모바일 센서공학
벡터

내용

- 벡터와 스칼라 **(Vector and Scalars)**
- 벡터의 합 -기하학적인 방법 **(Adding Vectors Geometrically)**
- 벡터의 성분 **(Components of Vectors)**
- 단위 벡터 **(Unit Vectors)**
- 벡터의 합 -성분법
- 벡터와 물리 법칙
- 벡터의 곱 **(Multiplying Vectors)**

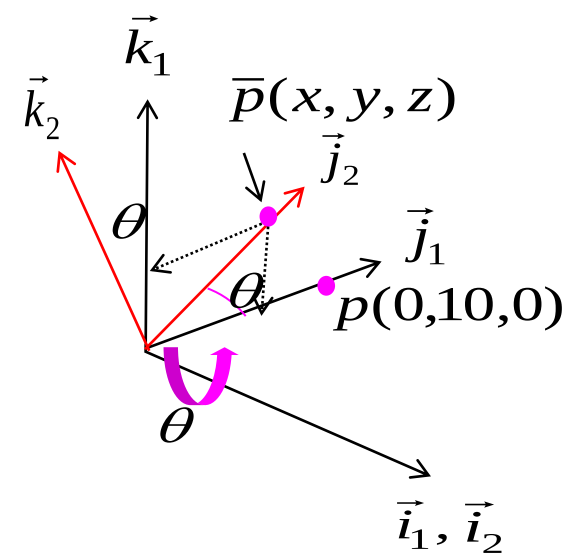
Ex. 회전 행렬(Rotation Matrix)

- $P(0,10,0) \Rightarrow \overline{p}(x,y,z)$

1. $\vec{i}$ 축을 중심으로 45°만큼 회전하였을 때 위치좌표

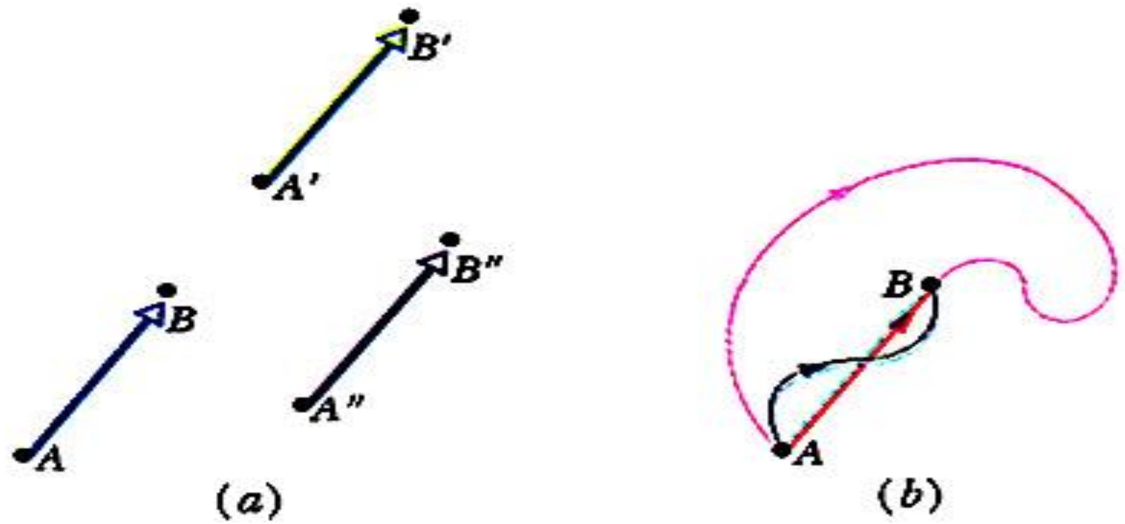
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = R(\vec{i},45°)\begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ 0 & \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}\begin{bmatrix} 0 \\ 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \frac{10}{\sqrt{2}} \\ \frac{10}{\sqrt{2}} \end{bmatrix}$$



벡터, 스칼라

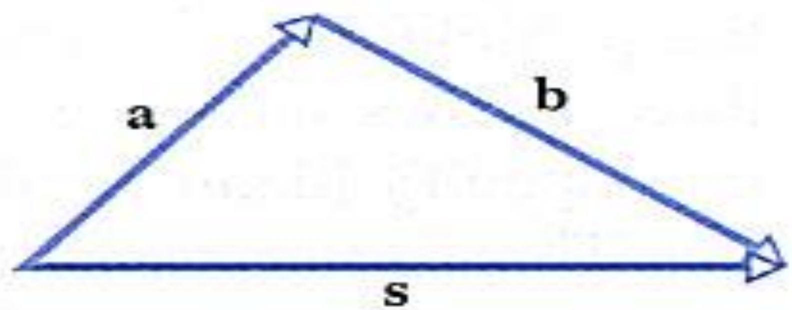
- 벡터 : 크기 + 방향 (예: 변위, 힘, 운동량 등)



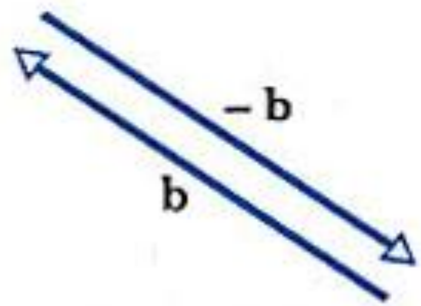
- 스칼라 : 크기 (예: 시간, 에너지 등)

벡터 합

- 벡터의 합 -기하학적인 방법 $\vec{s} = \vec{a} + \vec{b}$

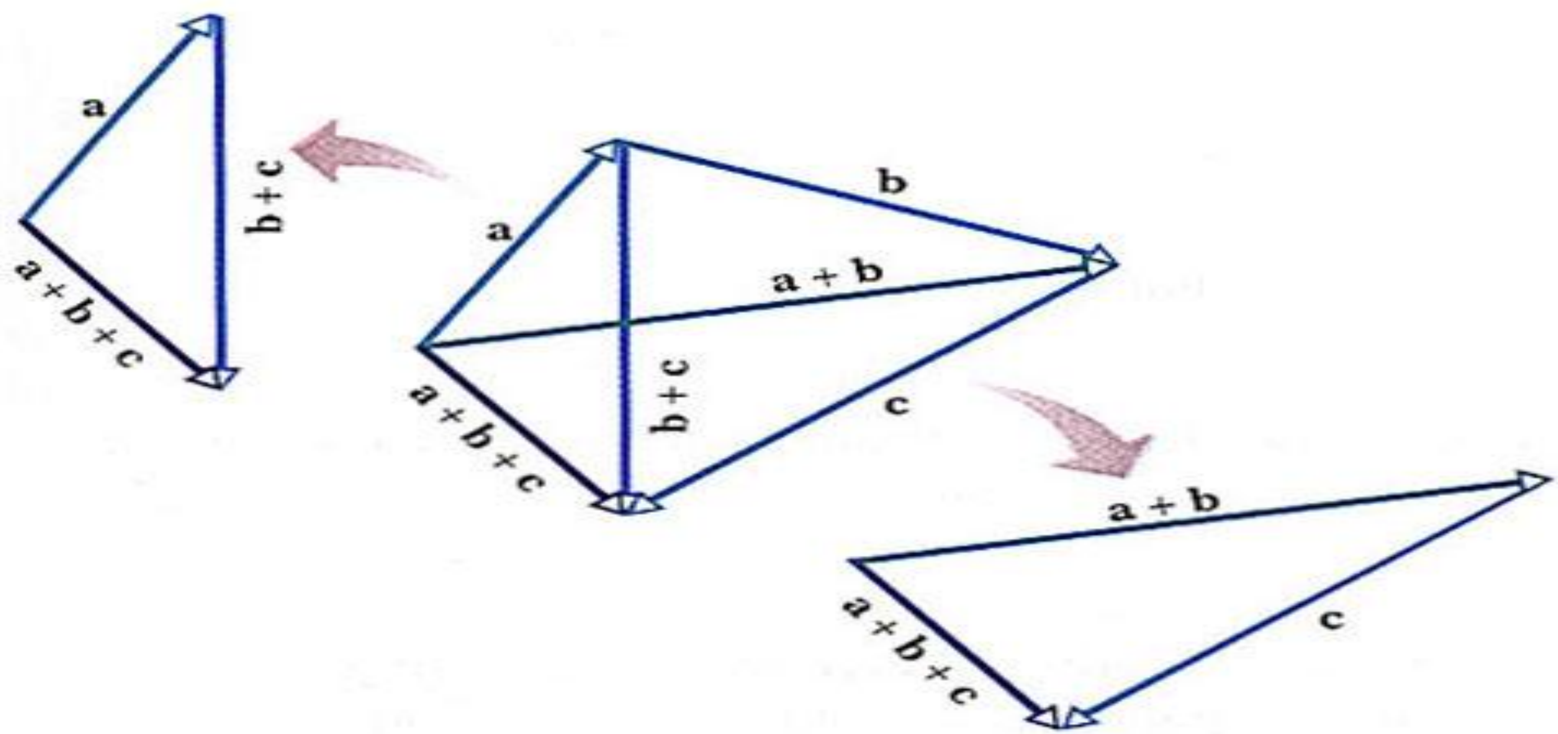


- 벡터의 차 (subtraction)



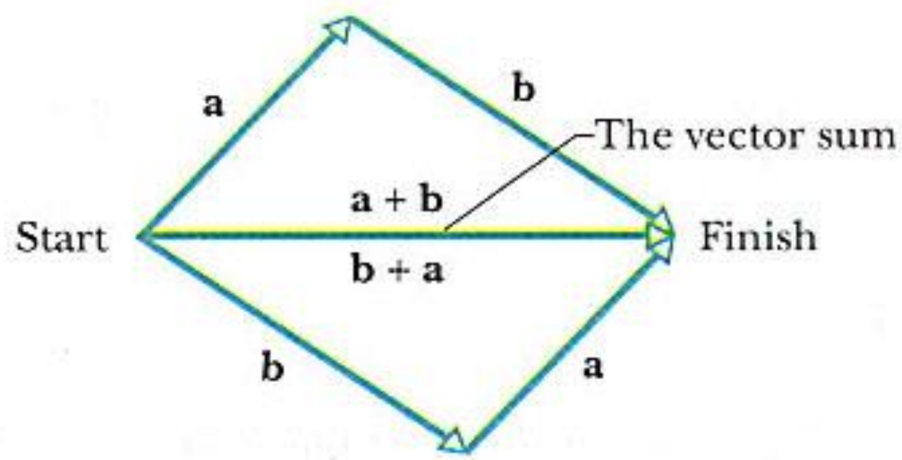
ii) 결합 법칙 (associative law) :

$$(\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$$



i) 교환 법칙 (commutative law) :

$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$



벡터의 곱 (Multiplying Vectors)

- 스칼라에 의한 곱 : $c\,\vec{a}$

- 스칼라 곱 (Scalar Product) : 내적

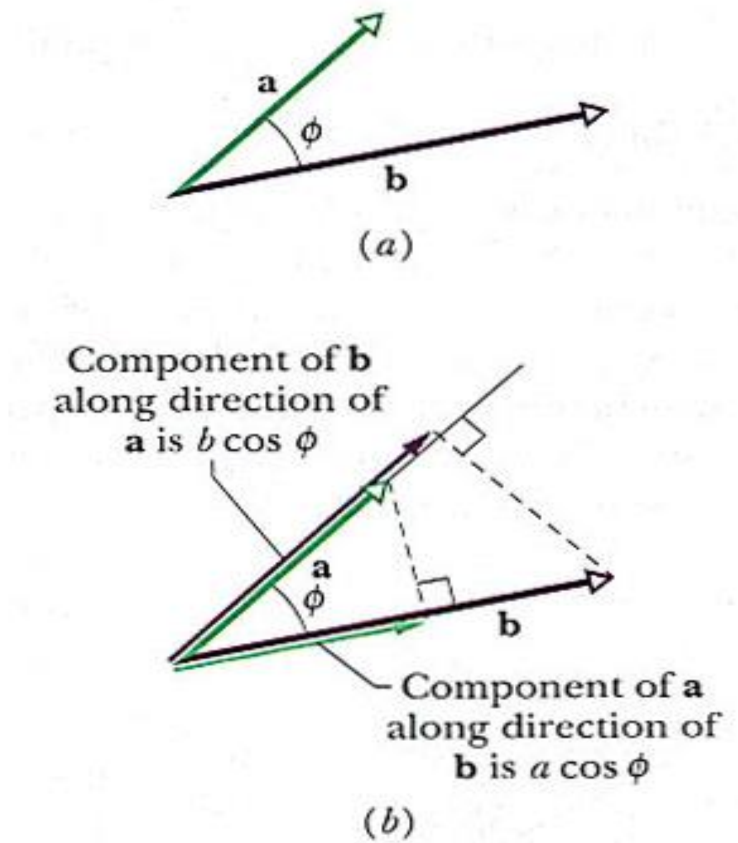
$$c = \vec{a} \cdot \vec{b}$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\phi$$

$$= (|\vec{a}|\cos\phi)\,(|\vec{b}|)$$

$$= (|\vec{a}|)\,(|\vec{b}|\cos\phi)$$

$$= \vec{b} \cdot \vec{a}$$

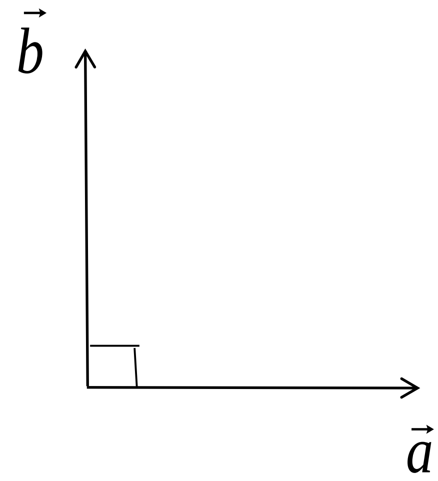


벡터의 곱 (Multiplying Vectors)

- 수직 벡터의 조건 : $\phi = 90^\circ$

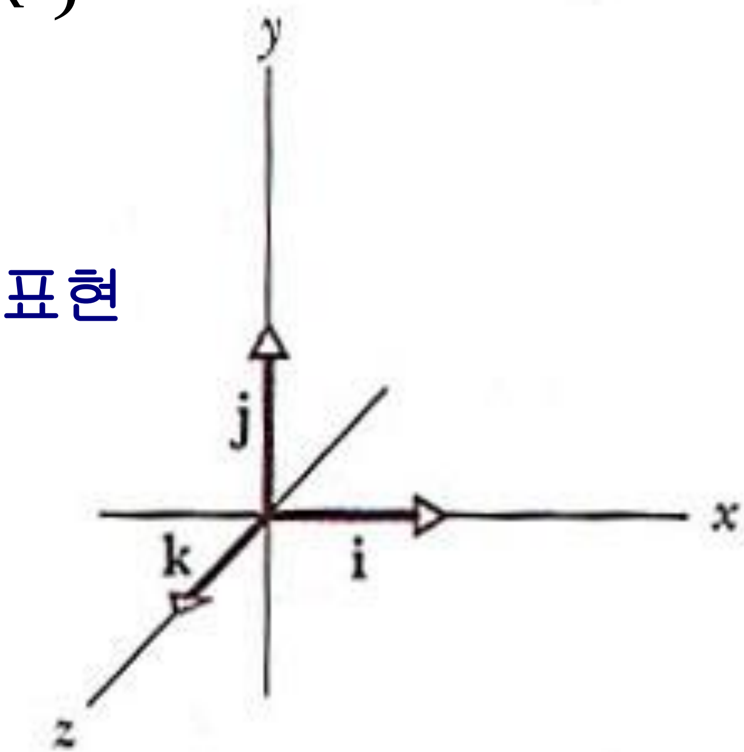
$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\phi \quad \Rightarrow \quad \boxed{\vec{a} \cdot \vec{b} = 0}$$

$$\vec{a} \perp \vec{b}$$



단위 벡터 (Unit Vector)

- **단위 벡터 (Unit Vector) :** $(\vec{i}, \vec{j}, \vec{k})$
  - 크기 : 1
  - 방향 : x, y, z 축 방향



- **단위 벡터 (Unit Vector) 의 수학적 표현**

$$\vec{i} = 1\vec{i} + 0\vec{j} + 0\vec{k}$$

$$\vec{j} = 0\vec{i} + 1\vec{j} + 0\vec{k}$$

$$\vec{k} = 0\vec{i} + 0\vec{j} + 1\vec{k}$$

- **단위 벡터 (Unit Vector) 간의 직교성**

$$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$$

단위벡터, 성분/ 성분 벡터

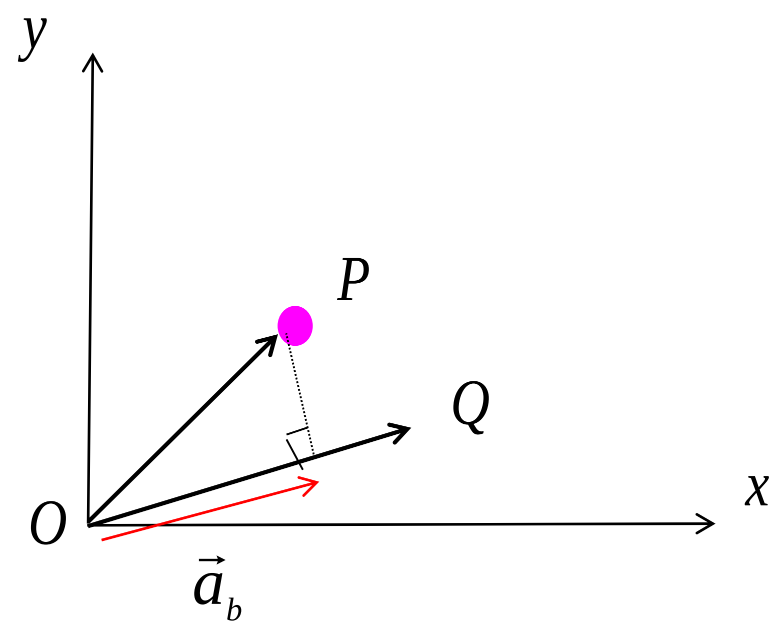
- **벡터 b 방향의 단위 벡터 (Unit Vector)**

$$\vec{u} = \frac{1}{|\vec{b}|}\vec{b}$$

- **성분**

  - 벡터 a 의 벡터 b 방향 성분

$$\vec{a}\cdot\vec{u} = |\vec{a}|\left|\frac{1}{|b|}\vec{b}\right|\cos\phi = |\vec{a}|\cos\phi$$



- **성분 벡터 (Component Vector)** $\vec{a}_b$

$$\vec{a}_b = (\vec{a}\cdot\vec{u})\vec{u} = (|\vec{a}|\cos\phi)\vec{u}$$

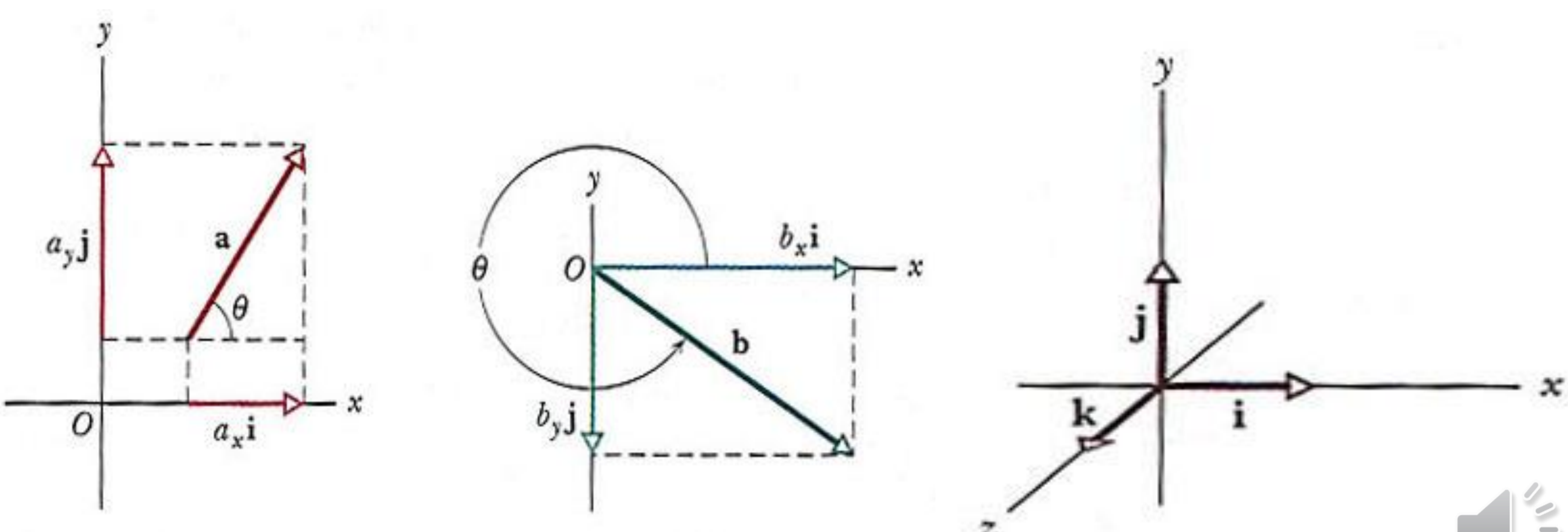
벡터의 수학적 정의

- **Vector 의 수학적 표현 : 벡터의 수치화**

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} \qquad \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$$

- **Vector 의 단위벡터 성분 : 단위벡터 방향 성분**

$$a_x = \vec{a} \cdot \vec{i} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \cdot \vec{i} = a_x \vec{i} \cdot \vec{i} + a_y \vec{j} \cdot \vec{i} + a_z \vec{k} \cdot \vec{i}$$



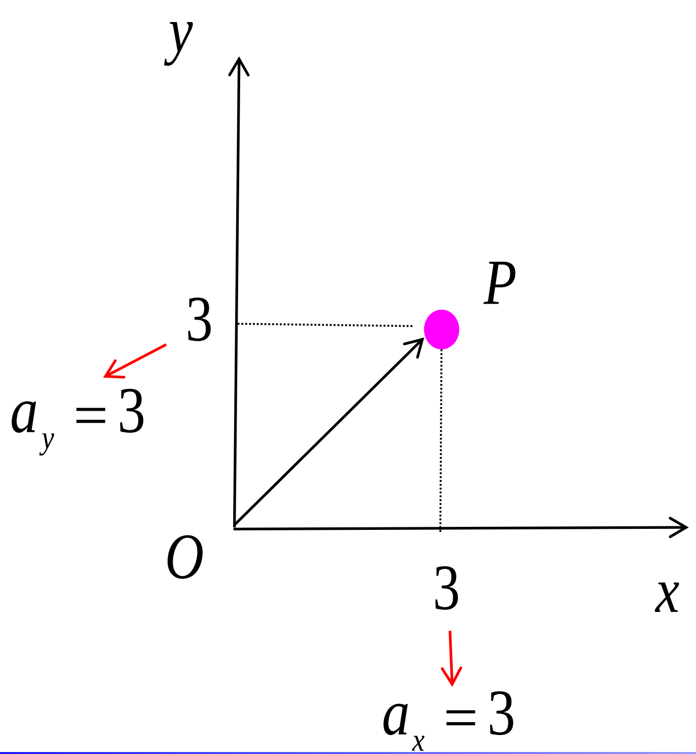
(a) (b)

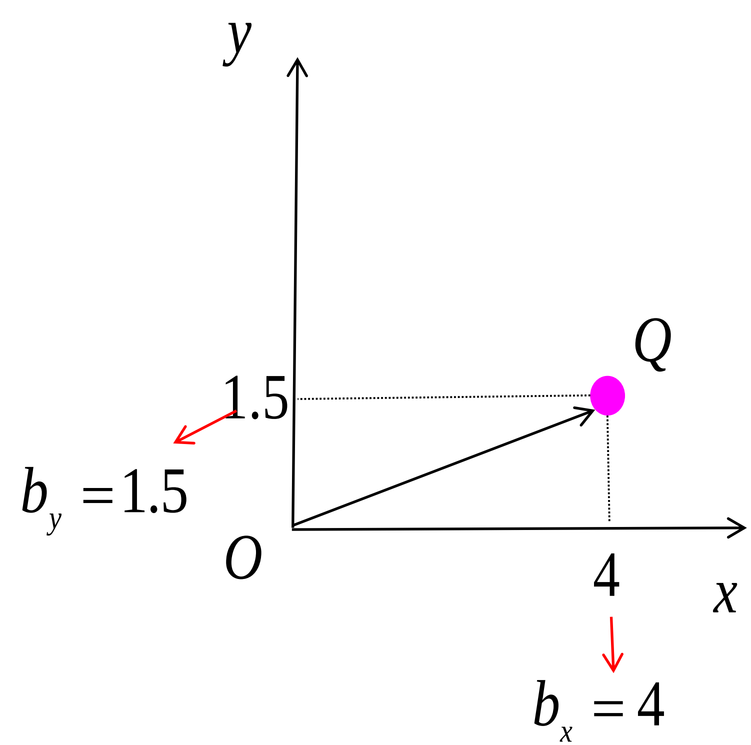
벡터의 수학적 정의

- **Vector 의 수학적 표현 : 벡터의 수치화**

$$\overrightarrow{OP} = \vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} = 3\vec{i} + 3\vec{j} + 0\vec{k}$$

$$\overrightarrow{OQ} = \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k} = 4\vec{i} + 1.5\vec{j} + 0\vec{k}$$

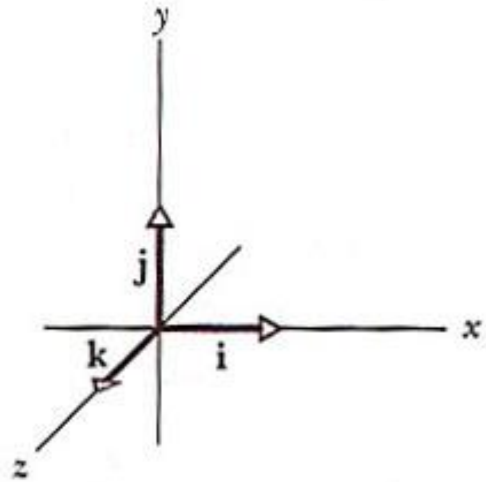




스칼라 곱의 계산

$$\vec{i}\cdot\vec{i}=\vec{j}\cdot\vec{j}=\vec{k}\cdot\vec{k}=1$$

$$\vec{i}\cdot\vec{j}=\vec{j}\cdot\vec{k}=\vec{k}\cdot\vec{i}=0$$



$$\vec{a}\cdot\vec{b}=(a_x\vec{i}+a_y\vec{j}+a_z\vec{k})\bullet(b_x\vec{i}+b_y\vec{j}+b_z\vec{k})$$

$$\begin{aligned}
&=a_x\vec{i}\cdot b_x\vec{i}+a_x\vec{i}\cdot b_y\vec{j}+a_x\vec{i}\cdot b_z\vec{k}\\
&\quad+a_y\vec{j}\cdot b_x\vec{i}+a_y\vec{j}\cdot b_y\vec{j}+a_y\vec{j}\cdot b_z\vec{k}\\
&\quad\quad+a_y\vec{j}\cdot b_x\vec{i}+a_y\vec{j}\cdot b_y\vec{j}+a_y\vec{j}\cdot b_z\vec{k}\\
&\quad\quad\quad+a_z\vec{k}\cdot b_x\vec{i}+a_z\vec{k}\cdot b_y\vec{j}+a_z\vec{k}\cdot b_z\vec{k}\\
&=a_xb_x+a_yb_y+a_zb_z
\end{aligned}$$

$$\boxed{\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z}$$

벡터의 합-성분법

$$\vec{s} = \vec{a} + \vec{b}$$

$$s_x \vec{i} + s_y \vec{j} = (a_x \vec{i} + a_y \vec{j}) + (b_x \vec{i} + b_y \vec{j})$$

$$= (a_x + b_x)\vec{i} + (a_y + b_y)\vec{j}$$

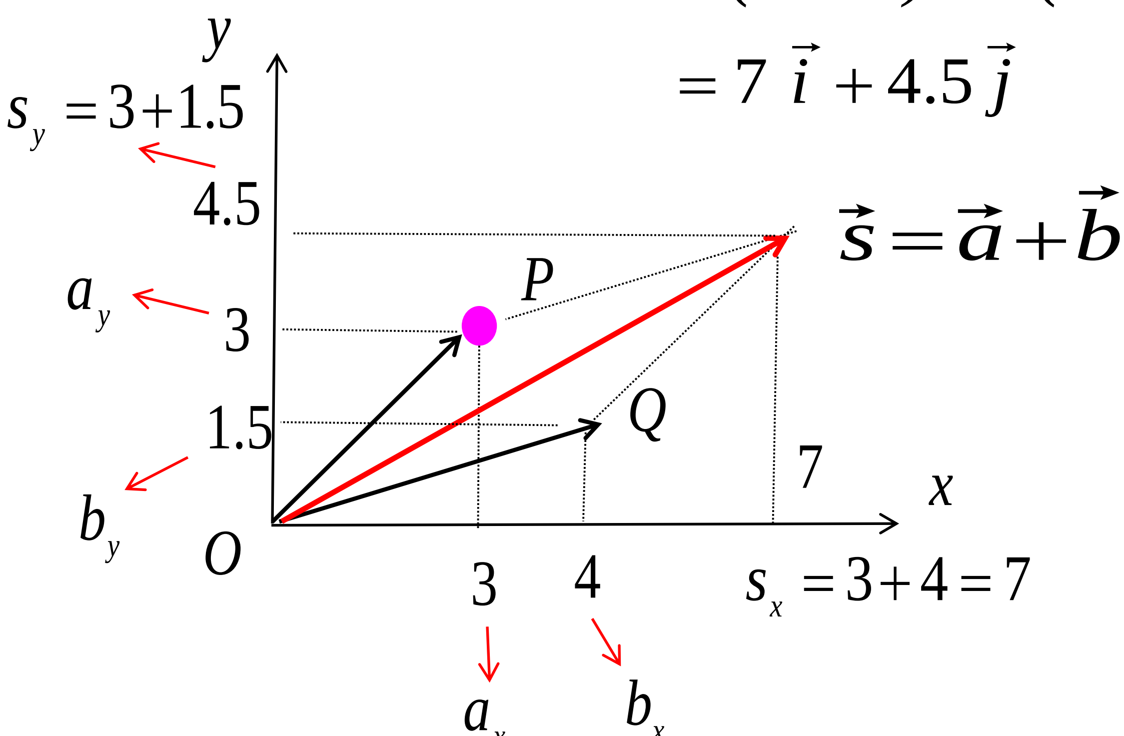
$$s_x = a_x + b_x$$

$$s_y = a_y + b_y$$

벡터의 합-성분법

$$\vec{s} = \vec{a} + \vec{b}$$

$$\vec{s} = s_x\vec{i} + s_y\vec{j} = (a_x\vec{i} + a_y\vec{j}) + (b_x\vec{i} + b_y\vec{j})$$
$$= (3+4)\vec{i} + (3+1.5)\vec{j}$$
$$= 7\vec{i} + 4.5\vec{j}$$



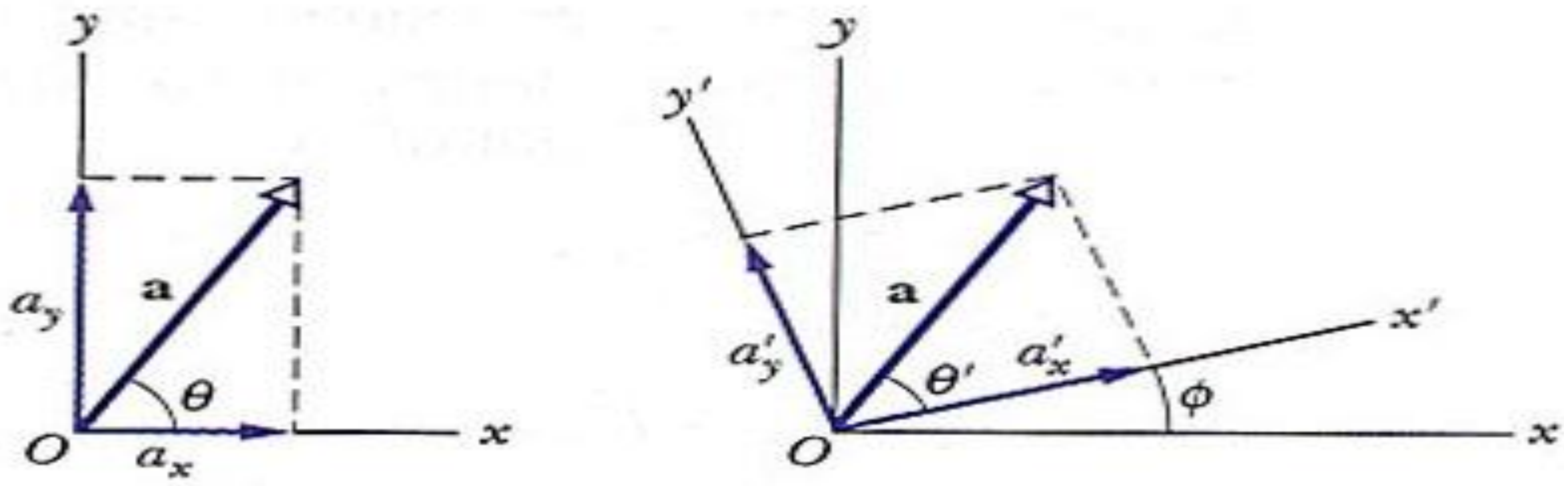
$$s_x = a_x + b_x$$
$$s_y = a_y + b_y$$

벡터의 크기

$$\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\cos\phi = |\vec{a}|^2 \Rightarrow \boxed{|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}}}$$

$$\boxed{\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2} = \sqrt{a_x^{'2} + a_y^{'2}}$$



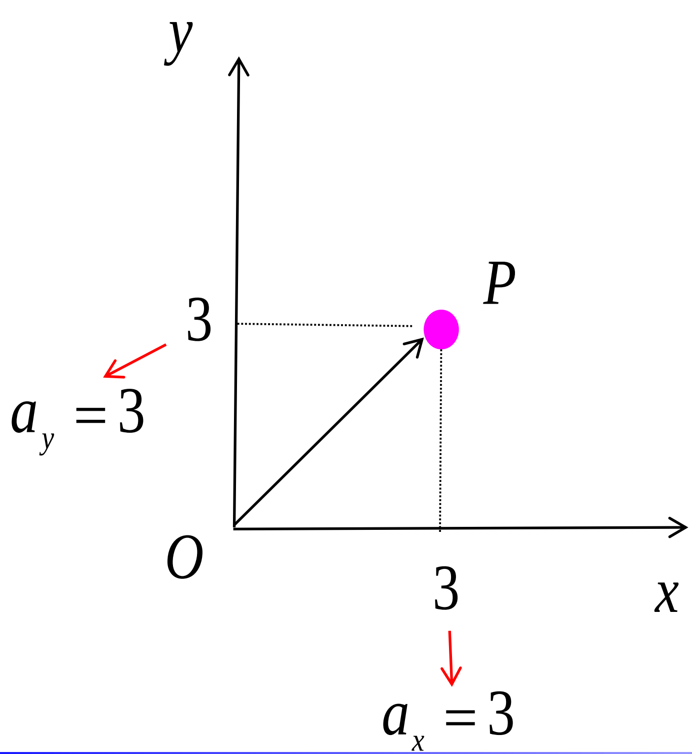
(a) (b)

벡터의 크기 계산

- **Vector 의 수학적 표현 : 벡터의 수치화**

$$\overrightarrow{OP} = \vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}$$
$$= 3\vec{i} + 3\vec{j} + 0\vec{k}$$

$$\boxed{\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z}$$



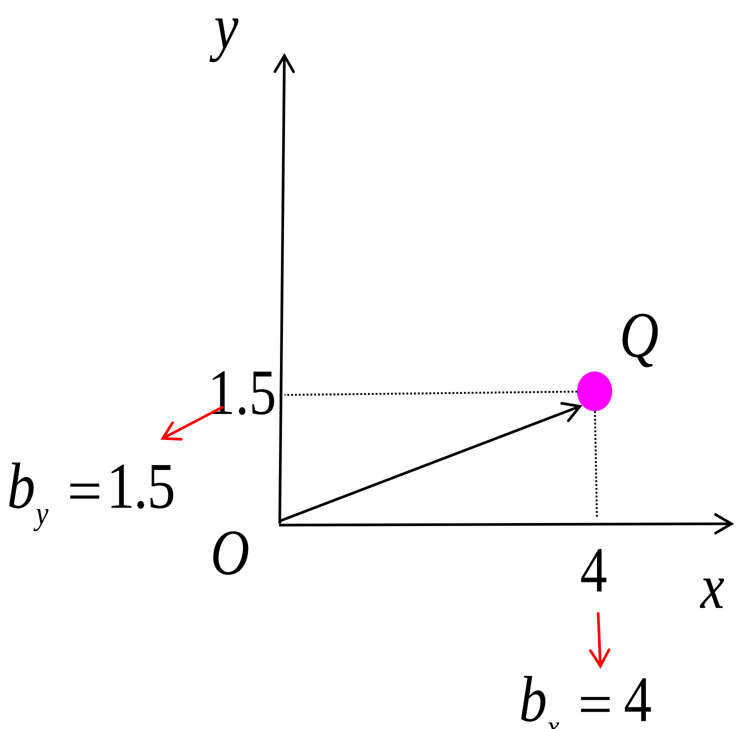
$$|\vec{a}| = \sqrt{\vec{a} \cdot \vec{a}}$$
$$= \sqrt{a_x^2 + a_y^2} = \sqrt{3^2 + 3^2}$$
$$= 3\sqrt{2}$$

벡터의 크기 계산

- **Vector 의 수학적 표현 : 벡터의 수치화**

$$\overrightarrow{OQ} = \vec{b} = b_x \vec{i} + b_y \vec{j} + b_z \vec{k}$$

$$= 4\vec{i} + 1.5\vec{j} + 0\vec{k}$$



$$|\vec{b}| = \sqrt{\vec{b} \cdot \vec{b}}$$

$$= \sqrt{b_x^2 + b_y^2} = \sqrt{4^2 + 1.5^2}$$

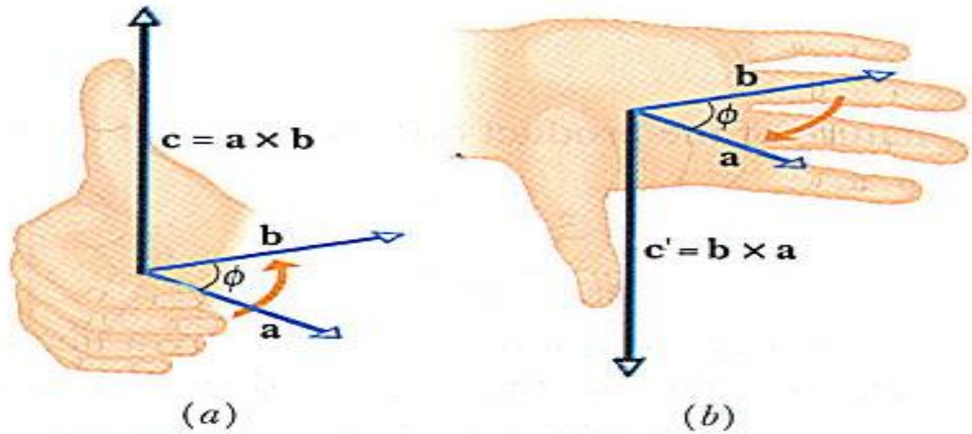
$$= \sqrt{\frac{73}{4}}$$

벡터 곱(Vector Product) : 외적

- 벡터 곱 : $\vec{c} = \vec{a} \times \vec{b}$

  - 크기 : $ab \sin \varphi$

  - 방향 : 오른손 법칙



(a) (b)

예

$$\vec{i} \times \vec{j} = \vec{k}$$

{ 크기 : 1
방향 : z 축 }

$$\vec{j} \times \vec{k} = \vec{i}$$

$$\vec{k} \times \vec{i} = \vec{j}$$

$$\vec{i} \times \vec{i} = \vec{j} \times \vec{j} = \vec{k} \times \vec{k} = 0$$

벡터 곱의 계산

$$\vec{c} = \vec{a} \times \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \times (b_x \vec{i} + b_y \vec{j} + b_z \vec{k})$$

$$= a_x \vec{i} \times b_x \vec{j} + a_x \vec{i} \times b_y \vec{j} + a_x \vec{i} \times b_z \vec{k}$$

$$+ a_y \vec{j} \times b_x \vec{i} + a_y \vec{j} \times b_y \vec{j} + a_y \vec{j} \times b_z \vec{k}$$

$$+ a_z \vec{k} \times b_x \vec{i} + a_z \vec{k} \times b_y \vec{j} + a_z \vec{k} \times b_z \vec{k}$$

$$= (a_y b_z - a_z b_y)\ \vec{i} + (a_z b_x - a_x b_z)\ \vec{j} + (a_x b_y - a_y b_x)\ \vec{k}$$

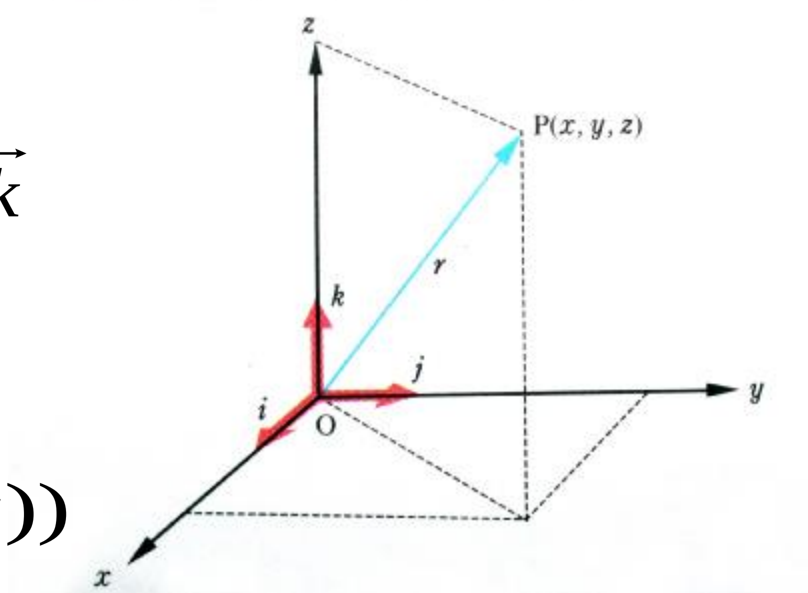
벡터 곱의 계산

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= (a_y b_z - a_z b_y)\vec{i} - (a_x b_z - a_z b_x)\vec{j} + (a_x b_y - a_y b_x)\bar{k}$$

위치 벡터

직교좌표계(Cartesian Coordinate System)



- 위치벡터

$$\vec{P}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$$

- 모션벡터 (Motion Vector) :

$$P(t) = (x(t), y(t), z(t))$$

$$\vec{P}(t) = \begin{bmatrix} x(t) \\ y(t) \\ z(t) \end{bmatrix}, \qquad \begin{aligned} x(t) &= \vec{P}(t)\cdot\vec{i}\,, \\ y(t) &= \vec{P}(t)\cdot\vec{j} \\ z(t) &= \vec{P}(t)\cdot\vec{k} \end{aligned}$$

연습문제

연습문제

1. 두 개의 벡터 $\vec{a} = 2\vec{i} + 3\vec{j} + 2\vec{k}$ 와 $\vec{b} = 1\vec{i} + 2\vec{j} - 1\vec{k}$, 에 대하여 답하시오.

(1) 각 벡터의 크기를 구하시오.

(2) 두 벡터의 내적을 구하시오.

(3) 두 벡터의 사이각을 구하시오.

(4) $\vec{b}$ 의 $\vec{a}$ 방향으로의 성분값 B 을 구하시오.

(5) $\vec{a} \times \vec{b}$ 와 $\vec{b} \times \vec{a}$ 를 구하시오.

(6) $\vec{a} \times \vec{b}$ 가 $\vec{a}$ 에 수직임을 보이시오.