) > F(x)$.
  (Disjunction of the Antecedent).

(36) Birds fly.
Fliers must have wings.

Birds have wings.

- $B(x) > F(x), \Box(F(x) \rightarrow W(x)) \mathrel{|\!\sim} B(x) > W(x)$.
  (Closure on the Consequent).

Simple Transformations

(35) Birds fly.
Airplanes fly.

Birds and Airplanes fly.

- $B(x) > F(x), A(x) > F(x) \mid\!\sim (B(x) \vee A(x)) > F(x)$. (Disjunction of the Antecedent).

(36) Birds fly.
Fliers must have wings.

Birds have wings.

- $B(x) > F(x), \Box(F(x) \rightarrow W(x)) \mid\!\sim B(x) > W(x)$. (Closure on the Consequent).

(37) Birds fly.

Birds fly or swim.

- $B(x) > F(x) \mid\!\sim B(x) > (F(x) \vee S(x))$. (Consequent Weak.).

The Nixon Diamond

(38) Richard Nixon is a Quaker.
Richard Nixon is a Republican.
Republicans are warmongers.
Quakers are pacifists.

~~Nixon is a warmonger~~.
~~Nixon is a pacifist~~.

– When in doubt, conclude neither.

The Nixon Diamond

(38) Richard Nixon is a Quaker.
Richard Nixon is a Republican.
Republicans are warmongers.
Quakers are pacifists.

~~Nixon is a warmonger~~.
~~Nixon is a pacifist~~.

- When in doubt, conclude neither.

- $Q(n), R(n), R(x) > W(x), Q(x) > P(x), \neg(W(x) \wedge P(x)) \not\mid\!\sim P(n)$

- $Q(n), R(n), R(x) > W(x), Q(x) > P(x), \neg(W(x) \wedge P(x)) \not\mid\!\sim W(n)$

The Penguin Principle

(39) Birds fly.
Penguins don't fly.
Penguins are birds (by definition).
Tux is a penguin.

~~Tux flies.~~
Tux doesn't fly.

- $B(x) > F(x), P(x) > \neg F(x), \Box(P(x) \rightarrow B(x)), P(t) \mathrel{|\!\sim} \neg F(t)$.
- The more specific inference wins.

The Penguin Principle

(39) Birds fly.
Penguins don't fly.
Penguins are birds (by definition).
Tux is a penguin.

~~Tux flies.~~
Tux doesn't fly.

- $B(x) > F(x), P(x) > \neg F(x), \Box(P(x) \rightarrow B(x)), P(t) \mathrel{|\!\sim} \neg F(t).$
- The more specific inference wins.

(25) Birds fly.
Penguins don't fly.
All Penguins are Birds.
Penguins with jetpacks fly.
Tux is a *jetpack* penguin

~~Tux doesn't fly.~~
Tux flies.

Antecedent Strengthening?

(40) $\dfrac{\text{Birds fly.}}{\text{Red Birds fly.}}$

(41) $\dfrac{\text{Birds fly.}}{\text{??Dead Birds fly.}}$

- It is possible to phrase a cautious version of this.

- $p > q \mathrel{|\!\sim} (p \wedge r) > q$, but not
- $p > q, (p \wedge r) > \neg q \mathrel{|\!\sim} (p \wedge r) > q$.

Truth–Conditions for $>$

– The idea is that $p > q$ is true if in all circumstances where p holds and these are normal circumstances for p, then q holds.

Truth–Conditions for $>$

- The idea is that $p > q$ is true if in all circumstances where p holds and these are normal circumstances for p, then q holds.
- We express this with the following modal semantics.

Commonsense Entailment Frames

A commonsense entailment frame is a triple $\langle W, *, R_\Box \rangle$ where W is a set of worlds (propositional models), $R_\Box \subseteq W \times W$ is an equivalence relation and $* : W \times \mathcal{P}(W) \to \mathcal{P}(W)$ is a function such that:

- for all $w \in W$, $*(w, X) \subseteq X$,
- for all $X \subseteq W$, $*(w, X) \subseteq \{v \mid wRv\}$.
- If $*(w, X) \subseteq Y$ and $*(w, Y) \subseteq X$), then $*(w, X) = *(w, Y)$.

The Logic of Commonsense Entailment (cont.)

Truth

A commonsense entailment model is a structure $\langle W, *, R_\Box, V\rangle$ such that $\langle W, *, R_\Box\rangle$ is a CE frame and $V : W \to \mathcal{P}(\mathrm{At})$ is a valuation.

- $M, w \Vdash p$ iff $p \in V(w)$ for atoms p.
- $M, w \Vdash \neg A$ iff $M, w \nVdash A$.
- $M, w \Vdash A \wedge B$ iff $M, w \Vdash A$ and $M, w \Vdash B$.
- $M, w \Vdash \Box A$ iff for all v with $wR_\Box v$, $M, v \Vdash A$.
- $M, w \Vdash A > B$ iff $*(w, [\![A]\!]) \subseteq [\![B]\!]$
  where: $[\![\varphi]\!] = \{w' \in W \mid M, w' \Vdash \varphi\}$.

- A proposition A roughly corresponds to a set of worlds $[\![A]\!]$.
- We interpret $*$ to select all the worlds where A is normal.
- So the truth–conditions of $A > B$ are circumscribed as "everywhere where A is normally true, B is true."

- The *Dudley Doorite* constraint (no, I don't know either) is: $*(w, X \cup Y) \subseteq *(w, X) \cup *(w, Y)$.
- Dudley Doorite corresponds to Disjunction of the Antecedent: $M, w \Vdash ((p > r) \wedge (q > r)) \rightarrow ((p \vee q) > r)$.

Monotonic Commonsense Entailment

- The *Dudley Doorite* constraint (no, I don't know either) is: $*(w, X \cup Y) \subseteq *(w, X) \cup *(w, Y)$.
- Dudley Doorite corresponds to Disjunction of the Antecedent: $M, w \Vdash ((p > r) \wedge (q > r)) \rightarrow ((p \vee q) > r))$.
- Proof: Suppose $M, w \Vdash (p > r) \wedge (q > r)$. Then: $*(w, [\![p \vee q]\!]) = *(w, [\![p]\!] \cup [\![q]\!]) \subseteq *(w, [\![p]\!]) \cup *(w, [\![q]\!]) \subseteq [\![r]\!]$.

$\square$

Monotonic Commonsense Entailment

- The *Dudley Doorite* constraint (no, I don't know either) is: $*(w, X \cup Y) \subseteq *(w, X) \cup *(w, Y)$.

- Dudley Doorite corresponds to Disjunction of the Antecedent: $M, w \Vdash ((p > r) \wedge (q > r)) \rightarrow ((p \vee q) > r))$.
- Proof: Suppose $M, w \Vdash (p > r) \wedge (q > r)$. Then: $*(w, [\![p \vee q]\!]) = *(w, [\![p]\!] \cup [\![q]\!]) \subseteq *(w, [\![p]\!]) \cup *(w, [\![q]\!]) \subseteq [\![r]\!]$.
  $\square$

Validity

$\Gamma \models A$ iff on all Dudley Doorite models M and for all $w \in W^M$: if $M, w \Vdash \Gamma$ then $M, w \Vdash A$.

- A standard Henkin/Lindenbaum construction shows that this is decidable for finite Γ.

Towards Defeasible Modus Ponens

– We now have a truth definition and a monotonic entailment relation that tells us from facts about generic statements further true generic statements.

Towards Defeasible Modus Ponens

- We now have a truth definition and a monotonic entailment relation that tells us from facts about generic statements further true generic statements.
- But we are missing an entailment relation that tells us what to infer from $A, A > B$.
- So we know that if “Birds fly” and “Fliers have wings” that “Birds have wings”, but we do not know that “Birds fly, Tux is a bird” (nonmonotonically) entails that “Tux flies.”

Towards Defeasible Modus Ponens

- We now have a truth definition and a monotonic entailment relation that tells us from facts about generic statements further true generic statements.
- But we are missing an entailment relation that tells us what to infer from $A, A > B$.
- So we know that if "Birds fly" and "Fliers have wings" that "Birds have wings", but we do not know that "Birds fly, Tux is a bird" (nonmonotonically) entails that "Tux flies."
- We need a definition of $\mid\!\sim$ that valildates $A, A > B \mid\!\sim B$ and $A, A > B, \neg B \mid\!\not\sim B$.

Towards Defeasible Modus Ponens

- We now have a truth definition and a monotonic entailment relation that tells us from facts about generic statements further true generic statements.
- But we are missing an entailment relation that tells us what to infer from $A, A > B$.
- So we know that if "Birds fly" and "Fliers have wings" that "Birds have wings", but we do not know that "Birds fly, Tux is a bird" (nonmonotonically) entails that "Tux flies."
- We need a definition of $\mid\!\sim$ that valildates $A, A > B \mid\!\sim B$ and $A, A > B, \neg B \not\mid\!\sim B$.
- We are inclined to just take all normal worlds and check what is going on there.

Towards Defeasible Modus Ponens

- We now have a truth definition and a monotonic entailment relation that tells us from facts about generic statements further true generic statements.
- But we are missing an entailment relation that tells us what to infer from $A, A > B$.
- So we know that if “Birds fly” and “Fliers have wings” that “Birds have wings”, but we do not know that “Birds fly, Tux is a bird” (nonmonotonically) entails that “Tux flies.”
- We need a definition of $\mid\!\sim$ that valildates $A, A > B \mid\!\sim B$ and $A, A > B, \neg B \mid\!\not\sim B$.
- We are inclined to just take all normal worlds and check what is going on there.
- However, $>$ embeds and thus this needs to be recursed. This is bonkers complicated.

Closure in the Consequent

- First, we can do a bit more in the logic we have already.
  - $\rightarrow$ The first to notice this is actually Michael Morreau, I think.
- $\models \big(\Box(B \rightarrow C) \wedge (A > B)\big) \rightarrow (A > C)$.

Closure in the Consequent

- First, we can do a bit more in the logic we have already.
  - $\rightarrow$ The first to notice this is actually Michael Morreau, I think.
- $\models \big(\Box(B \rightarrow C) \wedge (A > B)\big) \rightarrow (A > C)$.
- Proof:
  - $\rightarrow$ Fix a model M and a world w. Let $X = \{v \mid wR_{\Box}v\}$.
  - $\rightarrow$ By assumption $[\![B]\!] \cap X \subseteq [\![C]\!] \cap X$.
  - $\rightarrow$ Also, $*(w, [\![A]\!]) \subseteq [\![B]\!]$.
  - $\rightarrow$ And $*(w, [\![A]\!]) \subseteq X$.
  - $\rightarrow$ Thus $*(w, [\![A]\!]) \subseteq [\![B]\!] \cap X$.
  - $\rightarrow$ Hence $*(w, [\![A]\!]) \subseteq [\![C]\!] \cap X$.
  - $\rightarrow$ In particular, $*(w, [\![A]\!]) \subseteq [\![C]\!]$. $\Box$

Specificity

- Similarly:
- $\models \big(\Box(P \to B) \land (B > F) \land (P > \neg F)\big) \to (B > \neg P)$.
  (Penguins are birds, birds fly, penguins do not fly.
  Thus, normal birds are not penguins.)

Specificity

- Similarly:
- $\models \big(\Box(P \to B) \wedge (B > F) \wedge (P > \neg F)\big) \to (B > \neg P)$.
  (Penguins are birds, birds fly, penguins do not fly.
  Thus, normal birds are not penguins.)
- Proof:
  - $\longrightarrow$ Fix a model M and a world w. Wlog suppose that w sees all other worlds.
  - $\longrightarrow$ Then $[\![P]\!] \subseteq [\![B]\!]$. That is $[\![P]\!] \subseteq [\![B]\!]$, i.e. $[\![B]\!] = ([\![B]\!] \setminus [\![P]\!]) \cup [\![P]\!]$.
  - $\longrightarrow$ Then $*(w, [\![B]\!]) \subseteq *(w, [\![B]\!] \setminus [\![P]\!]) \cup *(w, [\![P]\!])$ by DD.
  - $\longrightarrow$ $*(w, [\![P]\!]) \subseteq [\![\neg F]\!]$ and $*(w, [\![B]\!]) \subseteq [\![F]\!]$.
  - $\longrightarrow$ Thus $*(w, [\![B]\!]) \subseteq *(w, [\![B]\!] \setminus [\![P]\!])$.
  - $\longrightarrow$ Hence $*(w, [\![B]\!]) \subseteq [\![B]\!] \setminus [\![P]\!] \subseteq [\![\neg P]\!]$. $\square$

Towards $\mid\!\sim$

– We want to nail down where we can use $A > B, A$ to conclude B. We try to systematically eliminate abnormal premises.

Towards $\mid\!\sim$

- We want to nail down where we can use $A > B, A$ to conclude B. We try to systematically eliminate abnormal premises.

- Let Γ be a finite set of formulae. Define:

$$\mathrm{Ant}(\Gamma) = \{A \mid \Gamma \models A > B \text{ for some } B\}.$$

For any $A \in \mathrm{Ant}(\Gamma)$ define:

$$\Gamma^A = \{(A > B) \rightarrow (A \rightarrow B) \mid \Gamma \models A > B\}.$$

Towards $\mid\!\sim$

- We want to nail down where we can use $A > B, A$ to conclude B. We try to systematically eliminate abnormal premises.

- Let Γ be a finite set of formulae. Define:

$$\mathrm{Ant}(\Gamma) = \{A \mid \Gamma \models A > B \text{ for some } B\}.$$

  For any $A \in \mathrm{Ant}(\Gamma)$ define:

$$\Gamma^A = \{(A > B) \rightarrow (A \rightarrow B) \mid \Gamma \models A > B\}.$$

- An immediate extension of Γ is any set Γ' such that $\Gamma' = \Gamma \cup \bigcup_{A \in T} \Gamma^A$ for some $T \subseteq \mathrm{Ant}(\Gamma)$.

Towards $\mid\!\sim$

- We want to nail down where we can use $A > B, A$ to conclude B. We try to systematically eliminate abnormal premises.

- Let Γ be a finite set of formulae. Define:

$$\mathrm{Ant}(\Gamma) = \{A \mid \Gamma \models A > B \text{ for some } B\}.$$

  For any $A \in \mathrm{Ant}(\Gamma)$ define:

$$\Gamma^A = \{(A > B) \to (A \to B) \mid \Gamma \models A > B\}.$$

- An immediate extension of Γ is any set Γ' such that $\Gamma' = \Gamma \cup \bigcup_{A \in T} \Gamma^A$ for some $T \subseteq \mathrm{Ant}(\Gamma)$.

- An extension of Γ is an immediate extension of Γ or an immediate extension of an extension of Γ.

Commonsense Entailment (finally)

Propositional Commonsense Entailment

$\Gamma \mathrel{|\!\!\sim} A$ iff $\Gamma^{\rightarrow} \models A$ for all maximally satisfiable extensions $\Gamma^{\rightarrow}$ of Γ.

– Recall that $\models$ is decidable; thus $\Gamma \mathrel{|\!\!\sim} A$ is decidable.

Commonsense Entailment (finally)

Propositional Commonsense Entailment

$\Gamma \mathrel{|\!\sim} A$ iff $\Gamma^{\rightarrow} \models A$ for all maximally satisfiable extensions $\Gamma^{\rightarrow}$ of Γ.

- Recall that $\models$ is decidable; thus $\Gamma \mathrel{|\!\sim} A$ is decidable.
- It is easy to see that Defeasible Modus Ponens holds:
  - → $A, A > B \mathrel{|\!\sim} B$ and $A, A > B, \neg B \mathrel{|\!\not\sim} B$.
  - → But $A, A > B, C \mathrel{|\!\sim} B$ if C is not a defeater for B.
  - → Because without a defeater, $(A > B) \rightarrow (A \rightarrow B)$ is in every consistent extension.

Commonsense Entailment (finally)

Propositional Commonsense Entailment

$\Gamma \mathrel{|\!\sim} A$ iff $\Gamma^{\rightarrow} \models A$ for all maximally satisfiable extensions $\Gamma^{\rightarrow}$ of Γ.

- Recall that $\models$ is decidable; thus $\Gamma \mathrel{|\!\sim} A$ is decidable.
- It is easy to see that Defeasible Modus Ponens holds:
  - $\rightarrow$ $A, A > B \mathrel{|\!\sim} B$ and $A, A > B, \neg B \mathrel{|\!\not\sim} B$.
  - $\rightarrow$ But $A, A > B, C \mathrel{|\!\sim} B$ if C is not a defeater for B.
  - $\rightarrow$ Because without a defeater, $(A > B) \rightarrow (A \rightarrow B)$ is in every consistent extension.
- Nixon Diamond:
  - $\rightarrow$ $A > B, C > \neg B, C, A \mathrel{|\!\not\sim} B$.
  - $\rightarrow$ $A > B, C > \neg B, C, A \mathrel{|\!\not\sim} \neg B$.
  - $\rightarrow$ Because there are consistent extensions with B and with $\neg B$.

The Penguin Principle

- $\Box(C \to A), A > B, C > \neg B, C \mid\!\sim \neg B.$

- $\Box(C \to A), A > B, C > \neg B, C \mid\!\sim \neg B$.
- Proof.
  - $\to$ Let $\Gamma = \{\Box(C \to A), A > B, C > \neg B, C)$.
  - $\to$ We know: $\models \big(\Box(C \to A) \wedge (A > B) \wedge (C > \neg B)\big) \to (A > \neg C)$.
  - $\to$ So $\Gamma \models A > \neg C$.
  - $\to$ So it is inconsistent to extend Γ with the antecedent A: $\Gamma \cup \{(A > \varphi) \to (A \to \varphi) \mid \Gamma \models A > \varphi\} \models C \wedge \neg C$.
  - $\to$ Thus A as an antecedent is defeated. We can however extend with C.
  - $\to$ thus, we arrive at $\neg B$. $\Box$

- Reading for Wednesday:
- Take a look at the list of SDRT discourse relations and their truth-conditions (I’ll send you the relevant scans from the book).