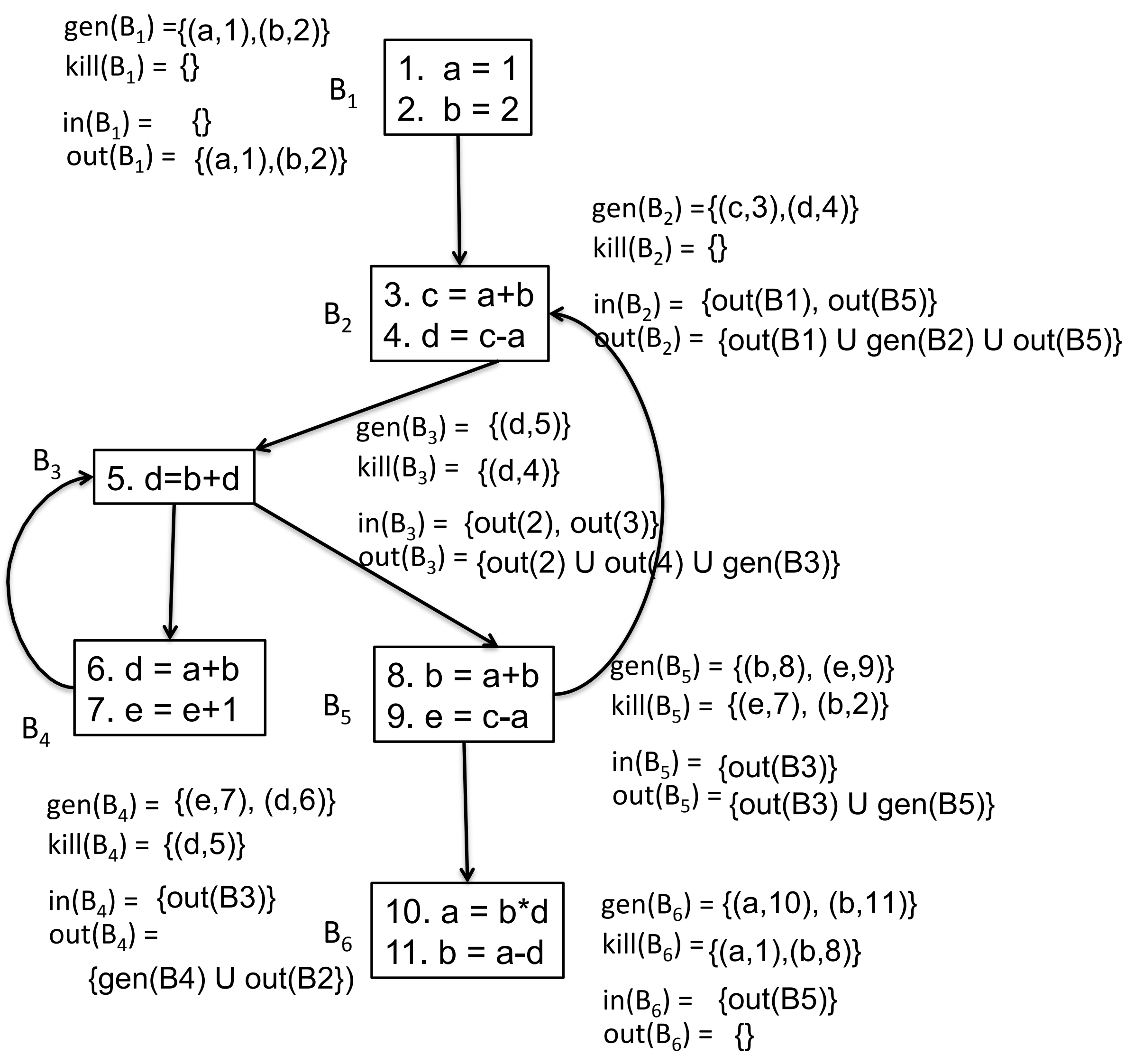
Homework 1

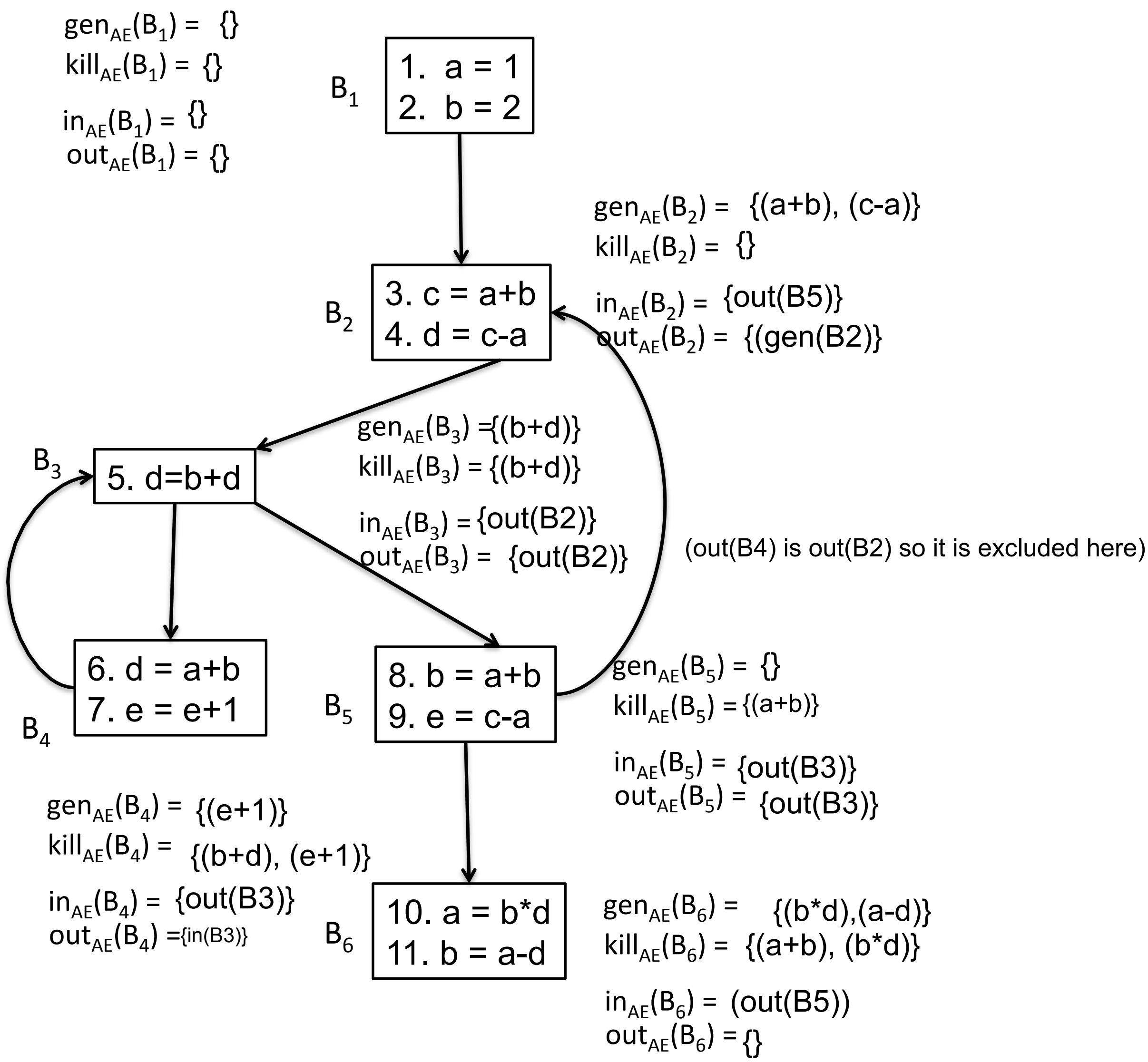
Posted Thursday January 16, Due Monday Thursday 30

50 points

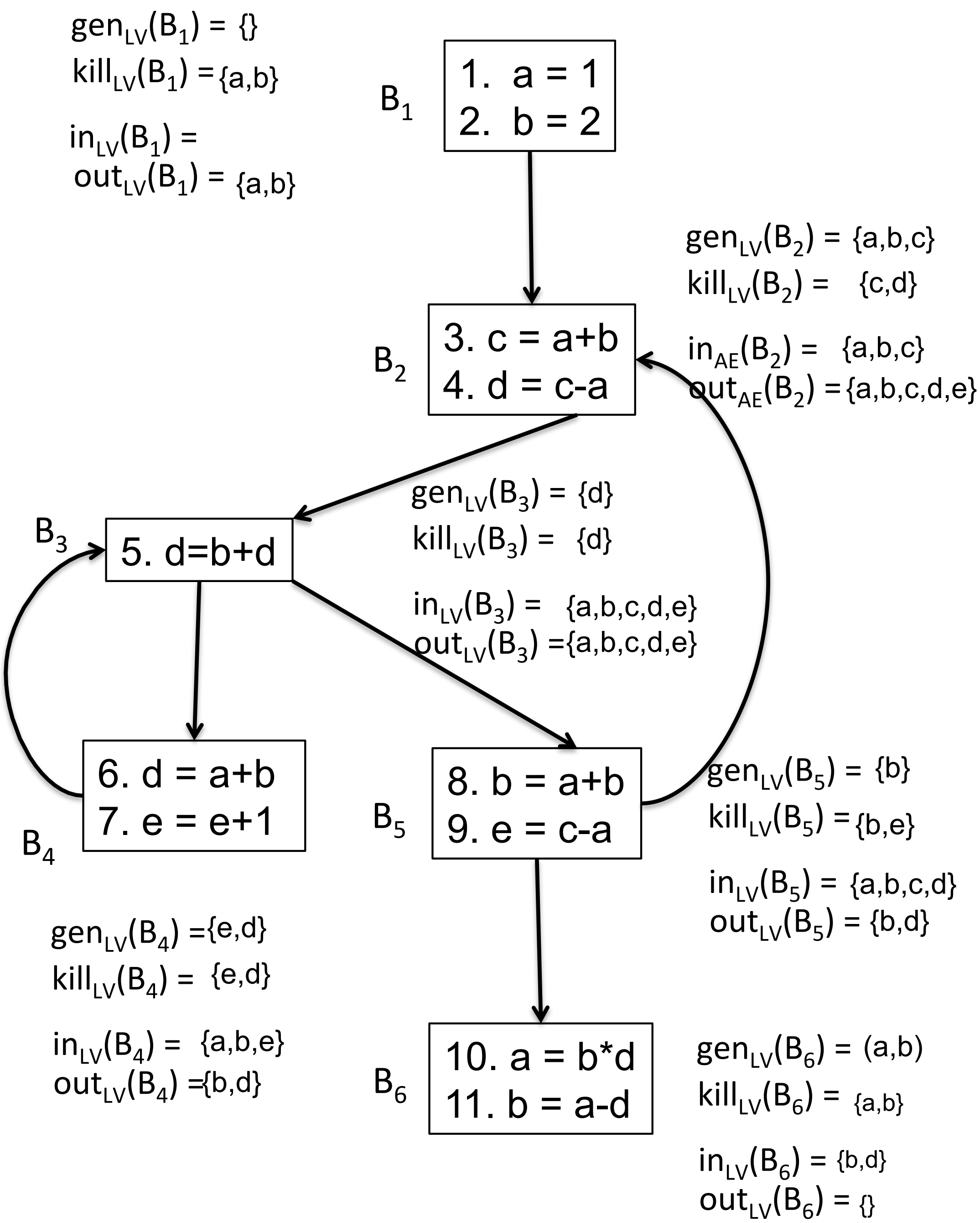
Problem 1 (6 pts). (From Aho, Lam, Sethi, Ullman.) For the CFG below fill in *Reaching Definitions* gen and kill sets for each block, and in and out sets for each block. (The in and out sets must show the final solution, not an intermediate value.)



Problem 2 (7 pts). (From Aho, Lam, Sethi, Ullman.) For the same CFG fill in *Available Expressions* $\mathsf{gen}_{\mathsf{AE}}$ and $\mathsf{kill}_{\mathsf{AE}}$ sets for each block, and $\mathsf{in}_{\mathsf{AE}}$ and $\mathsf{out}_{\mathsf{AE}}$ sets for each block.



Problem 3 (7 pts). (From Aho, Lam, Sethi, Ullman.) Now fill in *Live Variables* gen_{LV} and kill_{LV} sets for each block, and in_{LV} and out_{LV} sets for each block.



Problem 4 (10 pts, 2.5 pts each). (From Aho, Lam, Sethi, Ullman.) Let V be the set of complex numbers. Which of the following operations can serve as the meet operation for a lattice over V? For each of the choices below, if your answer is NO, explain why not. If your answer is YES, leave it at that.

a) Addition: $(a+ib) \wedge (c+id) = (a+c) + i(b+d)$

NO: meet(a,a) is not a --- does not follow idempotent property

b) Multiplication: $(a+ib) \wedge (c+id) = (ac-bd) + i(ad+bc)$

NO: meet(a,a) is not a --- does not follow idempotent property

c) Component-wise minimum: $(a+ib) \wedge (c+id) = min(a,c) + i\ min(b,d)$

Yes

d) Component-wise maximum: $(a+ib) \wedge (c+id) = max(a,c) + i\ max(b,d)$

Yes

Problem 5 (10 pts). The intraprocedural *Must-be-modified* problem is a backward dataflow problem solvable by fixpoint iteration. A variable is in the must-be-modified set on exit of CFG node n, if it is modified on *all paths* from n to exit. The problem statement is as follows: for each node n compute the set of variables that are in the must-be-modified set on exit from n.

a) (5 pts) Define the analysis as an instance of the dataflow framework. Specify

Lattice $L, \leq$: { x, y, z }

Merge operator: Intersect - "∩"

Transfer functions: in(j) = f(out(j)) = Use(j) U (out(j) - Def(j))
out(j) = ∩ in(i) *i is from succ(j)

b) (5 pts) Are the functions for this problem distributive or monotone? Show your proof.

Distributive: the functions are distributive if and only if f(x ∩ y) = f(x) ∩ f(y)

Use(j) U ((x ∩ y - Def(j)) = (Use(j) U (x - Def(j))) ∩ (Use (j) U (y - Def(j)))

- Use(j) and Def(j) are constants, and since they are the only modifiers on the resulting sets we can remove them as their effect on x and y being in the result is identical for both sides of the equation. This gives us:

x ∩ y = (x) ∩ (y)

- Which is indeed equal. ✓

Problem 6 (10 pts). (Modified from Nielson, Nielson and Hankin) A *bit vector* dataflow analysis is a special case of a monotone dataflow analysis where

I. The property space L is the lattice of the subsets over some finite set D, and $\leq$ is either $\subseteq$ or $\supseteq$ and

II. The transfer function space is $F = \{f : \mathcal{P}(D) \to \mathcal{P}(D) \mid f(Y) = (Y \cap Y_f^1) \cup Y_f^2$ where $Y_f^1 \subseteq D$ and $Y_f^2 \subseteq D$ are constants$\}$

Note: $\mathcal{P}(D)$ denotes the powerset of D (the powerset is also frequently denoted by 2^D). The above condition states that every transfer function f can be written as $f(Y) = (Y \cap Y_f^1) \cup Y_f^2$ where Y is the argument of the function (the in(j) set in a forward problem), and Y_f^1 and Y_f^2 are *constants* that do not depend on Y.

a) (5 pts) Briefly argue that the four classical dataflow analyses are bit vector dataflow analyses.

The dataflow analyses, Reaching Definitions, Live Variables, Available Expressions and Very Busy Expressions are bit vector dataflow analyses which can be proven by looking at the analyses' transfer function form, shared between them all:

out(j) = (in(j) - kill(j)) U gen(j) = (in(j) ∩ pres(j)) U gen(j)

In this case, pres(j) and gen(j) are constants since they only depend on the basic block contents rather than its input, and the structure of the equation follows precisely the form of f(Y) shown above.

b) (5 pts) Devise a distributive analysis that is *not* a bit vector analysis. Hint: Consider one of the non-distributive examples we studied in class, and drop one of the statements from the syntax.