

# [ matrix ]

\* transpose matrix  $(\begin{array}{ccc} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{array}) \rightarrow (\begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \end{array})$

\* diagonal matrix  $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}^P = \begin{pmatrix} 1^P & 0 & 0 \\ 0 & 2^P & 0 \\ 0 & 0 & 3^P \end{pmatrix}$

\* inverse matrix  $A \cdot B = B \cdot A = I_n \quad (B = A^{-1})$

(A: nonsingular matrix = invertible matrix  
 B: inverse matrix)

$$A^{-1} = \left( \frac{1}{\det(A)} \right) \text{adj}(A)$$

有 or 無  
 $\det(A) \neq 0 = (\backslash\backslash\backslash) - (\//\//)$   
 (determinant)  
 $\det(A) = |A|$

$$\begin{pmatrix} \bullet & \Delta \\ \square & \star \end{pmatrix} = (0 \times -\Delta \square)$$

adjoint matrix

$$\text{adj}(A) = (\underbrace{\text{cofactor matrix}}_{\downarrow})^T$$

$$C_{ij} = (-1)^{i+j} M_{ij}$$

$$= (-1)^{i+j} \det(\quad)$$

$$C = \begin{bmatrix} & \begin{pmatrix} \bullet & \Delta \\ \square & \star \end{pmatrix} & \begin{pmatrix} \bullet & \Delta \\ \square & \star \end{pmatrix} \\ & \begin{pmatrix} \bullet & \Delta \\ \square & \star \end{pmatrix} & \begin{pmatrix} \bullet & \Delta \\ \square & \star \end{pmatrix} \\ & \begin{pmatrix} \bullet & \Delta \\ \square & \star \end{pmatrix} & \begin{pmatrix} \bullet & \Delta \\ \square & \star \end{pmatrix} \end{bmatrix}$$

# [matrix product]

Inner product

= dot product

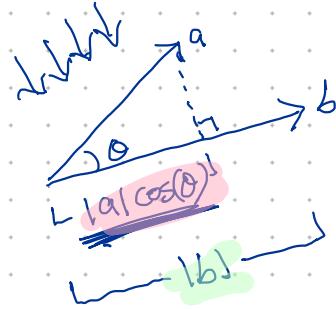
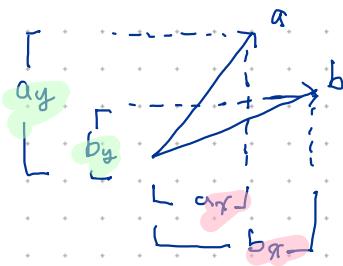
= scalar product

= projection product

$$a^T b = a \cdot b = (a, b)$$

$$= a_x \cdot b_x + a_y \cdot b_y$$

$$= |a| \times \cos(\theta) \times |b|$$



[ eigenvalue      eigenvector ]

$$A\mathbf{x} = \lambda \mathbf{x}$$

- $\lambda$  : eigenvalue = characteristic value  
= latent root
- $\mathbf{x}$  : eigenvector = characteristic vector

\* eigen decomposition  $\leftarrow m \times m$  matrix

$\leftarrow$  linearly independent : basis

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \Rightarrow \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = c \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \lambda_2 \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$$

eigenvalue,  $\cdot$  eigenvector,

eigenvalue<sub>2</sub>,  $\cdot$  eigenvector<sub>2</sub>

$$= \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix}^{-1}$$

$$\begin{bmatrix} \cdot & \cdot \end{bmatrix}^{(n)} = \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix}^n \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix}^{-1}$$

D(f)

$$Ax = \lambda x$$

$$Ax - \lambda x = \underline{(A - \lambda I)x} = 0$$

$$x = (A - \lambda I)^{-1} \cdot 0$$

$$\det(A - \lambda I) = \underline{\underline{|A - \lambda I|}} = 0 \quad \therefore x \neq 0$$

characteristic equation

we can take  $\underline{\lambda}$  from characteristic equation

$$\text{tr}(A) = \sum_{i=1}^n \lambda_i \quad (\text{trace})$$

$$|A| = \det(A) = \prod_{i=1}^n \lambda_i$$

$$|I+A| = \det(I+A) = \prod_{i=1}^n (1+\lambda_i)$$