

GG501

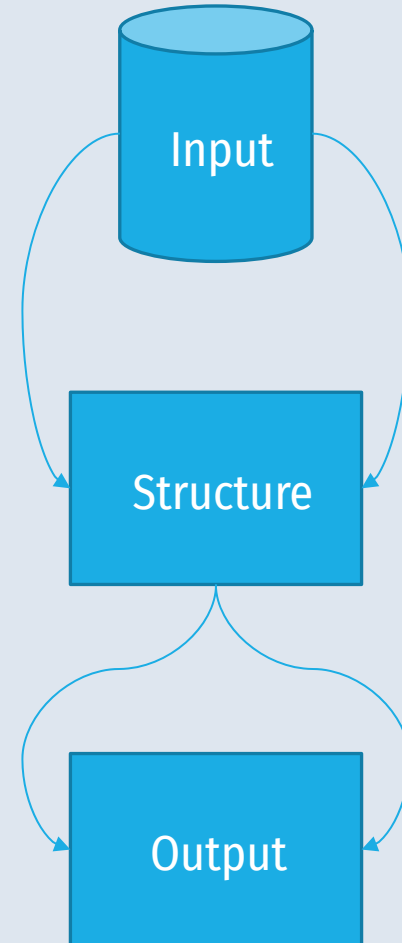
6. Model data

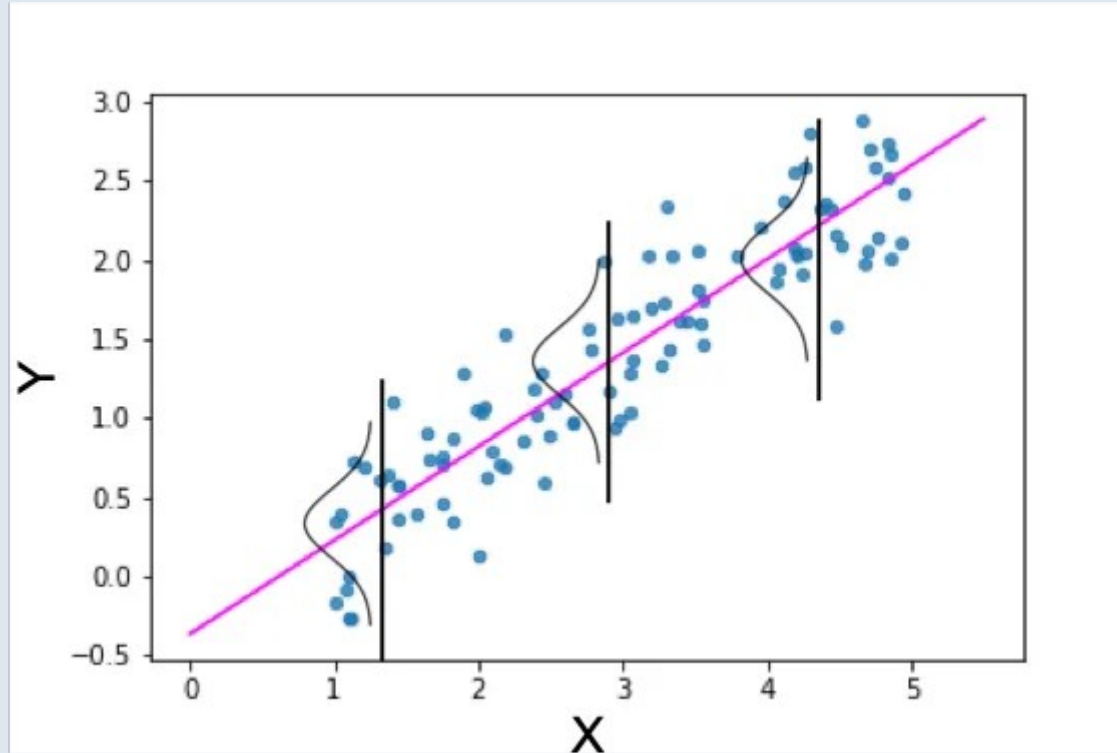
Models in R

- What do we mean by ‘model’?

Many variety of models

- Each take some input data
- Attempt to generalize about the underling data-generating-process
- Can be used for a variety of purposes
 - description
 - explanation
 - prediction





Statistical vs probability model

- Statistical model
 - Describes one or more variables & their relationship
- Probability model
 - Describes outcome of random *event*
 - Sometimes called a random variable



Describing randomness

- Random event/variable
 - Sample space - what are the possible outcomes?
- Probability model
 - Assign probability to each member of sample space
 - For a coin toss this is 0.5 for heads & 0.5 for tails
- Purely random
 - Probability model contains all the information
 - *No explanatory variables needed to account for variation*

Settings for probability models

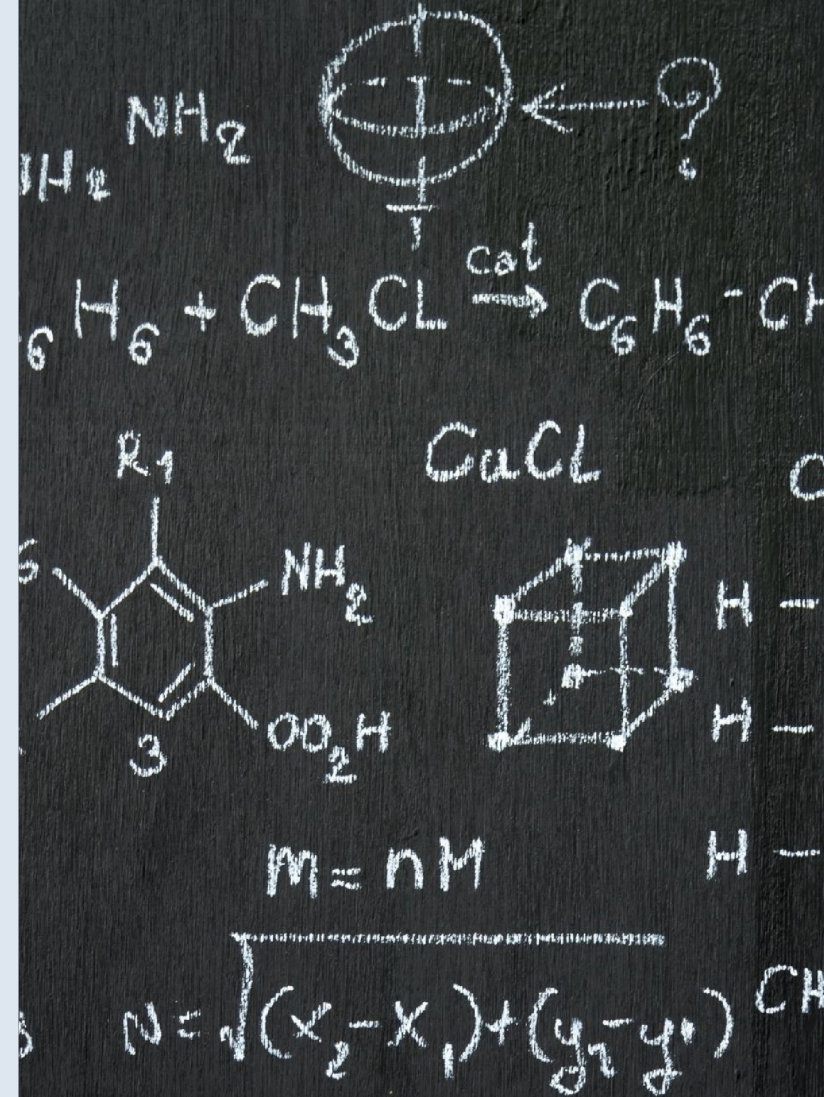
- How is the event configured/measured/represented?
- Examples
 - Number of radioactive particles per minute
 - Student test score on standardized test
 - Number of blood vessels in microscope slide
 - Number of people who support a candidate in a random sample
- Must pick form of model that fits setting
 - Combines expert knowledge & probability calculus

Discrete vs Continuous

- Helpful to distinguish between two kinds of sample spaces
 - Discrete numbers - outcome of rolling a die
 - Continuous numbers - any value in a range
- Possible to assign a probability to each outcome for discrete numbers
- Possible to assign probability to range of outcomes for continuous numbers

Probability density

- Can assign probability density to each outcome by dividing probability by extent of range
 - Usually treat this probability density as function of value of random value: $p(x)$
- Often use probabilities & probability densities in a similar way
 - For discrete sample space, assigned probabilities over all the members of space must add to 1
 - For continuous sample space, integral over assigned probability over possible values must be 1

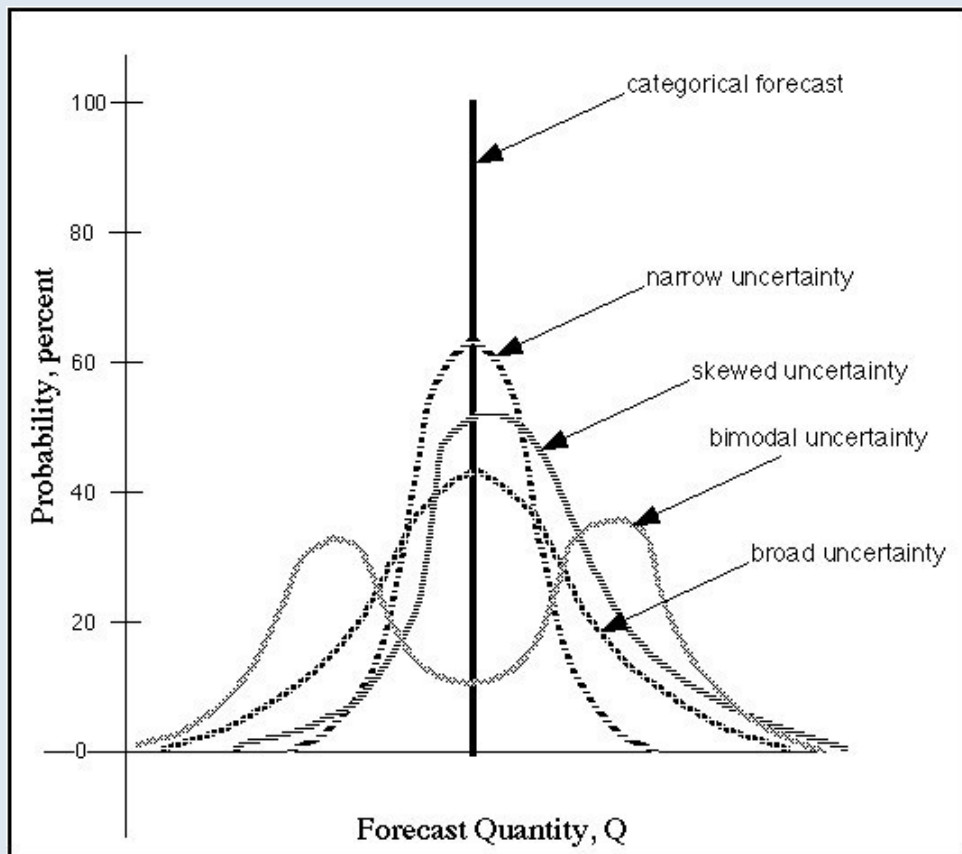


Multiple views of probability

- Frequentist view of probability
 - Describe how often outcomes occur
 - Example - 100 coin flips should lead to 50 heads
 - Based on large number of possible trials
- Subjectivist view of probability
 - Encodes modeller's assumptions/beliefs
 - Assess degree of belief
 - Probability is assigned to a hypothesis



“It will snow today ...”



http://www.hpc.ncep.noaa.gov/wwd/winter_wx.shtml

Standard probability models

- Small set of standard probability models apply to wide range of settings
- Don't need to derive them!
- Each model has parameters that need to be adjusted
 - Parameters are similar to coefficients from regression models



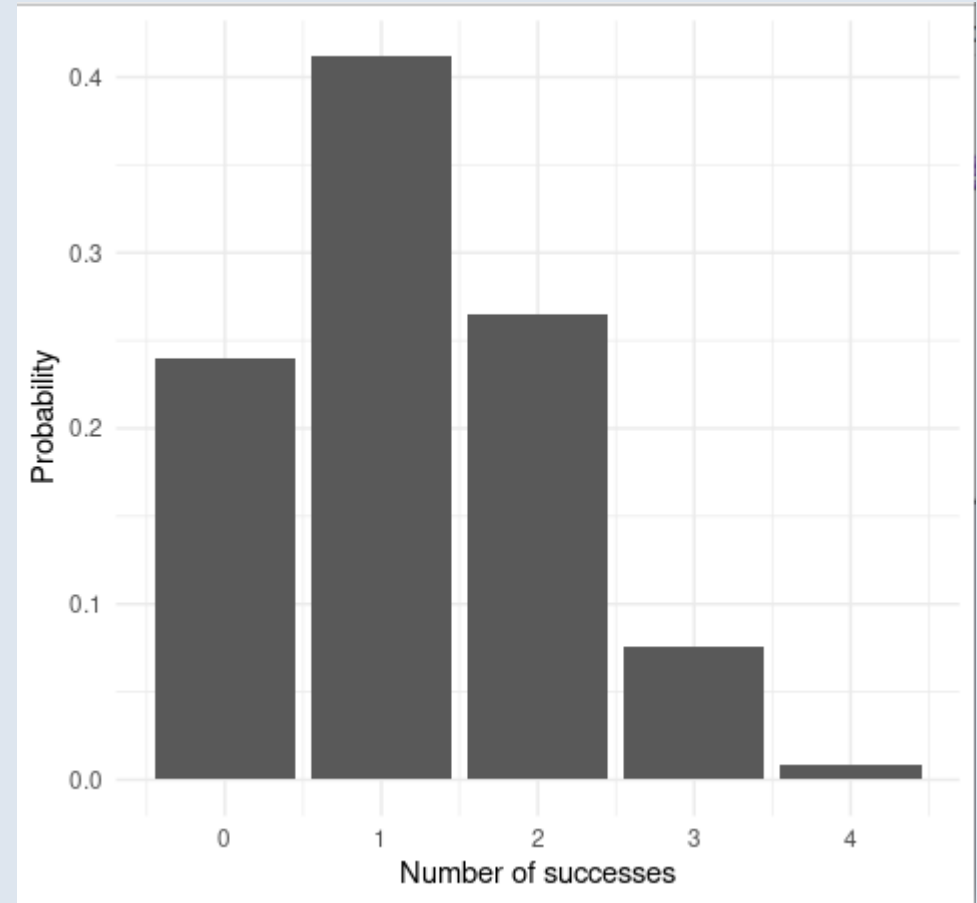
Discrete

- Equal probabilities
 - Examples - die toss, coin flip, distributions of ranks of any continuous variable
 - Parameter
 - size - how many possibilities

Discrete

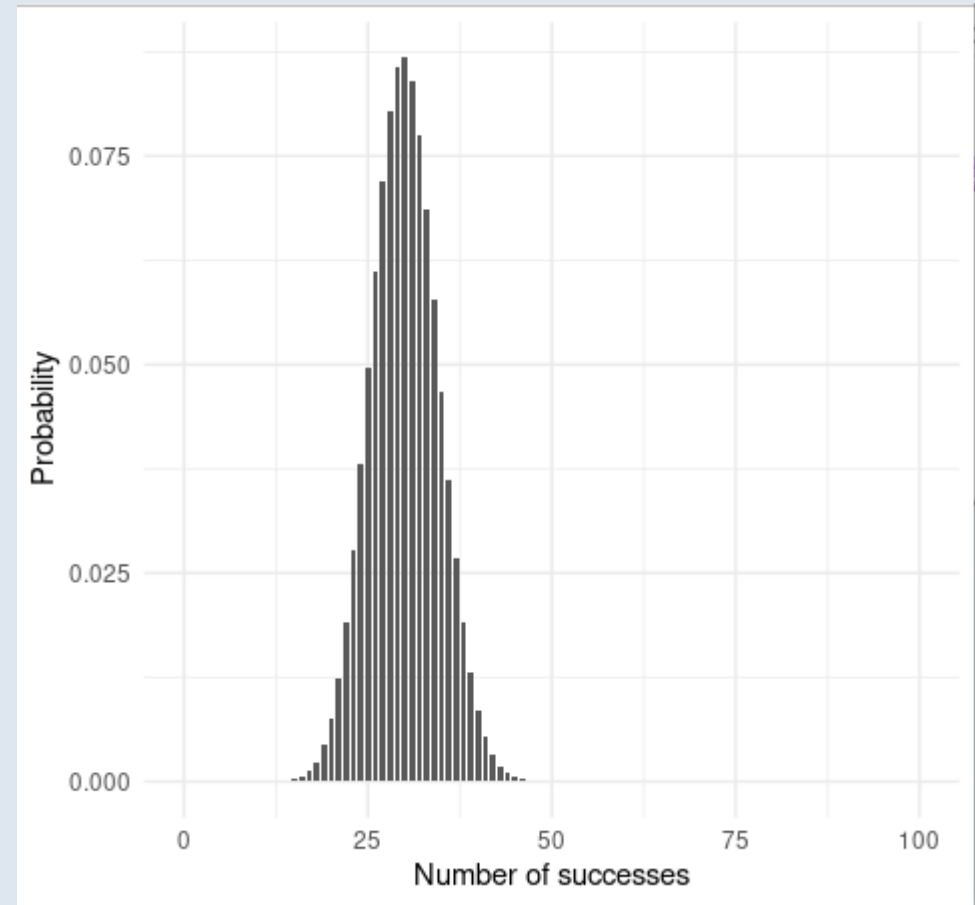
- Binomial
 - Example - trials of coin flip where outcome is count of “successes” or “heads” or “1s”
 - Parameters
 - size - number of trials
 - prob - probability of success on each trial

- Binomial
 - $n=4, p=0.30$



```
dbinom(0:4, size=4, p=0.3) %>% as_tibble() %>%  
ggplot(aes(x=0:4, y=value)) + geom_bar(stat="identity") +  
labs(x="Number of successes", y="Probability")
```

- Binomial
 - $n=100$, $p=0.30$

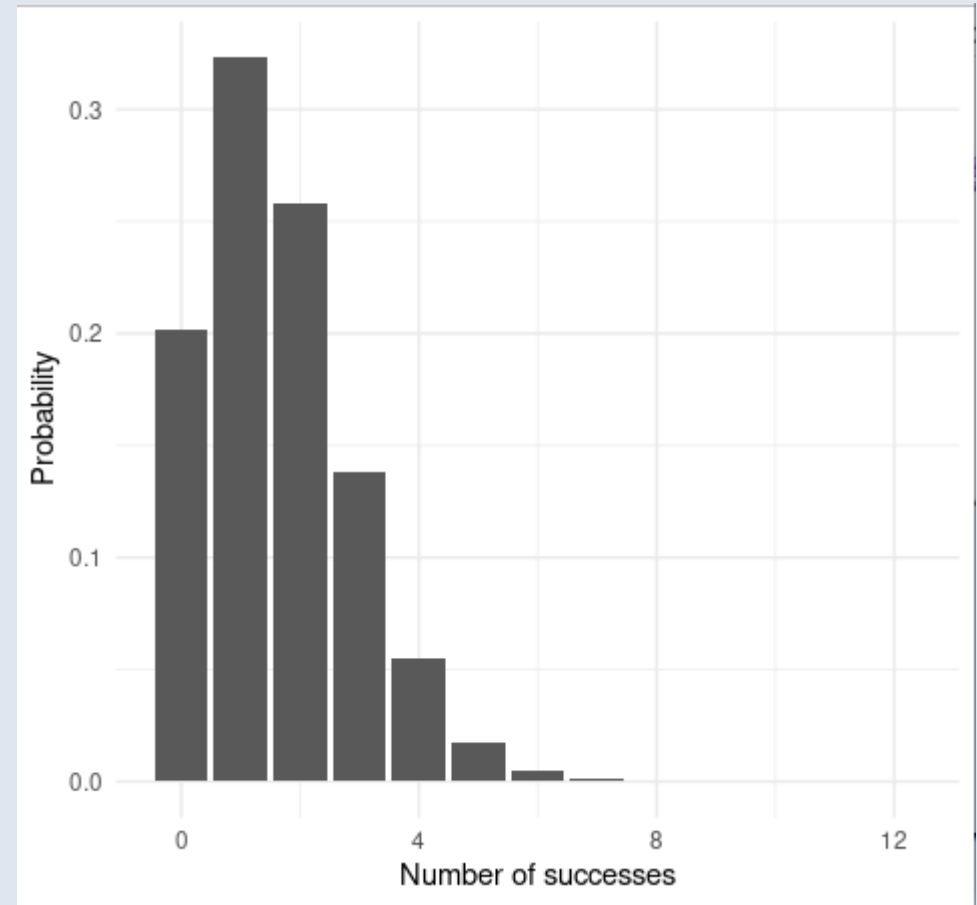


```
dbinom(0:100, size=100, p=0.3) %>% as_tibble() %>%  
ggplot(aes(x=0:100, y=value)) + geom_bar(stat="identity")  
+ labs(x="Number of successes", y="Probability")
```


Discrete

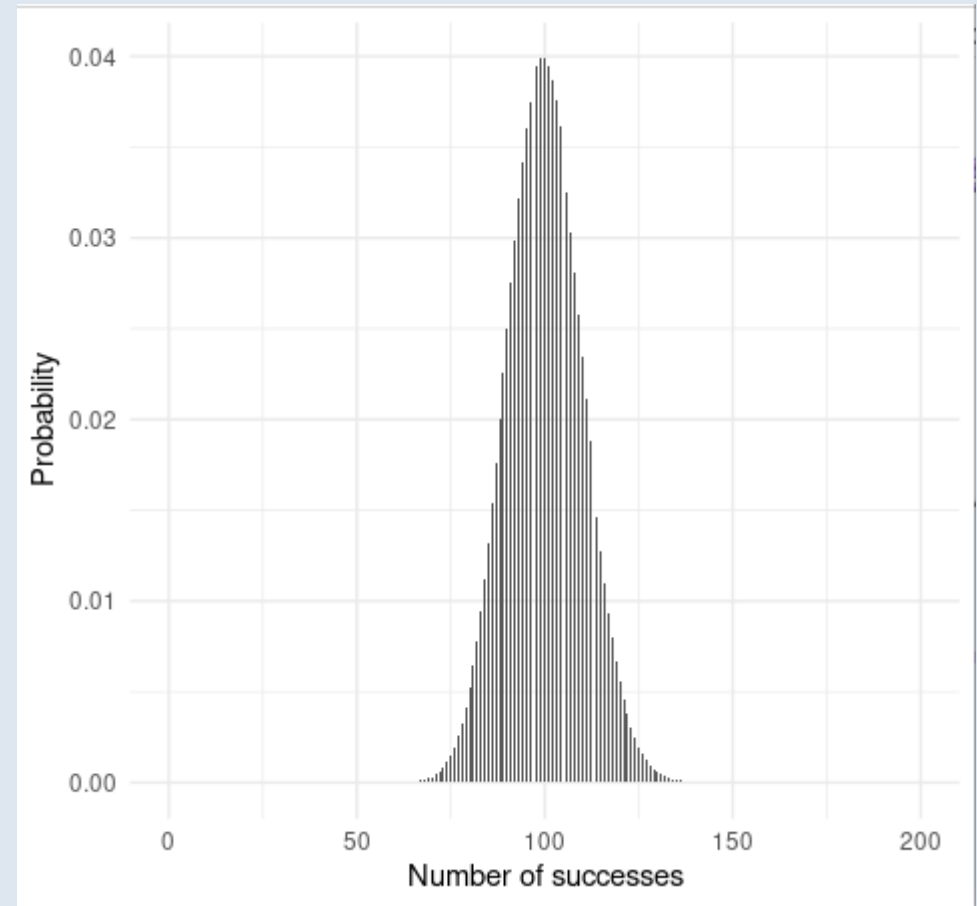
- Poisson
 - Number of events that happen in a given time
 - Example - number of cars passing by a point, number of shooting stars in minute, number of snowflakes that land on a glove in a minute
 - Parameter
 - λ - mean number of events

- Poisson
 - $\lambda(\text{rate})=0.1.6$



```
dpois(x=0:12,lambda=1.6) %>% as_tibble() %>%  
ggplot(aes(x=0:12, y=value)) + geom_bar(stat="identity")  
+ labs(x="Number of successes", y="Probability")
```

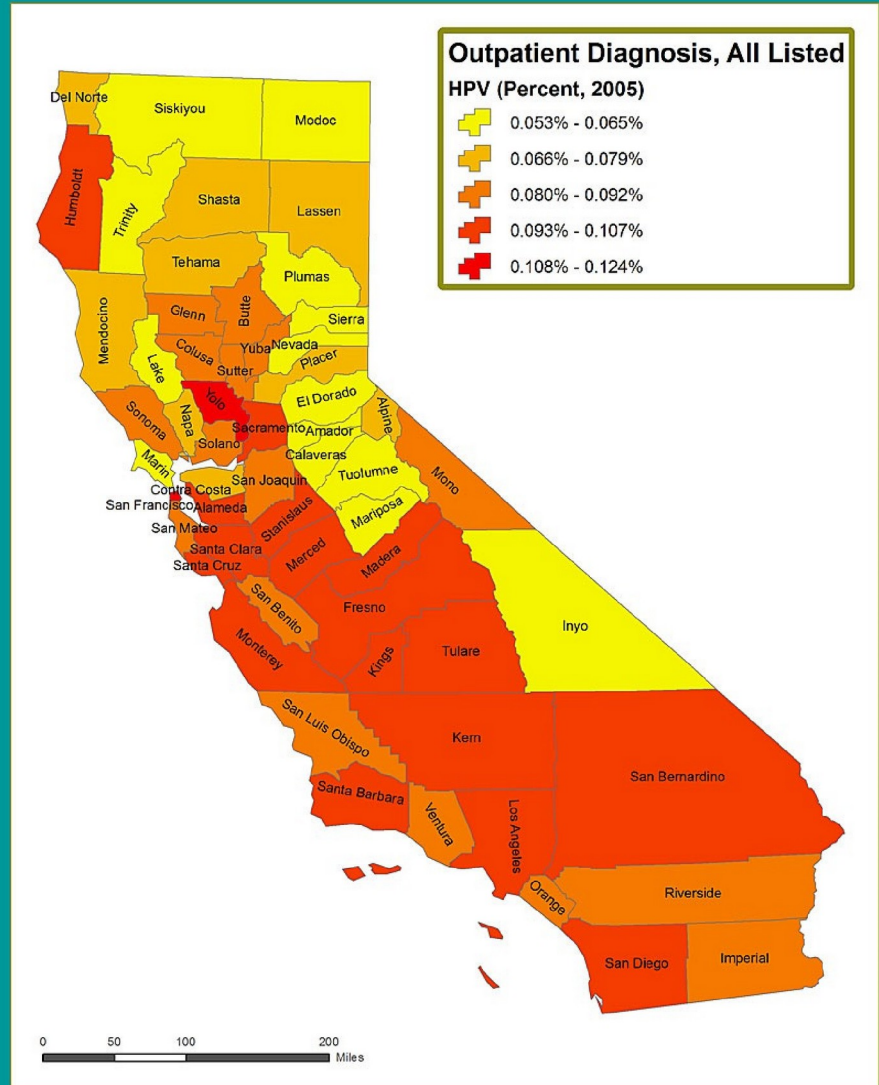
- Poisson
 - $\lambda(\text{rate})=0.1.6$



```
dpois(x=0:200,lambda=100) %>% as_tibble() %>%  
ggplot(aes(x=0:200, y=value)) + geom_bar(stat="identity")  
+ labs(x="Number of successes", y="Probability")
```

Map of California human papilloma virus (HPV) cases by outpatient diagnosis in each county

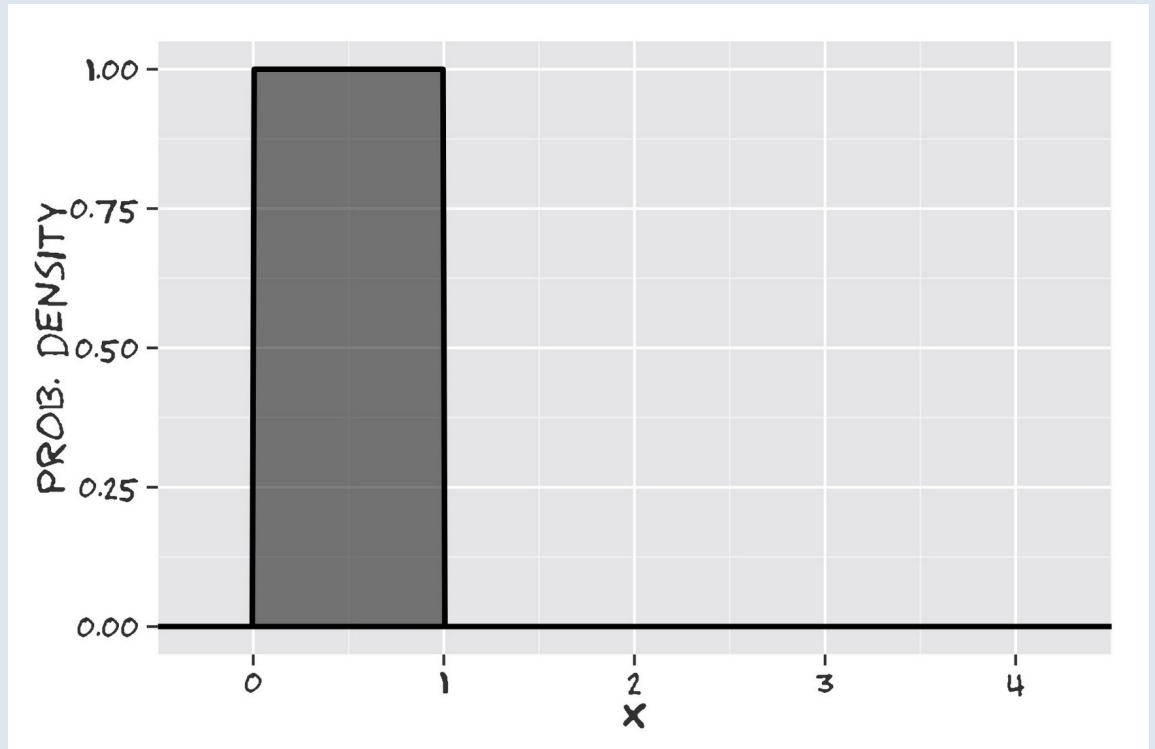
California Case Estimates by Count



Continuous

- Uniform
 - Parameters: Max and min
 - Models: spinners (angles in 2-d, but not higher), p-values under the Null Hypothesis

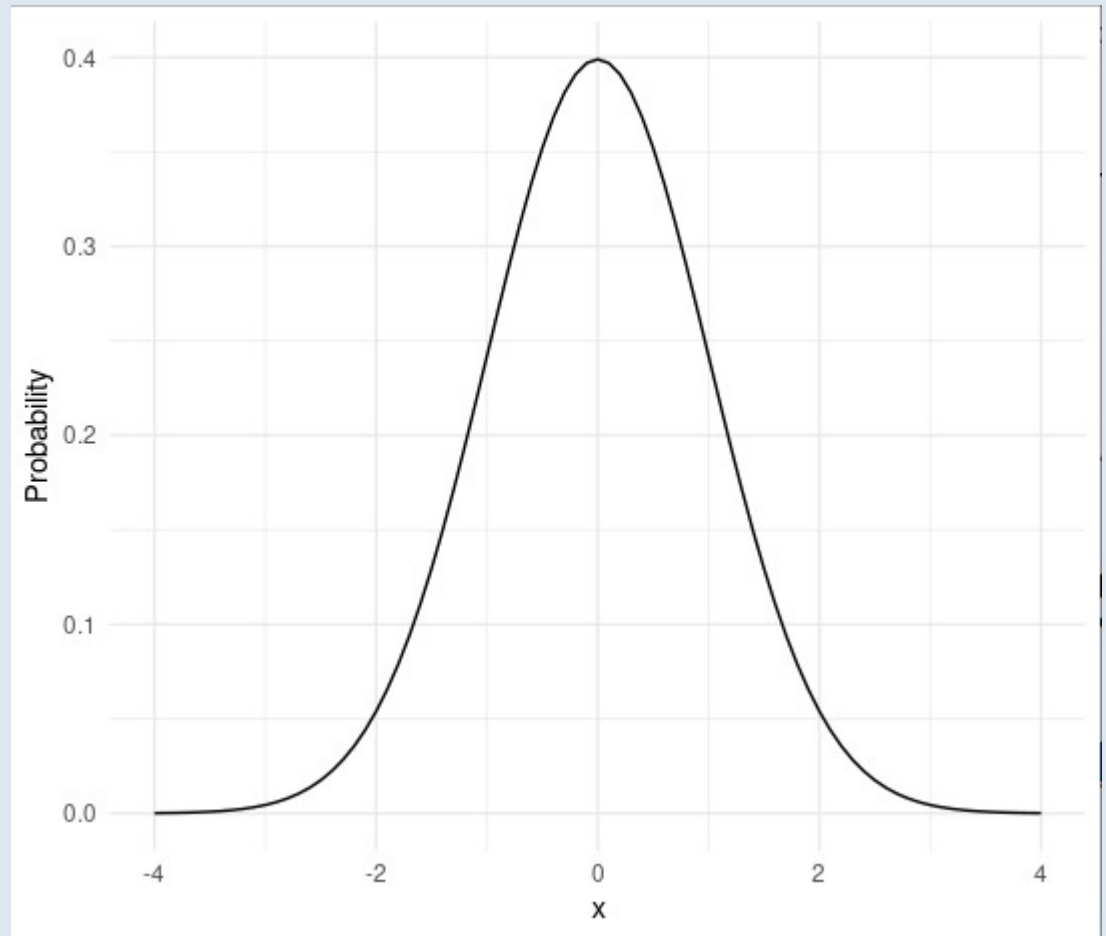
- Uniform
 - min=0, max=1



Continuous

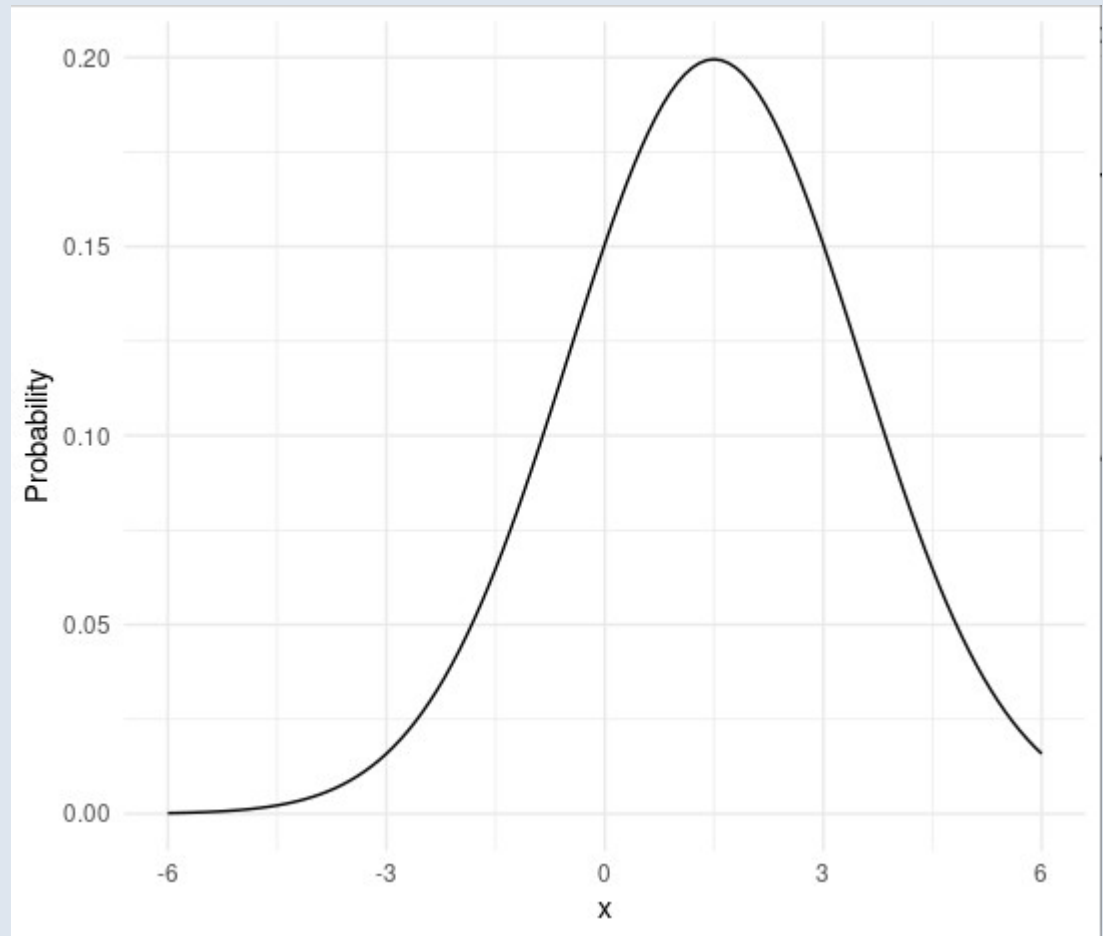
- Normal (gaussian)
 - Parameters: mean and sd
 - Models: your general purpose model

- Normal
 - mean=0, sd=1



```
dnorm(x=seq(-4,4,0.1), mean=0, sd=1) %>% as_tibble() %>%  
ggplot(aes(x=seq(-4,4,0.1), y=value)) + geom_line() +  
labs(x="x", y="Probability")
```


- Normal
 - mean=1.5, sd=2



```
dnorm(x=seq(-6,6,0.1), mean=0, sd=1) %>% as_tibble() %>%  
ggplot(aes(x=seq(-6,6,0.1), y=value)) + geom_line() +  
labs(x="x", y="Probability")
```

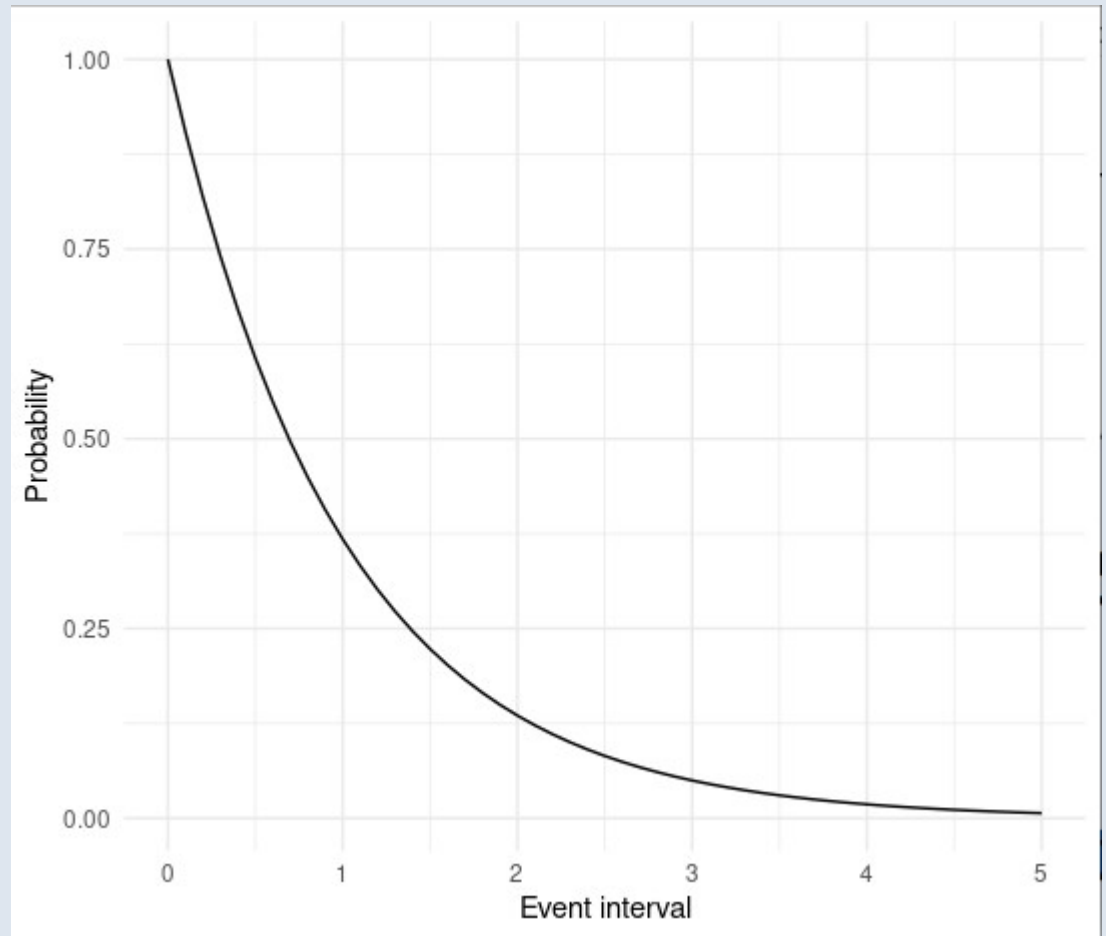
What can we do with probability models?

- Percentiles: what is the range of values within some probability range?
 - e.g., 90th percentile: value of the random variable such that 90% of the time values will be equal or smaller
- Quantiles: What is the percentile of a given outcome?
 - e.g., how unusual is this observation given the underlying probability model?

Continuous

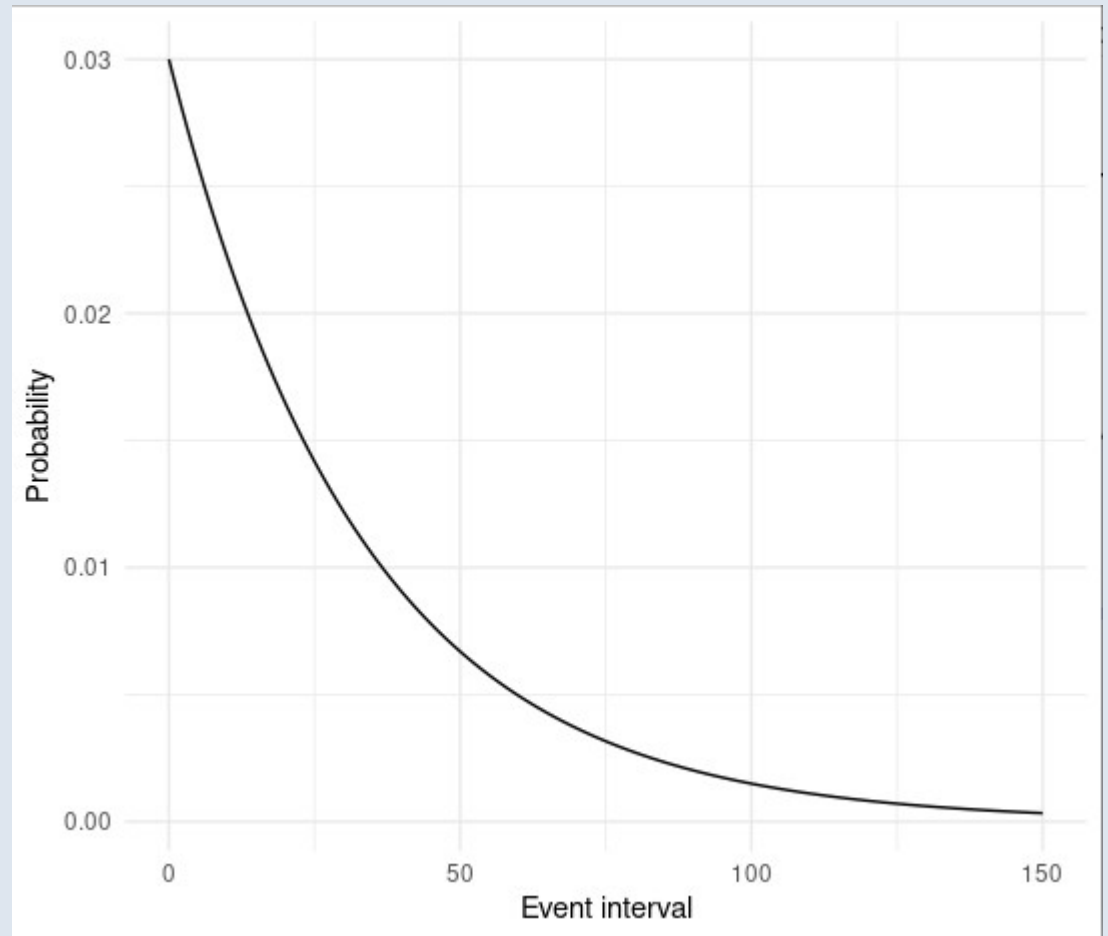
- Exponential
 - Times between random events (earthquakes, storms, floods)
 - Parameters: rate (mean time is $1/\text{rate}$)

- Exponential
 - rate=1



```
dexp(seq(0,5,0.1), rate=1)%>% as_tibble() %>%  
ggplot(aes(x=seq(0,5,0.1), y=value)) + geom_line() +  
labs(x="Event interval", y="Probability")
```

- Exponential
 - rate=1

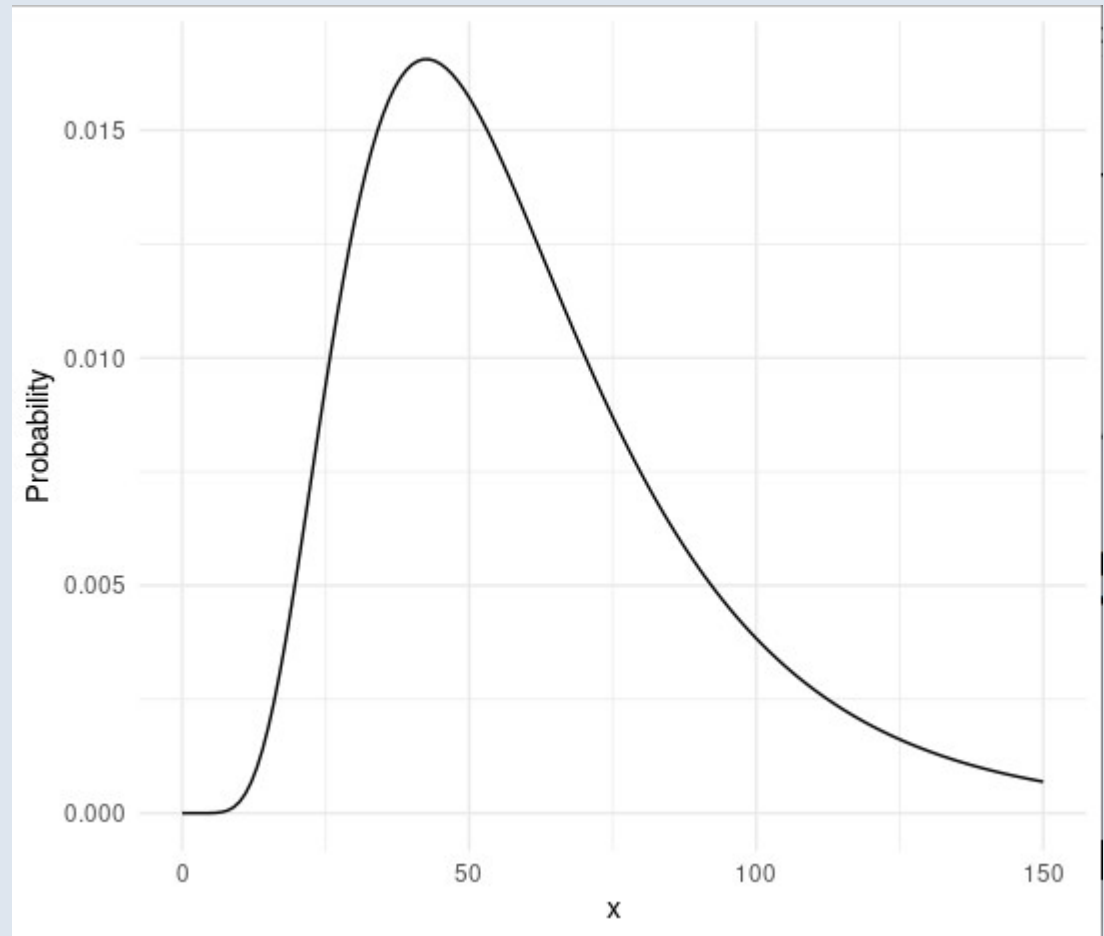


```
dexp(seq(0,150,1), rate=0.03)%>% as_tibble() %>%  
ggplot(aes(x=seq(0,150,1), y=value)) + geom_line() +  
labs(x="Event interval", y="Probability")
```

Continuous

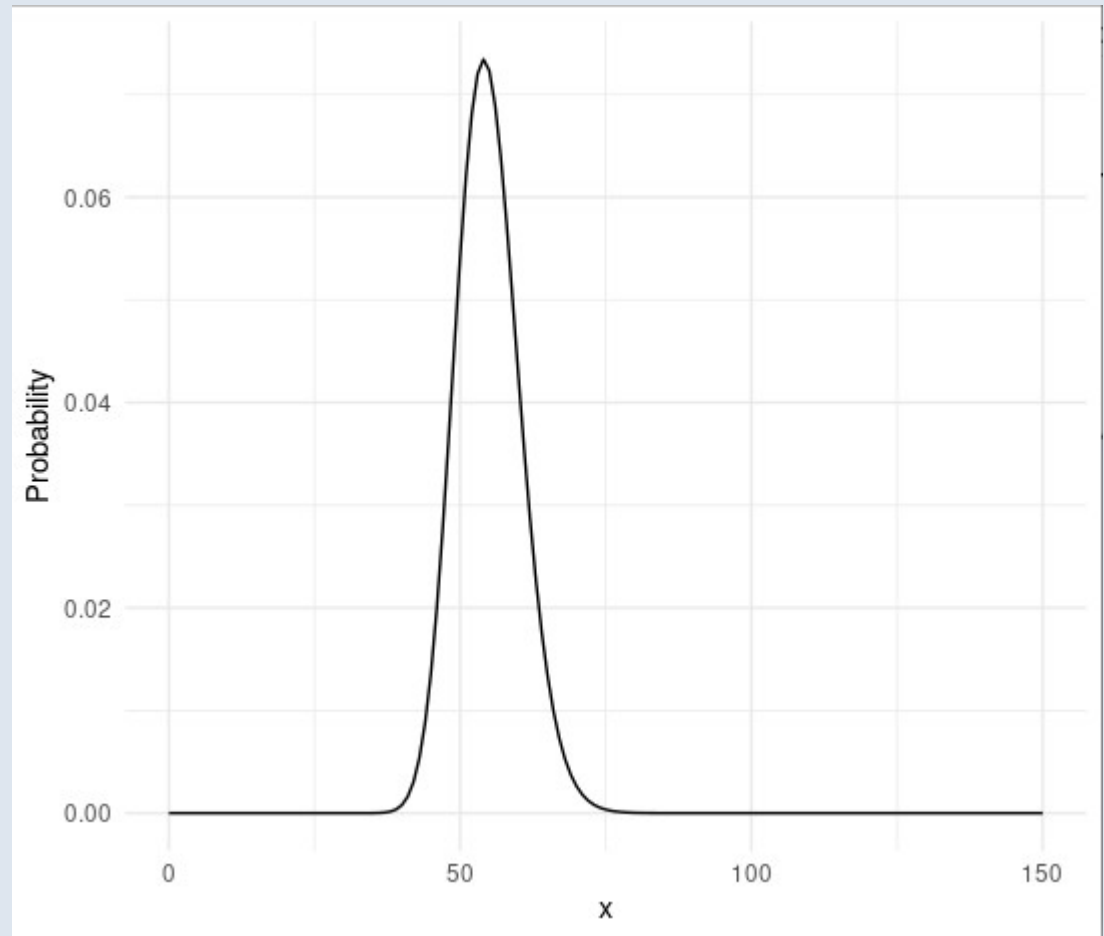
- Lognormal
 - Models: useful when skew is important
 - Parameters: mean, sd (of the log of the values)

- lognormal
 - meanlog=4, sdlog=0.5



```
dlnorm(seq(0,150,1), meanlog=4, sdlog=0.5) %>%  
as_tibble() %>% ggplot(aes(x=seq(0,150,1), y=value)) +  
geom_line() + labs(x="x", y="Probability")
```

- lognormal
 - meanlog=4, sdlog=0.1



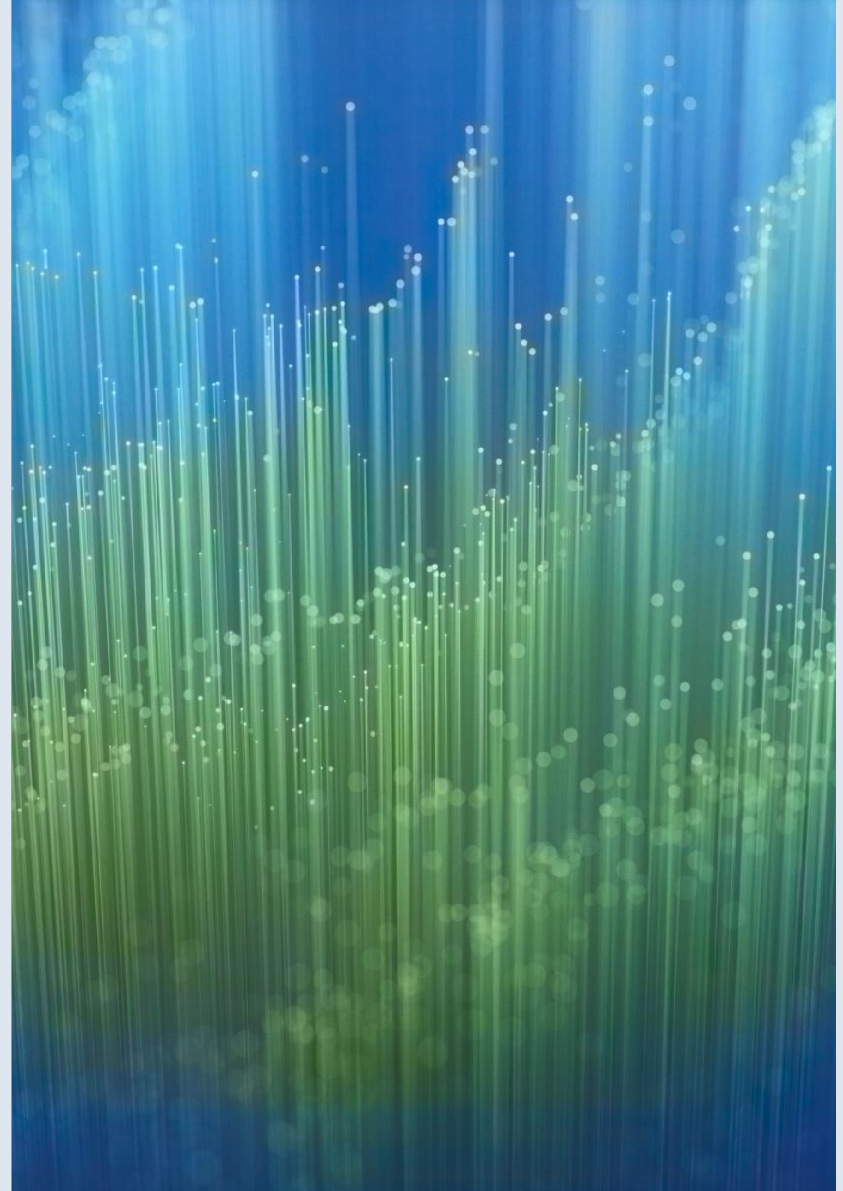
```
dlnorm(seq(0,150,1), meanlog=4, sdlog=0.1) %>%  
as_tibble() %>% ggplot(aes(x=seq(0,150,1), y=value)) +  
geom_line() + labs(x="x", y="Probability")
```


Why the normal distribution?

- Many chance phenomena are at least approximately described by a normal probability density function
- Example
 - Collect 1000 snowflakes & weigh them, would find distribution of weights accurately described by a normal curve
 - Measure the strength of bones in wildebeests; likely to find they are normally distributed

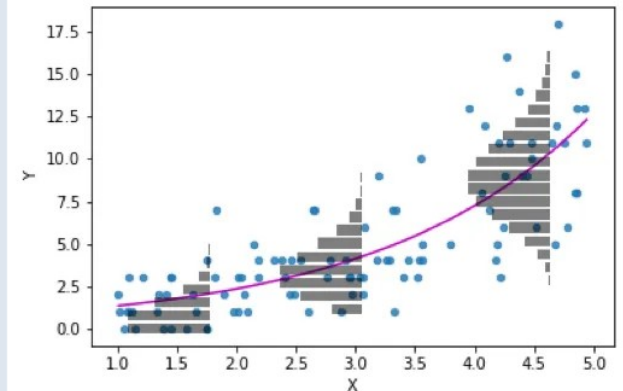
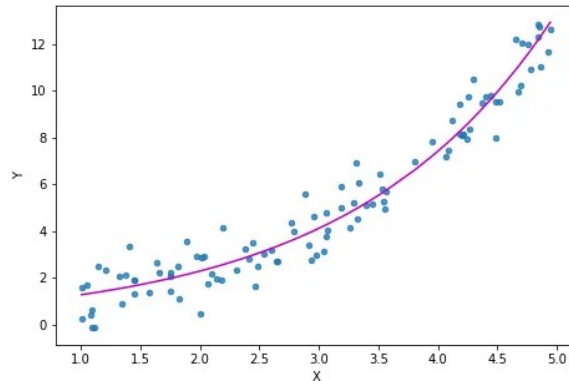
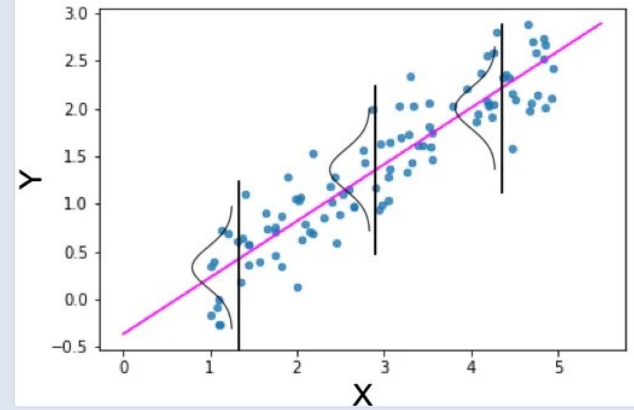
Normal nature

- Random biological or physical processes affected by *large* number of random processes with *individually* small effects...
- Sum of all these random components creates random variable that converges on normal distribution
- Regardless of the underlying distribution of processes causing the small effects!



What does this have to do with modelling?

- The relationships in models are based on sample data – therefore have randomness
- They could change if repeated
- Need to use randomness to evaluate precision of models
- We can use visualization to better understand the characteristics of a given model



Models are important

- Each take some data
- Attempt to generalize about the underlying data-generating-process
- Visualization of models is key to understanding what you can do with information embedded within them
 - regression trees
 - Kaplan-Meier plots, ROCs, marginal/conditional effects