Operational Amplifiers

CHAPTER 11

Operational amplifiers (usually shortened to simply "op-amps") are electronic devices that are the basic building blocks used for conditioning, manipulating, and preparing analog signals for presentation to a microcontroller. While the uses of op-amps can range from very simple to quite sophisticated circuits, the basic behavior of these devices can be described with only a few simple rules. With these rules, and the knowledge of a few basic circuit configurations, it is possible to construct many common circuits used to condition signals from sensors. This chapter will present the basic operation of the ideal op-amp, demonstrate how to analyze circuits that employ op-amps, and discuss the most important aspects of how real op-amps differ from ideal op-amps, including how to extract the necessary information from data sheets. It will also include a brief discussion of comparators, which are a subspecies of op-amps that are optimized for certain specific uses. After reading this chapter, the student should be able to:

1. recognize the basic op-amp circuit configurations,
2. use the Golden Rules of Op-Amps to analyze op-amp circuits and develop transfer functions to describe the input-to-output relationship,
3. explain the behavior of a comparator and how it differs from an op-amp,
4. explain the manifestations of the most common deviations from ideal op-amp behavior,
5. extract the relevant parameters from a data sheet and use them to choose among op-amps for a given application.

11.1 OP-AMP BEHAVIOR

Op-amps are the most common circuit elements used to manipulate analog signals. They are active electronic devices, requiring connection to a power supply in order to operate. Functionally, op-amps are 2-input, 1-output devices whose schematic representation is shown in Figure 11.1. One input is called the **inverting input** (labeled "–") and the other is called the **noninverting input** (labeled "+"). Analytically, the output voltage is equal to the difference between the noninverting input and the inverting input multiplied by the **gain** of the op-amp, ***G***,

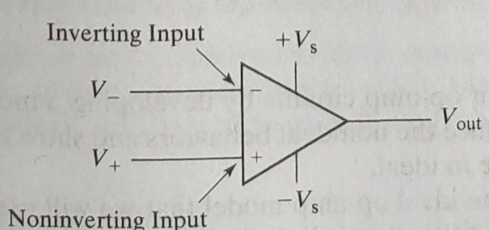


FIGURE 11.1
Schematic symbol for an op-amp.

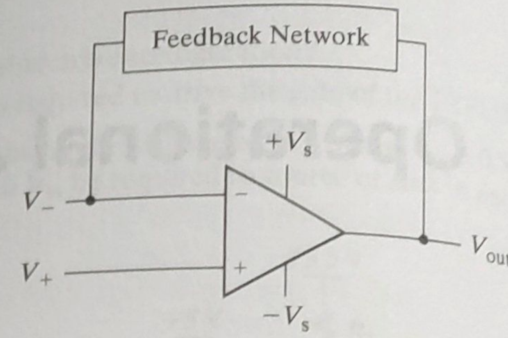


FIGURE 11.2
The closed-loop negative feedback configuration.

as shown in Eq. (11.1). For real devices, this gain is an extremely large value (more than 10^5 for typical real devices), and the output voltage is bounded by the power supply voltages (no matter what is happening at the inputs, the output voltage is always somewhere between $-V_s$ and $+V_s$).

$$V_{out} = G\,(V_+ - V_-) \tag{11.1}$$

When drawing generic op-amp circuits, it is common to omit the power supply connections. However, when preparing a schematic diagram for a design that you want to build, it is essential that they be included; otherwise it is very likely that power will not actually be connected, and the op-amp will not operate without it.

11.2 NEGATIVE FEEDBACK

The use of op-amps as simple differential amplifiers, as shown in Figure 11.1, is very unusual. This configuration, known as **open loop** (i.e., no feedback), is almost never used. The magnitude of amplification, which is the gain *G*, is so large that almost any finite voltage difference at the inputs would be amplified so much that the output voltage would be driven to one of the two power supply voltages. Instead, op-amps are most often used in a configuration known as **closed-loop negative feedback**, shown generically in Figure 11.2. The closed-loop portion of the term comes from the fact that there is a connection from the output back to the input, hence "closing a loop." The negative feedback portion of the term refers to the fact that the connection from the output is made back to the inverting input (as opposed to the noninverting input).

When operated in closed-loop negative feedback, the characteristics of the overall circuit are determined almost exclusively by the details of the feedback network and other components connected to the op-amp. For typical circuits, the exact open-loop gain of the op-amp has little impact on the actual gain of the resulting op-amp circuit. This is a very good thing, since it allows us to pick the circuit's gain so that it has a value that's useful in a particular application, with minimal dependence on the op-amp characteristics.

11.3 THE IDEAL OP-AMP

We will begin our study of op-amp circuits by developing a model for an ideal op-amp. In Chapter 12, we will introduce the nonideal behaviors and show that, under many conditions, modern op-amps are close to ideal.

Figure 11.3 shows the ideal op-amp model that we will use. The output voltage is created by an ideal voltage source that is linked to an ideal measurement of the difference in

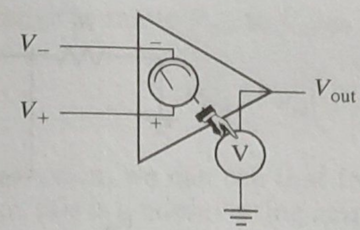


FIGURE 11.3
The ideal op-amp model.

the voltages present at the two inputs. This model requires three assumptions that we will make whenever we use the ideal op-amp model:

1. The gain $\left[V_{out}/(V_{+} - V_{-})\right]$ is infinite.
2. The inputs draw no current. This is necessary for an ideal voltage measurement.
3. The output impedance is zero. This is necessary for an ideal voltage source.

The use of this model for an ideal op-amp and acceptance of these three assumptions will greatly simplify the analysis of op-amp circuits. Even when we know that the nonideal aspects of an op-amp will impact a design, it is common to begin with the assumption of an ideal op-amp and then calculate the impact of the important nonidealities after the basic analysis.

11.4 ANALYZING OP-AMP CIRCUITS

11.4.1 The Golden Rules[1]

We can use the ideal op-amp assumptions to develop a pair of rules that have become known as **The Golden Rules** that will allow us to reduce the complexity of the equations we will develop during our analysis of op-amp circuits.

Rule #1: The inputs draw no current.

Rule #2: When operated with negative feedback, the output will do whatever is necessary to make the two input voltages the same.

Rule #1 comes directly from our assumptions about the behavior of an ideal op-amp. Rule #2 is a result of assuming that the op-amp has infinite gain. If the output is connected to the inverting input (even if there are other circuit elements in this path, such as resistors) and the noninverting input is at a higher voltage than the inverting input, then the output voltage will be driven upwards until the voltage at the inverting and noninverting inputs match. In cases where the inverting input is initially at a higher voltage than the noninverting input, the output voltage will be driven lower—also resulting in the voltages at the inputs achieving the same value. Since the gain is infinite, they will match exactly.

11.4.2 The Noninverting Op-Amp Configuration

Figure 11.4 shows what is referred to as the **noninverting amplifier** configuration. The reason for that name will become clear as we analyze this circuit. Before we get into the analysis of the circuit though, it is helpful to walk through the circuit on an imagined, very small time

---

[1]This usage comes from *The Art of Electronics*, Horowitz, P., and Hill, W., 2nd ed., Cambridge University Press, 1989.

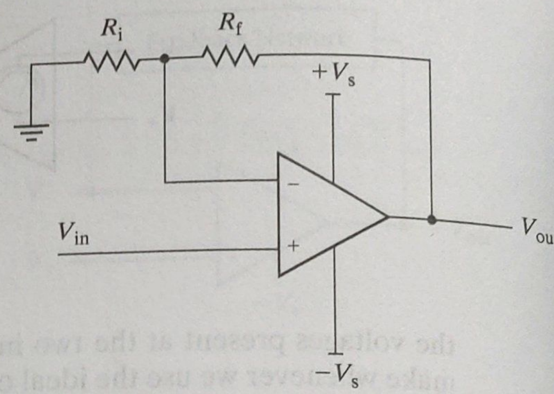


FIGURE 11.4
The noninverting amplifier configuration.

scale. Let's begin the walkthrough by imagining that V_{in} is initially at 1 V and V_{out} is at 0 V. With V_{out} at 0 V, the connection through the voltage divider formed by R_f and R_i will place the inverting input also at 0 V. We therefore have an initial condition in which the noninverting input is at a voltage greater than that at the inverting input. The model of the ideal op-amp tells us that this will result in the output voltage rising (since $V+$ is greater than $V-$). The rising output voltage, acting through the voltage divider, will cause the voltage at the inverting input to also rise. The output voltage will continue to rise until the voltage at the inverting input matches the 1 V at the noninverting input. The resistor values, R_f and R_i, determine the current flowing in the feedback loop and the voltage required at the output to make the input voltages match.

We can analyze the behavior of the circuit symbolically through the use of the Golden Rules. Applying Golden Rule #2 allows us to state that the voltage at Node A (Figure 11.5) is V_{in}. With this information, we can calculate the current i_1 as

$$i_1 = \frac{V_{out} - V_{in}}{R_f} \tag{11.2}$$

And the current i_2 as

$$i_2 = \frac{V_{in} - 0}{R_i} \tag{11.3}$$

Golden Rule #1 tells us that there is no current going into the inverting input of the op-amp, therefore $i_1 = i_2$.

$$\frac{V_{out} - V_{in}}{R_f} = \frac{V_{in} - 0}{R_i} \tag{11.4}$$

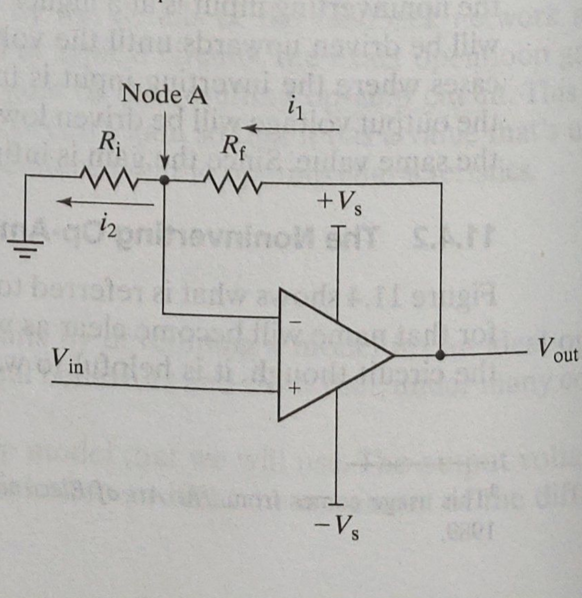


FIGURE 11.5
Noninverting op-amp circuit showing current flows for analysis.

which we can rearrange to relate V_{out} to V_{in} as

$$V_{out} = V_{in}\left(1 + \frac{R_f}{R_i}\right) \tag{11.5}$$

From this expression, we can see that the output voltage is the same polarity as the input voltage (hence, this is a noninverting amplifier) and that the input and output voltages are related by the gain for the noninverting amplifier: $(1 + R_f/R_i)$.

11.4.3 The Inverting Op-Amp Configuration

Figure 11.6 shows a variation on the noninverting amplifier configuration called the **inverting amplifier**. As you may have guessed by its name, this circuit gives a very different result. To create the inverting amplifier circuit, we swapped the connections to the noninverting input and the left-hand side of R_i. The analysis for this circuit follows much the same sequence as before: we start by invoking Golden Rule #2 to assert that the voltage at Node A is 0 V. With that helpful simplification, we can write expressions for i_1 and i_2:

$$i_1 = \frac{V_{out} - 0}{R_f} \tag{11.6}$$

$$i_2 = \frac{0 - V_{in}}{R_i} \tag{11.7}$$

Now we can invoke Golden Rule #1 to assert that since there is no current flowing into the inverting input of the op-amp, again $i_1 = i_2$.

$$\frac{V_{out} - 0}{R_f} = \frac{0 - V_{in}}{R_i} \tag{11.8}$$

which we can rearrange to relate V_{out} to V_{in} as

$$V_{out} = -V_{in}\left(\frac{R_f}{R_i}\right) \tag{11.9}$$

For the inverting configuration, we can see that the output voltage is the opposite polarity as the input voltage (hence the name "inverting") and that the input and output voltage amplitudes are related by the factor R_f/R_i. We combine the change in polarity with the amplitude factor to say that the gain for the inverting configuration is $-R_f/R_i$.

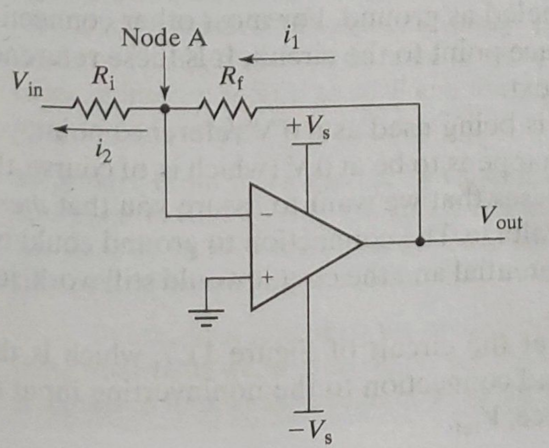


FIGURE 11.6
The inverting amplifier configuration.

As we saw before, with the noninverting configuration, the gain of closed-loop op-amp circuits is governed by the ratio of the feedback resistor to the input resistor. In the ideal case, it is only the *ratio* that matters and not the absolute values of these resistors. For practical circuits, the values of these resistors should be kept in the range of a few kilohms to a few hundreds of kilohms. These guidelines are rooted in the nonideal behavior of real op-amps. We cover op-amp nonidealities and point out which of them contribute to the guidelines in Section 12.1.

Here, it is helpful to take a look at a concrete example. Suppose that in the circuit of Figure 11.6, the input voltage, V_{in}, is 1 V and that $R_f = R_i = 10\ \text{k}\Omega$. Using Eq. (11.7), we can calculate the current i_2 as

$$i_2 = \frac{0 - V_{in}}{R_i} = \frac{-1\ \text{V}}{10\ \text{k}\Omega} = -0.1\ \text{mA}$$

The fact that the calculated current is negative tells us that the direction of current flows that we had assumed in writing the equations is opposite to the actual current flow in this situation.

With a value for i_2, we can use Golden Rule #1 to assert that $i_1 = i_2$, then rearrange Eq. (11.6) to write an expression for V_{out}:

$$V_{out} = R_f i_1 = 10\ \text{k}\Omega \times -0.1\ \text{mA} = -1\ \text{V}$$

This value for V_{out} agrees with the result that we would get from applying Eq. (11.9) directly.

11.4.3.1 The Virtual Ground

It is interesting to note that Node A in Figure 11.6 is held at ground potential by the action of the op-amp. It is not actually connected to ground, but is always at ground potential. This means that, for example, when analyzing the current through resistor R_i, we can treat the right-hand side of the resistor as if it were connected to ground. However, the current flowing through R_i is not going directly to ground. This can be a useful characteristic and this has been given the name **virtual ground**. Node A would be said to be "at a virtual ground."

11.4.3.2 There Is Nothing Magic about Ground

The role of ground in electronic circuits is often somewhat of a mystery to beginning circuit designers. Ground in electronic circuits serves two purposes: 1) it is the common voltage reference point across the circuit and 2) it provides a return path for current to flow back to the power source. It is important to learn to tell which of these purposes is being served by any given connection to ground. The return path function is of greatest importance when dealing with the power being supplied to devices. This will be primarily the power connections to device pins that are explicitly labeled as ground. For most other connections to ground, the purpose is to supply a 0 V reference point to the circuit. It is these reference voltage connections that we want to deal with next.

In instances where ground is being used as a 0 V reference point, you should think of this as a voltage source that just happens to be at 0 V (which is, of course, the same voltage as ground potential). It is in these cases that we want to assure you that *there is nothing magic about ground*. It is just another voltage. The connection to ground could be substituted with a voltage source at some other potential and the circuit would still work, just with a different transfer function.

To explore this, let's look at the circuit of Figure 11.7, which is the same circuit as Figure 11.6 except that the ground connection to the noninverting input has been replaced by a connection to a voltage source, V_{ref}.

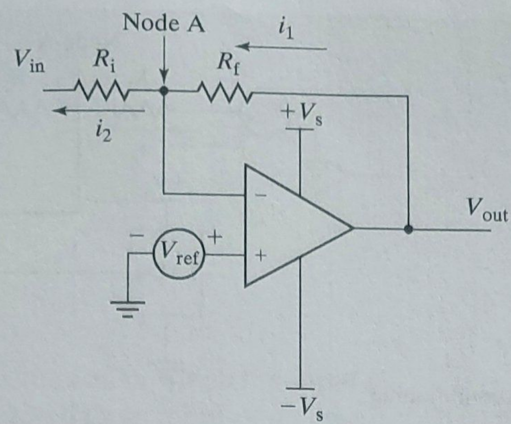


FIGURE 11.7

Inverting op-amp with offset voltage.

We can proceed just as we did in developing Eqs. (11.6)–(11.9) to get an expression relating V_{out} to V_{in}. This time, Node A will be at V_{ref}:

$$i_1 = \frac{V_{out} - V_{ref}}{R_f} \tag{11.10}$$

$$i_2 = \frac{V_{ref} - V_{in}}{R_i} \tag{11.11}$$

$$\frac{V_{out} - V_{ref}}{R_f} = \frac{V_{ref} - V_{in}}{R_i} \tag{11.12}$$

$$V_{out} = (V_{ref} - V_{in})\left(\frac{R_f}{R_i}\right) + V_{ref} \tag{11.13}$$

In Eq. (11.13), we can see the manifestation of the basic differential amplifier behavior of the op-amp in the fact that the gain is now applied to the difference between V_{ref} and V_{in}. We can see that the output has been shifted (offset) by the magnitude of the reference voltage, V_{ref}. V_{in} still appears in the equation with a negative sign, indicating that increases in V_{in} will result in reductions in V_{out}, which is the basic inverting configuration behavior. Whether or not here is an actual polarity reversal in the output will depend on the relative values of V_{in} and V_{ref}. Finally, we see that if $V_{ref} = 0$, then we get back the transfer function for the basic inverting op-amp configuration.

Example 11.1

As a concrete example of what we have seen so far, consider the design of the amplification circuit for an LM34 temperature sensor to be used to measure room temperatures. The LM34 is a precision temperature sensor that outputs a voltage proportional to temperature in Fahrenheit, with a scale factor of 10 mV/°F. It is to be used in an application that measures room temperature, where the range of interest is 50°F to 90°F and the goal is to make this temperature range correspond to an output voltage range from 0.5 to 4.5 V.

Solution: From these specifications, we can determine that we need a gain of 10 [(10 mV/°F × 40°F)/(4.5 V–0.5 V) = 10]. The specifications also tell us that we need to use a noninverting amplifier since 0.5 V is to correspond to 50°F and 4.5 V to 90°F. With that information, we can draw a first-pass schematic (Figure 11.8).

To achieve a gain of 10, we need to make the ratio of R_f to R_i = 9:1. What remains is to determine what value we need to supply for V_{ref} in order to meet the system requirements. To do that, it is easiest to look at the situation when the temperature is at 50°F and the LM34's output will be

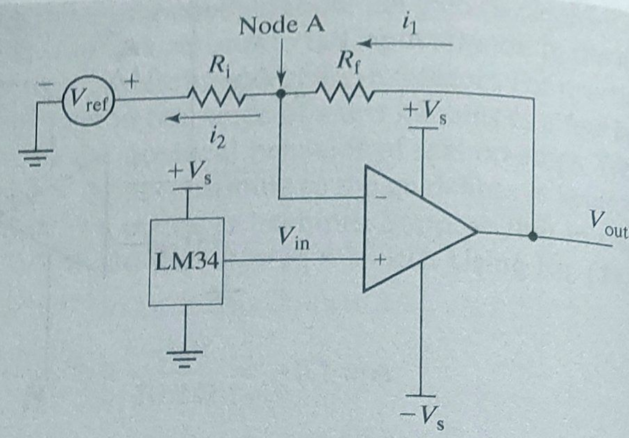


FIGURE 11.8

First-pass signal conditioning for LM34.

50°F × 10 mV/°F = 500 mV. The second Golden Rule tells us that Node A will also be at 0.5 V. The specifications require that at this temperature the output voltage also be 0.5 V. If both Node A and V_{out} are at 0.5 V, then i_1 will be 0 and therefore, i_2 will be 0, so V_{ref} must also be at 0.5 V.

We can test this solution at the upper end of the temperature range, when the output of the LM34 will be 90°F × 10 mV/°F = 0.9 V. Node A will also be at 0.9 V, and the output should (according to the specifications) be at 4.5 V, yielding 4.5 V − 0.9 V = 3.6 V across R_f and forcing i_1 to be 3.6 V/R_f. Because of the first Golden Rule, i_2 will also be 3.6 V/R_f. This current will flow across R_i (which is 1/9R_f) to create a voltage drop of $3.6\ \text{V}/R_f \times R_f/9 = 3.6\ \text{V}/9 = 0.4$ V. Since Node A was at 0.9 V, this places V_{ref} at 0.5 V, which agrees with our original calculation. Therefore, the solution is the circuit of Figure 11.8 with V_{ref} = 0.5 V. We cover how to create the 0.5 V source for V_{ref} in the next section.

11.4.4 The Unity Gain Buffer

There are many situations in which you would like the characteristics of an amplifier with a gain of exactly 1. For example, suppose that you would like to use a voltage divider to generate a reference voltage, but need the reference voltage to be able to source or sink significant amounts of current (a few milliamps). When we analyzed the voltage divider in Chapter 9, we assumed that no current was flowing into or out of the V_{ref} node in Figure 11.9. Currents flowing into or out of V_{ref} would disturb the reference voltage by changing the voltage divider expression. In fact, it would be very difficult to keep the disturbances down to only a few percent. This would require that the normal current flowing in R_1 and R_2 be approximately 50–100 times larger than the current flowing into or out of V_{ref}. If the V_{ref} current were 2 mA, that would mean 100–200 mA would be required to flow in the voltage divider. This is a relatively large amount of current that represents wasted power that is dissipated as heat in the resistors of the voltage divider. And in any case, this would be an inelegant approach: the resistors R_1 and R_2 would be dissipating somewhere between 0.5 and 1.0 W, which would require the use of power resistors.

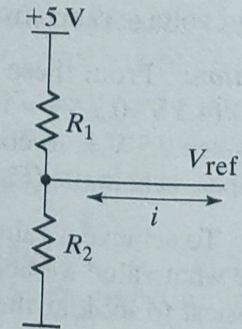


FIGURE 11.9

A voltage divider used to provide a reference voltage.

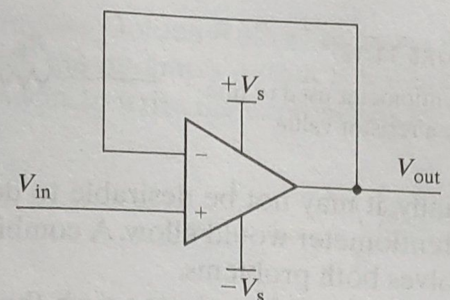


FIGURE 11.10

The unity gain buffer.

This is a situation in which the input characteristics of the op-amp can be very useful. If V_{ref} were connected to an input of an op-amp, Golden Rule #1 tells us that no current will be drawn and therefore, there would be no disturbance in V_{ref}. The current provided to the load would be delivered by the output of the op-amp. This isolation of currents between the input and the output is referred to as **buffering**.

Of the circuits that we have discussed so far, our needs would seem to be best met by the noninverting configuration, since in that circuit the input voltage is tied directly to the op-amp input and there is no inversion of the input voltage. However, at first glance, it appears that the gain expression for the noninverting amplifier, $(1 + R_f/R_i)$, indicates that it is only capable of gains greater than 1. A bit of outside-the-box thinking would bring us to the conclusion that if we were to make $R_f = 0$ and $R_i = \infty$, the resulting gain would be exactly 1. Actually, from a theoretical standpoint, we would achieve a gain of exactly 1 if either $R_f = 0$ or $R_i = \infty$, but from a practical standpoint, it's easiest to make both modifications. Modifying the noninverting amplifier circuit in this way yields a circuit known as a **unity gain buffer** (Figure 11.10).

By combining the circuits of Figures 11.9 and 11.10 (with one additional embellishment), we arrive at a practical reference voltage source (Figure 11.11).

The capacitor, C_1, has been added to combat fluctuations in the power supply. Without C_1, any disturbance in $+V_s$ will be immediately reflected in V_{ref}. Adding C_1 to the voltage divider creates a low-pass filter that will reduce the magnitude of disturbances on $+V_s$ that are seen at the input to the op-amp. Since $+V_s$ should not be changing at all (a desirable characteristic for a power supply), the value of C_1 is usually chosen to give a relatively long time constant (tens to hundreds of milliseconds, for example).

The circuit of Figure 11.11 works well when we can achieve the desired voltage using standard resistor values and are willing to tolerate the uncertainty associated with standard resistor tolerances. For those situations when this does not produce an acceptable result, it is possible to replace either R_1 or R_2 with an adjustable resistance. In order to have fine control over the adjustable resistance, it is often a good idea to implement this as a combination of a fixed value resistor and a potentiometer wired to act as a variable resistor (Figure 11.12). A single potentiometer could be used, but the resolution of the adjustment would be much

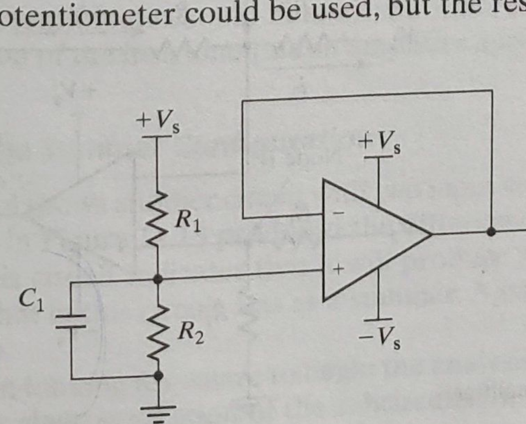
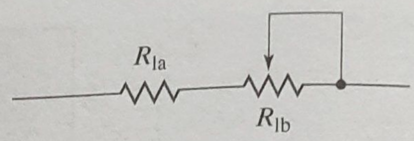


FIGURE 11.11

A practical voltage reference.

FIGURE 11.12
Potentiometer used to fine tune a resistor value.



more coarse. Additionally, it may not be desirable to decrease the resistance all the way to 0 Ω, which a single potentiometer would allow. A combination of a fixed resistor (R_{1a}) and a potentiometer (R_{1b}) solves both problems.

The fixed resistor, R_{1a}, should be chosen such that at the maximum range of its tolerance it is less than the minimum required total resistance. The value of the variable resistor, R_{1b}, should be chosen so that when R_{1a} is at its minimum possible value, based on tolerance, the sum of that minimum value plus the minimum value of R_{1b}, when the wiper is positioned for maximum resistance (i.e., labeled resistance of the potentiometer minus its worst case tolerance), slightly exceeds the maximum required value of the combination. By choosing the values of the resistors in this way, we maximize the resolution with which we can adjust the combined resistance and never have a situation when the combination has zero resistance (which often creates problems). This approach to achieving an exact resistance can be applied in place of any resistor in any circuit. Having said that, it is exceedingly rare to need this kind of adjustability for gain resistors outside the realms of prototype circuits and measurement instruments. Most mechatronics systems can be designed to meet requirements despite the tolerance in resistors. In some instances, it will be necessary (and far preferable) to specify resistors with tighter tolerances (2%, 1%, or better) to meet requirements.

11.4.5 The Difference Amplifier Configuration

Figure 11.13 shows a circuit in which there are two input voltages. The caption claims that this is a **difference amplifier**. Let's work out the details of what the transfer function is for this circuit to see if the caption writer knows what he or she is talking about.

To start the analysis, it is helpful to look for parts of the circuit with minimal dependencies on other parts of the circuit. The subcircuit composed of V_2, R_i, Node B, and R_f forms a voltage divider with the only other connection being to the noninverting input to the op-amp. Because of Golden Rule #1 (the inputs draw no current), we can treat this as an ideal voltage divider and write the expression for the voltage at Node B.

$$V_B = V_2\left(\frac{R_f}{R_i + R_f}\right) \tag{11.14}$$

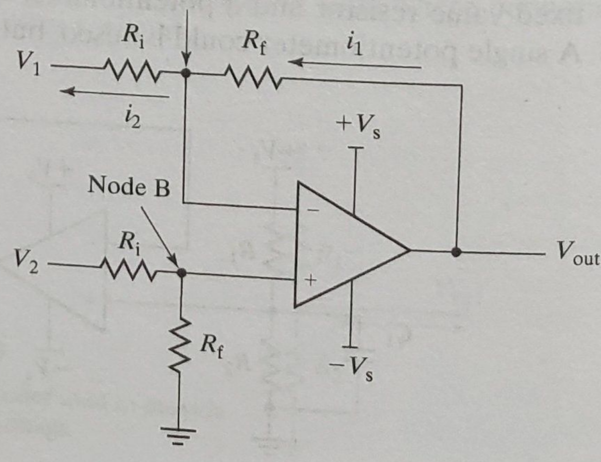


FIGURE 11.13
The difference amplifier.

We can then invoke Golden Rule #2 to assert that the voltage at Node A is the same as that at Node B, since the op-amp's output is fed back to the inverting input (negative feedback). This will allow us to write the expressions for i_1 and i_2 as

$$i_1 = \frac{V_{\text{out}} - V_2\left(\dfrac{R_f}{R_i + R_f}\right)}{R_f} \tag{11.15}$$

$$i_2 = \frac{V_2\left(\dfrac{R_f}{R_i + R_f}\right) - V_1}{R_i} \tag{11.16}$$

Applying Golden Rule #1 to the inverting input allows us to say that $i_1 = i_2$ and hence,

$$\frac{V_{\text{out}} - V_2\left(\dfrac{R_f}{R_i + R_f}\right)}{R_f} = \frac{V_2\left(\dfrac{R_f}{R_i + R_f}\right) - V_1}{R_i} \tag{11.17}$$

Now a bit of algebra allows us to get to an expression relating the output voltage to the two input voltages:

$$V_{\text{out}} = (V_2 - V_1)\frac{R_f}{R_i} \tag{11.18}$$

Eq. (11.18) shows that the caption was indeed correct. This circuit produces an output voltage that is proportional to the difference between the two input voltages with an overall gain of R_f/R_i.

From a practical standpoint, some care is necessary in using the circuit of Figure 11.13 to produce a signal that is the difference between two other signals. The first issue relates to the accuracy of the subtraction. The derivation of Eq. (11.18) assumed that both resistors labeled R_i were *exactly* the same value (that's how we were able to make many of the algebraic simplifications) and that both resistors labeled R_f also had *exactly* the same value. Any differences in the actual values of these resistors in a real circuit will change the transfer function, resulting in a more complex expression that will only be an approximation of the difference. The second issue relates to the absolute values of the resistors used for R_f and R_i. To minimize the current that must be supplied by V_1 and V_2, we would like to make the resistors as large as possible. As we shall see in Chapter 12, this is at odds with minimizing errors due to the nonideal behavior of real op-amps. As a result, the circuit of Figure 11.13 is typically only used for low precision subtraction when the voltage sources are capable of delivering significant currents (i.e., they have a low output impedance). For more accurate difference amplification, you should use what is known as an **instrumentation amplifier**. A discussion of the instrumentation amplifier appears in Chapter 14 on Signal Conditioning.

11.4.6 The Summer Configuration

Figure 11.14 shows another circuit with two input voltages and a single output voltage. Whereas the circuit in Figure 11.13 produced the difference between the two input voltages, the caption for this circuit indicates that it will produce an output that is the sum of the two input voltages: that is, this circuit acts as a **summer**. Again, let's double-check the work of the caption writer.

When looking for where to begin the analysis of this circuit, we find that it doesn't offer an obvious clean separation of the subcircuits for the two input voltages that we enjoyed in

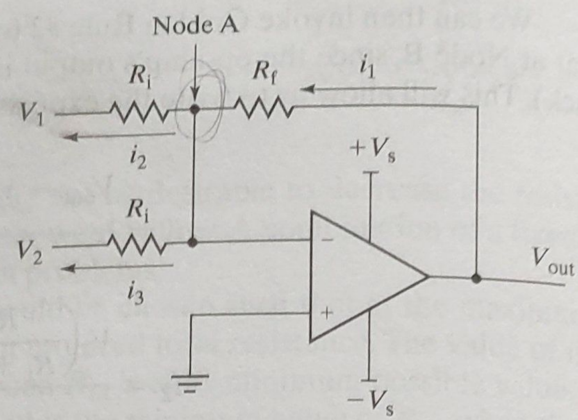


FIGURE 11.14
The summer.

our analysis of the circuit in Figure 11.13. However, the fact that we have two independent input voltages suggests that this might be a good place to apply the principle of superposition. In Chapter 9, we saw that we could analyze circuits with multiple sources by treating each source separately and then summing the results from each of the sources. That's what we'll do here.

We begin the analysis by removing one of the sources. As with Thevenin equivalent circuits and analysis, we remove voltage sources by replacing them with a short circuit, and we remove current sources by replacing them with an open circuit. Let's start by removing V_2. This is the same as setting this voltage to 0 V. This circuit shares a characteristic with the inverting amplifier configuration in that the noninverting input is tied to ground, producing a virtual ground at Node A. With Node A at 0 V and V_2 at 0 V, there is no voltage difference across the lower input resistor R_i and so $i_3 = 0$. With $i_3 = 0$, this circuit is identical to the inverting amplifier that we analyzed in Section 11.4.3. The output that results from V_1 is therefore,

$$V_{out} = -V_1 \frac{R_f}{R_i} \tag{11.19}$$

Now we can go back and reinstate V_2 in the circuit, and remove V_1 by setting it equal to 0 V. Having done that, we find that there is no longer a voltage difference across the upper input resistor R_i and therefore, $i_2 = 0$. With $i_2 = 0$, we have a circuit that is very similar to the inverting amplifier configuration shown in Figure 11.6, with the only difference being that i_2 has been replaced with i_3. That will give us a similar output description, with the term V_2 replacing the term V_1 from Eq. (11.9):

$$V_{out} = -V_2 \frac{R_f}{R_i} \tag{11.20}$$

Having calculated the results from each of the input voltages independently, we obtain the total response by adding the two results together:

$$V_{out} = -V_2 \frac{R_f}{R_i} + \left(-V_1 \frac{R_f}{R_i}\right) = -\frac{R_f}{R_i}(V_2 + V_1) \tag{11.21}$$

Eq. (11.21) shows that the caption writer was not being entirely truthful. The circuit of Figure 11.14 is actually an **inverting summer**—the sign of the resulting sum is changed by this circuit.

11.4.7 The Trans-Resistive Configuration

The circuits that we've looked at so far have all had voltage inputs and voltage outputs. While a voltage output is the most useful type of output when interfacing to microcontrollers, not all interesting inputs appear as voltages. Prime examples of devices that produce

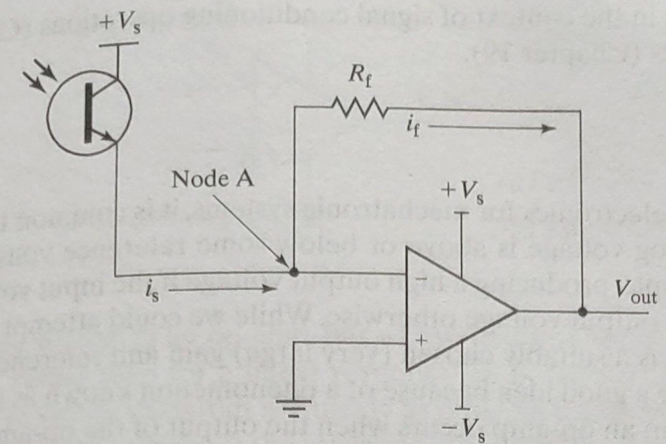


FIGURE 11.15
Trans-resistive amplifier for a photo-transistor.

an output as a current rather than a voltage are the photo-diode and photo-transistor introduced in Chapter 10. Both of these devices produce current flows that are proportional to the amount of light incident on the sensor. Both of these types of devices also produce the most linear transfer function from light to current when a constant voltage is maintained across their terminals. Using an op-amp, we can build a simple circuit that achieves both of these goals: the **trans-resistive amplifier** circuit.

In the circuit of Figure 11.15, Node A is once again a virtual ground node (held by the op-amp at 0 V). This fixes the voltage across the photo-transistor to $+V_s$, independent of how much light is falling on the photo-transistor. This will yield the most linear light-to-current transfer function from the device. The photo-current, i_s, flows into Node A. Because of Golden Rule #1, $i_f = i_s$ and therefore, $V_{out} = -i_f R_f$. This amplifier circuit's output voltage is the product of the amount of current flowing through the sensor, i_s, times a gain term which is the value of the feedback resistor, R_f. Note too, that the resulting sign is inverted.

The circuit is called trans-resistive because it performs a function similar to that of a resistor: it converts a current into a voltage. The circuit is superior to a simple resistor in a number of respects. Since Node A is held at a constant voltage, the current is being delivered into a constant voltage, independent of the amount of current flowing. Also (and this is a subtler point), the fact that Node A is held at a constant voltage means that any stray capacitance associated with the circuit is not being charged or discharged by changes in the photo-current. This, in turn, means that the rise and fall times of the photo-current will not be limited by the RC charge and discharge times that would result from using a simple resistor.

11.4.8 Computation with Op-Amps

So far we have discussed op-amp circuits that will amplify (multiply), sum (add), and take the difference (subtract). If we add in the voltage divider (divide), we have the basic mathematical operations. Indeed, in the past, analog computers were built to quickly perform mathematical transforms on analog signals because they could do the operations much more quickly than the digital computers available at that time. As part of analog computation field, op-amp circuits were also developed to perform the calculus operations of integration and differentiation. In this text, we do not deal directly with op-amp circuits to create integrators and differentiators, though in Chapter 14, we cover low-pass filters (integrators) and high-pass filters (differentiators) in the context of their frequency domain behavior. Today the integration and differentiation computations are most commonly performed in software executed by microcontrollers. While it is true that addition and subtraction of signals are also done digitally, implementing these processes digitally requires more inputs to the microcontroller, a very limited resource. Understanding the summer and difference amplifier circuits is also important in the context of signal conditioning operations (Chapter 14) and digital to analog converters (Chapter 19).

11.5 THE COMPARATOR

In designing the electronics for mechatronic systems, it is common to need to know whether a particular analog voltage is above or below some reference voltage. This would typically involve, for example, producing a high output voltage if the input voltage were above the reference and a low output voltage otherwise. While we could attempt to do this with an amplifier circuit that has a suitably chosen (very large) gain and reference voltage (Figure 11.16), that would not be a good idea because of a phenomenon known as saturation in the op-amp.

Saturation in an op-amp occurs when the output of the op-amp is not able to force the two inputs to the same voltage. The output goes to the highest (or lowest) voltage that it can produce, and the output transistors enter the saturated state as they try to drive the output voltage further. While this does not harm the op-amp, driving the transistors into saturation takes them outside of their normal operating regime. If, in the circuit of Figure 11.16, the reference voltage were 1 V and the input voltage was 0.999 V (1 mV below the reference), Eq. (11.13) predicts that the output voltage would be

$$V_{\text{out}} = (V_{\text{ref}} - V_{\text{in}})\left(\frac{R_{\text{f}}}{R_{\text{i}}}\right) + V_{\text{ref}} = (1\text{ V} - 0.999\text{ V})(10{,}000) + 1\text{ V} = 11\text{ V}$$

However, since the circuit has only a 5 V positive supply, the output would actually be driven into a state of saturation as it attempted to drive the output to a positive voltage that it cannot achieve.

If the input voltage then rose from 0.999 to 1.001 V (from 1 mV below to 1 mV above the reference), Eq. (11.13) predicts that the output voltage should go to

$$V_{\text{out}} = (V_{\text{ref}} - V_{\text{in}})\left(\frac{R_f}{R_i}\right) + V_{\text{ref}} = (1\text{ V} - 1.001\text{ V})\,(10{,}000) + 1\text{ V} = -9\text{ V}$$

Ideally, the output of the op-amp would immediately switch to the low output voltage. However, the saturated transistors in the op-amp take considerable time (often microseconds) to come out of saturation, and then more time to drive the output voltage to the new voltage level (which is again in saturation at the voltage limit imposed by the negative power supply). This produces an undesirable delay in the response to the change in the input voltage. To get around this problem, integrated circuit designers have produced another type of device

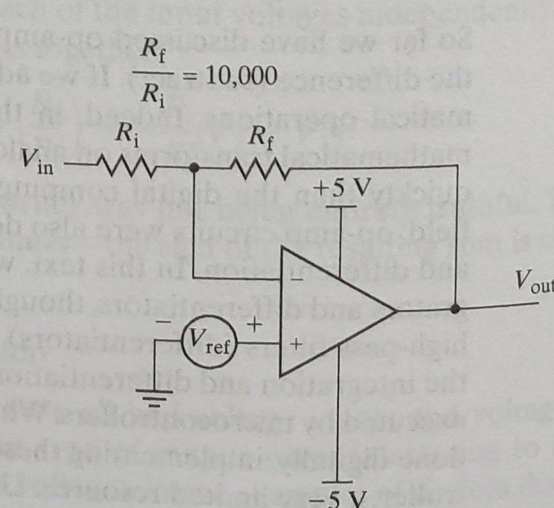


FIGURE 11.16

Attempting to use an amplifier circuit to compare two voltages (a bad idea).

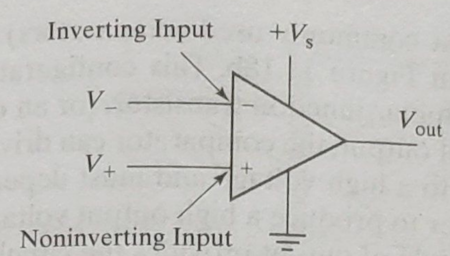


FIGURE 11.17
The schematic symbol for a comparator.

that is very similar to an op-amp, but whose internal design is optimized for a high speed of response, rather than for a linear response (as is the goal for op-amps). The comparator is designed to switch its output as quickly as possible whenever the voltage at the noninverting input exceeds the voltage at the inverting input, and vice versa. This response is largely independent of the magnitude of the difference between these two voltages. Figure 11.17 shows the schematic symbol for a comparator.

Comparing the schematic symbol for the comparator (Figure 11.17) with the schematic symbol for the op-amp (Figure 11.1) shows that they are identical except for the change of the lower supply voltage from $-V_s$ in the op-amp to ground in the comparator. This reflects the fact that the vast majority of comparators are designed to operate on a single supply voltage.

So, how do you tell the difference between an op-amp and a comparator in a schematic diagram? There are several ways. First and foremost, each device in a schematic should be labeled with a part number. With experience, you will begin to recognize the most common part numbers for op-amps and comparators. When faced with a part number that you don't recognize, you could look up the part in any number of online references or you could look at how the device is used in the circuit. Op-amps are almost always used in one of the closed-loop negative feedback configurations. Comparators are *never* used in negative feedback (they make lousy amplifiers) and are most commonly used with positive feedback (e.g., the output is connected to the noninverting or "positive" input) or in open-loop mode (e.g., no connection between the output and either of the inputs).

Most comparators are designed with a different type of output than op-amps. Op-amps are designed with what is known as a **totem-pole output** (sometimes called a **push-pull output**) as shown in a simplified form in Figure 11.18a. This type of output is capable of driving the output voltage high and sourcing current to the output by turning on Q_1 and turning Q_2 off. It is also capable of driving the output low and sinking current by turning on Q_2 and turning Q_1 off. Comparators are sometimes designed with totem-pole style outputs but much

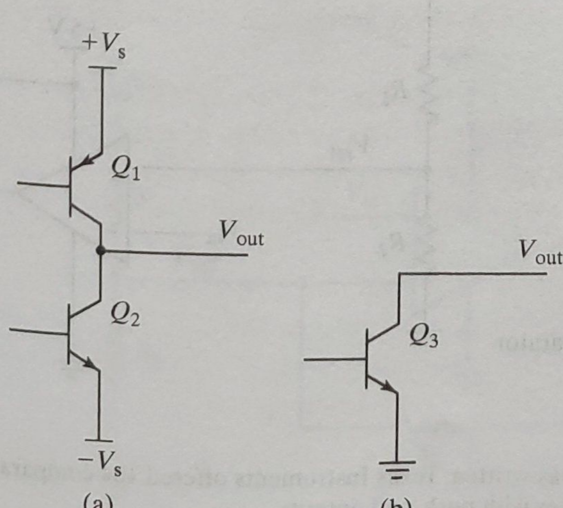


FIGURE 11.18
Totem-pole (a) vs. open-collector (b) outputs.

more often[2] (including the most commonly used comparators) they are designed with an output stage like that shown in Figure 11.18b. This configuration is known as an **open-collector output** when Q_3 is a bipolar junction transistor, (or an **open-drain output** when Q_3 is a MOSFET). With this style of output, the comparator can drive the output low by turning on Q_3, but it cannot drive V_{out} to a high voltage and must depend on external devices that the circuit designer adds in order to produce a high output voltage at the output when Q_3 is turned off. The open-collector style of output provides the circuit designer with a great deal of flexibility in what the high output voltage will be (e.g., it could come from a voltage source other than the power supply for the comparator) as well as in how this output is connected with other outputs. We will cover totem-pole and open-collector outputs in more detail in Chapter 17 on Digital Outputs and Power Drivers.

11.5.1 Comparator Circuits

The simplest practical comparator application circuit is shown in Figure 11.19. We have assumed that this comparator has an open-collector output, as evidenced by the **pull-up resistor**, R_{PU}, connected between the output and the power supply. The output, V_{out}, will be at approximately 5 V whenever V_{in} is greater than the voltage, V_{ref}, set by the voltage divider formed by R_1 and R_2. Otherwise, the output voltage will be at approximately 0 V. This circuit behaves as a **noninverting comparator**. In order to construct an **inverting comparator**, we can simply reverse the connections to the two inputs. When we do this, the circuit output will be low when V_{in} is above V_{ref}, and otherwise, the output will be high.

The circuit of Figure 11.19 works well as long as V_{in} transitions quickly across the reference voltage level. If the input voltage transitions relatively slowly, this circuit will be subject to the same chatter issue that we saw with the software comparator discussed in Chapter 5, Section 5.3. A slowly changing signal with even a little bit of electrical noise will cross back and forth across the threshold voltage multiple times as it passes completely over the threshold, as shown in Figure 11.20, and this will cause multiple output transitions for what should, in fact, be treated as a single input transition.

The cure for this problem is to take the same approach as for the software comparator: add some hysteresis to the system. For the hardware comparator, this means feeding the output signal back to the reference level input so that the reference level is shifted depending on the output level. This is most simply accomplished by modifying the inverting comparator circuit to produce the circuit shown in Figure 11.21, which gives us an **inverting comparator**

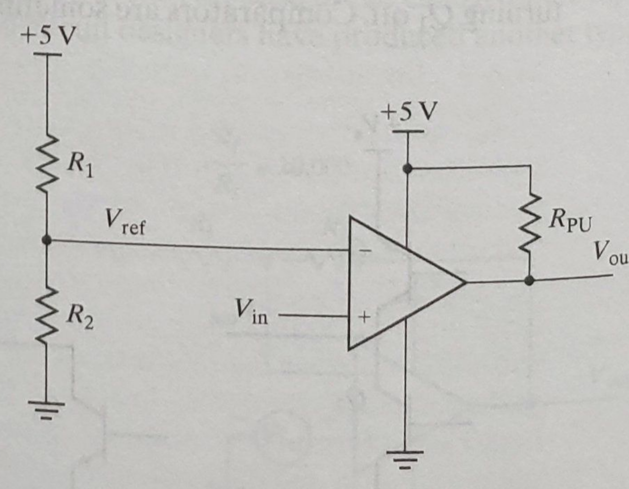


FIGURE 11.19

Simple noninverting comparator circuit.

[2]For example, at the time that this was written, Texas Instruments offered 104 comparators with open-collector or open-drain outputs but only 26 devices with push-pull outputs.

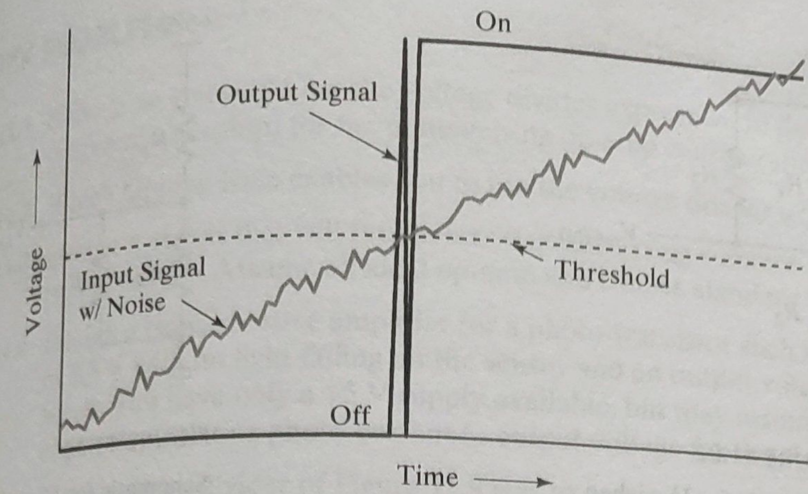


FIGURE 11.20
Chatter in the output of a comparator.

with hysteresis. A comparator with hysteresis (inverting or noninverting) also goes by the name **Schmitt trigger**.

The value of R_3, for reasonable levels of hysteresis, is usually over 100 times larger than R_{PU}, so we can simplify the analysis here by ignoring R_{PU}. For our circuit, when V_{in} is below V_{ref}, the output will be high and V_{ref} will be given by Eq. (11.22), which was derived from the simplified schematic form in Figure 11.22.

$$V_{ref(hi)} = V_{CC}\frac{R_2}{R_2 + (R_1 \| R_3)} \tag{11.22}$$

When V_{in} rises above this value of $V_{ref(hi)}$, the output will go low. This will ground the right-hand end of R_3 putting it in parallel with R_2 (Figure 11.23) and giving an expression for $V_{ref(low)}$ as

$$V_{ref(low)} = V_{CC}\frac{(R_2 \| R_3)}{R_1 + (R_2 \| R_3)} \tag{11.23}$$

The magnitude of the hysteresis is the difference between $V_{ref(hi)}$ and $V_{ref(low)}$. Once the input voltage V_{in} rises above $V_{ref(hi)}$, it must fall below $V_{ref(low)}$ before it can trigger another change in the output state. If the amplitude of the noise on the signal is less than the magnitude of the hysteresis, then there will be only one transition in the output as the input signal rises through the hysteresis band.

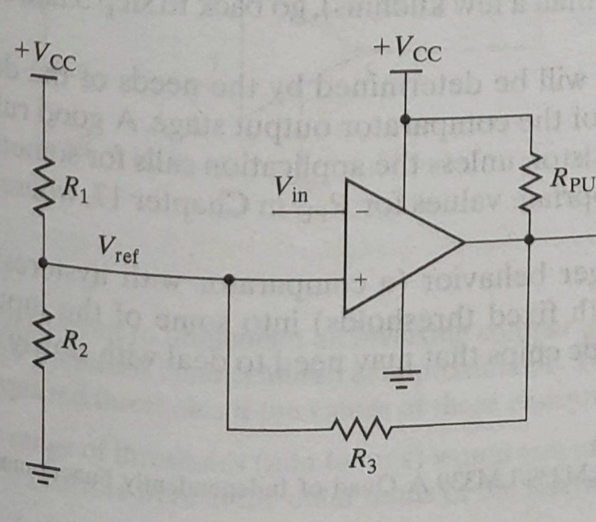


FIGURE 11.21
Inverting comparator with hysteresis.

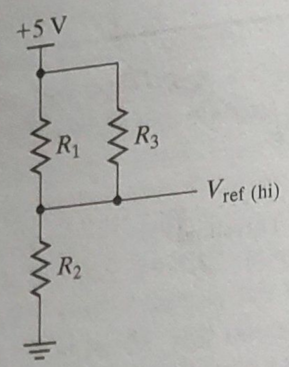


FIGURE 11.22

Schematic for V_{ref} when comparator output is high.

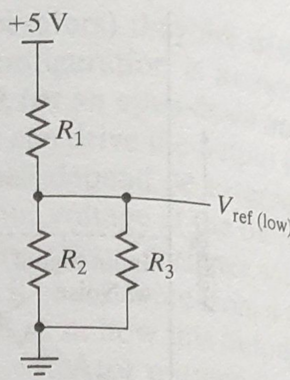


FIGURE 11.23

Schematic for V_{ref} when comparator output is low.

From a circuit analysis standpoint, we can use Eqs. (11.22) and (11.23) to calculate the upper and lower threshold voltages for an existing circuit. From a circuit design standpoint, though, we are in a bit of a quandary. We have two thresholds that we are trying to set and three resistor values (i.e., three variables) to determine. The system is under-constrained. To address this issue, we offer the following design procedure[3] for determining values for R_1, R_2, and R_3.

1. Let the upper threshold be VA_1.
2. Let the lower threshold be VA_2.
3. Calculate ΔVA as $VA_1 - VA_2$.
4. Calculate n as $n = \dfrac{\Delta VA}{VA_2}$.
5. Choose $R_3 = 1\ \text{M}\Omega$.
6. Calculate R_1 as $R_1 = nR_3$.
7. Calculate R_2 as $R_2 = \dfrac{R_1 \| R_3}{\dfrac{V_{CC}}{VA_1} - 1}$.
8. If the values for R_1 and R_2 are deemed to be too large (more than a few hundred kilohms), go back to step 5 and choose a smaller value for R_3. If the values for R_1 and R_2 are too small (less than a few kilohms), go back to step 5 and choose a larger value for R_3.

The value of R_{PU} will be determined by the needs of the device(s) connected to V_{out} and the characteristics of the comparator output stage. A good rule of thumb, however, is to use a 3.3 kΩ pull-up resistor, unless the application calls for something else. We deal with the determination of appropriate values for R_{PU} in Chapter 17, where we go into more detail on open-collector outputs.

The Schmitt trigger behavior (a comparator with hysteresis) is so useful that it has been incorporated (with fixed thresholds) into some of the inputs on microcontrollers as well as other digital logic chips that may need to deal with slowly changing inputs.

---

[3]Based on AN-74 "LM139/LM239/LM339 A Quad of Independently Functioning Comparators," National Semiconductor Corp., 1973.