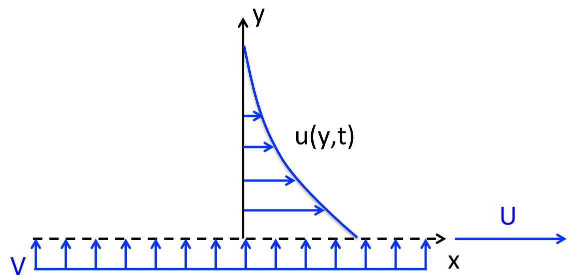
AEM 8202 Fluid Mechanics II Spring 2023

Problem Set #3
Due Fri., Apr. 7 by 11:59pm

Please hand in your completed problem sets as PDF files to the course Canvas site. Working together with others is fine (and encouraged), but you must write up each solution yourself.

1. Consider the unsteady two-dimensional flow of incompressible viscous fluid above a flat, porous plate of infinite extent. The plate lies along the x-axis and is initially at rest. At time t = 0, the plate suddenly starts moving to the right at velocity U. At the same time, fluid starts to be injected uniformly through the plate with velocity V.



a) Find an equation that governs the x-component of velocity u(y, t) of the fluid above the plate.

b) If the injection velocity V is constant in time (and non-zero), show that no similarity solution can exist.

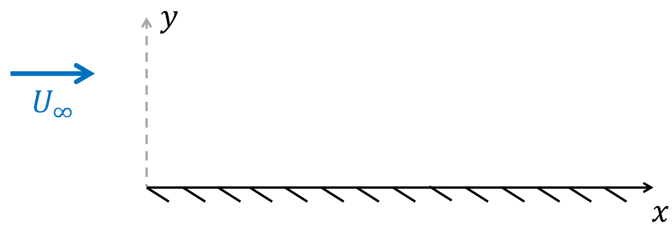
c) Find a time-varying injection velocity V(t) that will admit a similarity solution $f(\eta)$ for this problem. In this case, reduce the PDE of part (a) to an ODE for $f(\eta)$.

2. The Falkner-Skan solutions to the laminar boundary layer equations produce self-similar profiles for a whole range of different freestream velocity variations of the form $U = Cx^{\alpha}$. The objectives of this problem are to reproduce the profiles shown in figure 10.8 in Kundu's textbook for relevant α values, and to verify that $\alpha = -0.0904$ corresponds to incipient separation. For this purpose, use a shooting method to numerically solve the Falkner-Skan governing equation (since it has no known analytical solutions). Generate and plot profiles corresponding to $\alpha = -0.0904,\ 0,\ \frac{1}{3},\ \frac{1}{9},\ 1$. Evaluate the wall-shear stress $f''(0)$ and the shape factor $H = \frac{\delta^*}{\theta}$ for each of these cases.

3. Consider the two-dimensional boundary layer that develops in a steady flow past a semi-infinite flat plate. The fluid is incompressible, but *non-Newtonian,* meaning that the dynamic viscosity is not constant, but rather is a function of strain rate. In particular, suppose that the viscosity varies with strain-rate such that:

$$\mu = K\left(\frac{\partial u}{\partial y}\right)^{\frac{1}{2}},$$

where K is a constant known as the flow consistency index. (Note: Similar shear-thickening effects occur in hypersonic boundary layers, where the strain-rate elevates the temperature close to the wall because of viscous dissipation, and this in turn increases viscosity.)



a) How does the thickness of the non-Newtonian boundary layer scale with distance from the leading edge of the plate? Does this non-Newtonian boundary layer grow faster or slower than an ordinary Blasius boundary layer for a Newtonian (constant viscosity) fluid?

b) Show that a similarity solution exists for this non-Newtonian boundary layer by finding an ODE for $f(\eta)$, where the streamfunction has the form $\psi = Afx^m$ and the similarity variable $\eta = \frac{By}{x^n}$, where A, B, m, and n are constants.

c) Use a shooting method to solve the ODE you obtained in part (b), and use your solution to plot a velocity profile for the non-Newtonian boundary layer. How does this profile compare to that of a Newtonian Blasius boundary layer?