$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$\sin(x-y) = \sin x \cos y - \cos x \sin y$$
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$
$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$
$$\operatorname{tg}(x+y) = \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y}$$
$$\operatorname{tg}(x-y) = \frac{\operatorname{tg} x - \operatorname{tg} y}{1 + \operatorname{tg} x \operatorname{tg} y}$$
$$\operatorname{ctg}(x+y) = \frac{\operatorname{ctg} x \operatorname{ctg} y - 1}{\operatorname{ctg} x + \operatorname{ctg} y}$$
$$\operatorname{ctg}(x-y) = \frac{-\operatorname{ctg} x \operatorname{ctg} y - 1}{\operatorname{ctg} x - \operatorname{ctg} y}$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$
$$\operatorname{ctg} 2x = \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x}$$
$$\sin x + \sin y = 2 \sin\frac{x+y}{2} \cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2 \sin\frac{x-y}{2} \cos\frac{x+y}{2}$$
$$\cos x + \cos y = 2 \cos\frac{x+y}{2} \cos\frac{x-y}{2}$$
$$\cos x - \cos y = -2 \sin\frac{x+y}{2} \sin\frac{x-y}{2}$$

Funkcija	Odvod	Nedoločeni integrali
$f(x) = x^n$	$f'(x) = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
$f(x) = \cos x$	$f'(x) = -\sin x$	$\int \frac{dx}{x} = \ln\lvert x\rvert + C$
$f(x) = \sin x$	$f'(x) = \cos x$	$\int e^x dx = e^x + C$
$f(x) = \operatorname{tg} x$	$f'(x) = \frac{1}{(\cos x)^2}$	$\int \cos x dx = \sin x + C$
$f(x) = \operatorname{ctg} x$	$f'(x) = -\frac{1}{(\sin x)^2}$	$\int \sin x dx = -\cos x + C$
$f(x) = e^x$	$f'(x) = e^x$	$\int \operatorname{ch} x dx = \operatorname{sh} x + C$
$f(x) = \ln x$	$f'(x) = \frac{1}{x}$	$\int \operatorname{sh} x dx = \operatorname{ch} x + C$
$f(x) = \arcsin x$	$f'(x) = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + C$
$f(x) = \operatorname{arctg} x$	$f'(x) = \frac{1}{1+x^2}$	$\int \frac{dx}{1+x^2} = \operatorname{arctg} x + C$

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right) + C, \quad a > 0,$$
$$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + C, \quad a > 0.$$

$$V = \pi \int_a^b f(x)^2 dx$$
$$P = 2\pi \int_a^b f(x)\sqrt{1 + f'(x)^2} dx$$
$$l = \int_a^b \sqrt{1 + f'(x)^2} dx$$

Parametricna
$$S = \frac{1}{2}\int_{t_1}^{t_2} (x\dot{y} - \dot{x}y)\, dt.$$

polarna:
$$S = \frac{1}{2}\int_{\phi_1}^{\phi_2} r^2\, d\phi.$$

$$l = \int_{t_1}^{t_2} \sqrt{\dot{x}^2 + \dot{y}^2}\, dt.$$

l = int(a->b) (sqrt(r^2 + r'^2))

l = int(a->b) (sqrt(1+f '(x)))

Integral s kot. fun: R(cosx,sinx), R racionalna

$$dx = \frac{2\, dt}{1+t^2},$$
$$\cos x = \frac{1-t^2}{1+t^2},$$
$$\sin x = \frac{2t}{1+t^2}, \qquad \operatorname{tg}\frac{x}{2} = t.$$

Racionalna enacba:

$R(x) = p(x) + \frac{r(x)}{q(x)}$, q razcepimo, in integriramo parcialne ulomke:

- $\frac{1}{(x-a)^k} \rightsquigarrow \frac{A_1}{x-a} + \frac{A_2}{(x-a)^2} + \cdots + \frac{A_k}{(x-a)^k}$,
- $\frac{1}{(x^2+bx+c)^l} \rightsquigarrow \frac{B_1+C_1x}{x^2+bx+c} + \frac{B_2+C_2x}{(x^2+bx+c)^2} + \cdots + \frac{B_l+C_lx}{(x^2+bx+c)^l}$.

R(x) kjer je v imenovalcu kvadratna f visje stopnje:

- Zapišemo $R(x) = p(x) + \frac{r(x)}{q(x)}$ in faktoriziramo q.
- Uporabimo nastavek:
  - $\frac{1}{(x-a)^k} \rightsquigarrow A \ln\lvert x-a\rvert$,
  - $\frac{1}{(x^2+bx+c)^l} \rightsquigarrow B \ln(x^2+bx+c) + C \operatorname{arc\,tg} \frac{2x+b}{\sqrt{4c-b^2}}$,
  - $\frac{\tilde{r}(x)}{\tilde{q}(x)}$, kjer polinom $\tilde{q}$ dobimo iz polinoma q z znižanjem potence vsakega faktorja za ena, polinom $\tilde{r}$ pa ima stopnjo za eno nižjo kot $\tilde{q}$.
- Odvajamo obe strani in izračunamo koeficiente.

Iracionalne f:

Rešitev: Integrale tipa $\int \frac{p(x)\,dx}{\sqrt{ax^2+bx+c}}$ integriramo na naslednji način:

(1) Če je polinom p konstanten, integral prevedemo na enega izmed integralov:

- $$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\left(\frac{x}{a}\right) + C, \quad a > 0,$$
- $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + C, \quad a > 0,$$
- $$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right) + C, \quad a > 0.$$

(2) Če je p poljuben polinom, uporabimo nastavek

$$\int \frac{p(x)}{\sqrt{ax^2+bx+c}} dx = \tilde{p}(x)\sqrt{ax^2+bx+c} + \int \frac{C}{\sqrt{ax^2+bx+c}} dx,$$

kjer je C konstanta, polinom $\tilde{p}$ pa ima stopnjo eno manjšo kot p.

(~p = A. Odvajamo obe strani in poracunamo A in B.)

S koreni:

Rešitev: Integrale tipa $\int R\left(x, \sqrt[n]{\frac{ax+b}{cx+d}}\right) dx$ integriramo s pomočjo substitucije

$$t = \sqrt[n]{\frac{ax+b}{cx+d}} \quad \text{oziroma} \quad t^n = \frac{ax+b}{cx+d},$$

ki nam integral prevede na integriranje racionalnih funkcij.

E^x, ln(x)

Integrale tipa $\int R(e^x)\, dx$ integriramo s pomočjo substitucije

$$e^x = t \Longrightarrow dx = \frac{dt}{t},$$

Integrale tipa $\int p(\ln x)\, dx$ integriramo s pomočjo substitucije

$$\ln x = t \Longrightarrow dx = e^t\, dt,$$

Izlimitirani integral

(a) Naj bo g zvezna funkcija na intervalu $[a, b]$.

- $\int_a^b \frac{g(x)}{(x-a)^s} dx$ konvergira, če je $s < 1$.
- $\int_a^b \frac{g(x)}{(x-a)^s} dx$ divergira, če je $s \geq 1$ in $g(a) \neq 0$.

(b) Naj bo g zvezna in omejena funkcija na intervalu $[a, \infty)$.

- $\int_a^\infty \frac{g(x)}{x^s} dx$ konvergira, če je $s > 1$.
- $\int_a^\infty \frac{g(x)}{x^s} dx$ divergira, če je $s \leq 1$ in $\lvert g(x)\rvert > m > 0$ za vse x od nekje dalje.