UPPSALA
UNIVERSITET

U.U.D.M. Project Report 2011:26

Investigation of some tests for homogeneity of intensity with applications to insurance data

Sara Gustin

Examensarbete i matematik, 15 hp
Handledare och examinator: Jesper Rydén
December 2011

Department of Mathematics
Uppsala University

Abstract

In this thesis, a review is made of some statistical tests for detecting trends in stochastic processes. The aim is to apply the tests on specified data and by means of the test results draw conclusions of whether or not there is a trend in the underlying process of the data. Most of the tests are built on the assumption that the observed process is a Poisson process. Data considering the worst catastrophes in two distinct regards will be investigated; the worst catastrophes in terms of costs and the worst catastrophes in terms of victims. In order to apply the tests there was a need to calculate the times of the events and the times between successive events, which were done in the program Excel. The tests were then implemented in the statistical program R. All of the tests in the first regard were statistically significant and the conclusion was that there is an increasing trend, whereas none of the tests was statistically significant in the second regard, which implicates a lack of trend.

Acknowledgement

Thank you Jesper Rydén.

Contents

1 Introduction

There are various situations when it is of interest to know if there are systematic changes in the pattern of events when a stream of events is regarded. For example if the observed stream of events describes failures in a system it is important to find out if the failures occur randomly or if they occur more frequently or less frequently after some time point. If the failures increase over time, the system is called a deteriorating system and if the failures decrease over time the system is called an improving system [1].

Trend tests are useful tools when investigating possible systematic changes in the pattern of events. The pattern of events is said to contain trend if the times between the events are not identically distributed [1]. A trend test is, as the name reveals, a test for detecting trends. The null hypothesis in such tests is that the underlying process of the events is stationary [2]. The null hypothesis usually assumes that the process is a renewal process or a homogeneous Poisson process since these processes do not contain trends [1].

Investigating trends of failures in systems is a vital and common use of trend tests. Other areas where trend tests may be applied are for example in finance and medicine. This paper will concentrate on examining trends in natural catastrophes and other extreme catastrophes that cause huge costs and large numbers of victims. The catastrophes will be regarded in two cases; the worst catastrophes in terms of costs and the worst catastrophes in terms of victims.

First the definitions of some processes that are used to describe streams of events will be stated. After the basic definitions, various tests that can be performed to test for trend are presented and finally the tests are applied in order to examine whether the catastrophes in the two distinct respects contain trend. Figures showing the corresponding counting process and the times between the events in respective case will be displayed and the final conclusions are drawn by means of both the figures and the test results.

2 Definitions

Before going into the concepts of the tests and applying them, there are some important definitions and notations that need to be stated.

2.1 Basic notation

As a stream of events is observed (in the time interval $(a, b]$) we keep track of the *arrival times* $T_1, T_2, \dots$ i.e. the times when the events occur. Thus the time when the first event occurred is named T_1, the time for the second event T_2 and so on. In this work we will only consider streams of events where T_0 is defined to be 0. We also keep track of the times between the events, of the so-called *interarrival times*. The interarrival times are denoted $X_1, X_2, \dots$ where

$$X_n = T_n - T_{n-1}, \qquad n = 1, 2, \dots \tag{1}$$

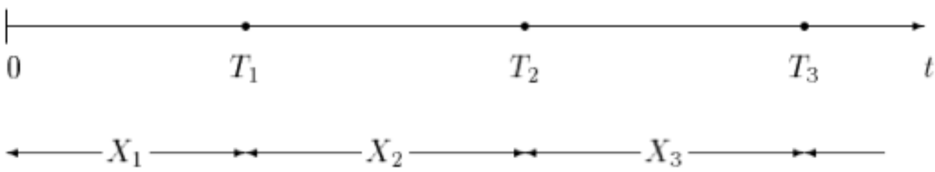

Figure 1: Arrival times T_n and interarrival times X_n

Furthermore we have that

$$T_n = X_1 + X_2 + \cdots + X_n. \tag{2}$$

An equivalent way of expressing the stream is by the *counting process* representation

$$N(t) = \text{number of events in } (0, t]. \tag{3}$$

Often when these kinds of observations are made one talks about a *failure censored* system or a *time censored* system. If a system is failure censored or time censored depends on what determines for how long the system is observed. A system that is observed until a certain time τ is called time censored and a system that is observed until a certain number n of failures (here failures corresponds to events) has occurred is called failure censored [1]. For a process with n observed events, we let

$$\hat{n} = \begin{cases} n, \text{if the process is time censored} \\ n-1, \text{if the process is failure censored} \end{cases}$$

2.2 Renewal process

A *renewal process* (RP) is a counting process for which the times between the events are non-negative, independent and identically distributed with a mutual distribution F. The process is denoted RP(F). In addition to equation (2) it also holds that the number of events up to time t is equal to the largest number n such that the n^{th} arrival time, T_n, is less than or equal to t. From this it is possible to derive the following mathematical relations:

$$T_n = \sum_{i=1}^{n} X_i \quad \text{and} \quad N(t) = \sup\{\, n : T_n \leq t \,\} \tag{4}$$

Note that the time horizon for a renewal process can be either discrete: $\{\, 0, 1, 2, \dots \}$ or continuous: $[0, \infty)$. [1, 2]

These processes are called renewal processes because of their probabilistic renewal at each arrival time. Imagine a renewal process is observed until a certain time τ, when the n^{th} event occurs. Then consider everything happening after τ as a new process and count the events occurring in this process. Then the new process would be a counting renewal process, as was the original process. In addition, the new process would have iid interarrival intervals of identical distribution as the process regarded from the beginning. To be convinced that this is really the fact, one can study the arrival time for the k^{th} event counted from τ. The arrival time turns out to be $T_{n+k} - T_n = X_{n+1} + X_{n+2} + \dots + X_{n+k}$. The reader can easily notice that the arrival time of the k^{th} event will be the sum of the interarrival times in the same way as defined for a renewal process above. [3]

Thus, if the time of the n^{th} event, τ, is given, the process $\{\, N(\tau + t) - N(\tau);\ \mathrm{t} \geq 0 \,\}$ will be a renewal counting process for which the interarrivals are iid with the same distribution as the initially observed renewal process [3]. Consequently, “at any given event time $T_n = \tau$, the future outlook of the events is the same as that from time 0” [4]. This property, that the system is renewed to its original condition after each arrival time, is often called a “perfect repair”.

2.3 Homogeneous Poisson process

A *homogeneous Poisson process* (HPP) is specified with a constant intensity λ and is a renewal process for which the interarrival distribution is an Exponential with mean value $1/\lambda$. This process is denoted HPP(λ). For a homogeneous Poisson process the number of events in disjoint time intervals are independent and Poisson distributed, i.e. when the time intervals $(s, t]$ and $(l, k]$ are disjoint, the random variables $N(t) - N(s)$ and $N(k) - N(\mathrm{l})$ are independent and $N(t)$ has the distribution Po(λt). [5-6]

It also holds that the increments are stationary, i.e. the number of events in a certain interval depends only on the length of the interval [5]. $N(0)$ is defined to be 0, i.e. at time 0 no events has occurred [6]. Since we know that $N(t) \sim$ Po(λt) we can calculate the probability of n events occurring up to time t;

$$P(N(t) = n) = \frac{(\lambda t)^{n}}{n!} e^{-\lambda t}. \tag{5}$$

2.4 Non-homogeneous Poisson process

The homogeneous Poisson process can be generalized to a *non-homogeneous Poisson process* (NHPP) by letting the intensity take different values at different time points [5]. The non-homogeneous Poisson process is denoted NHPP($\Lambda(t)$). Recall that the HPP has a constant intensity λ and that the distribution of the interarrivals only depends on the length of the interval. Since the intensity varies in time for the NHPP we need to define a function that is deterministic and describes how the intensity varies. This function is called the *intensity function*, $\lambda(t)$. Then we denote the *cumulative intensity function* [6] as

$$\Lambda(t) = \int_0^t \lambda(u)\, du. \tag{6}$$

The distribution of $N(t)$ becomes $\mathrm{Po}(\int_0^t \lambda(u)\, du)$ [5] and we can calculate the probability of n events occurring up to time t;

$$P(N(t) = n) = \frac{(\Lambda(t))^n}{n!} e^{-(\Lambda(t))}. \tag{7}$$

A major difference between HPP and NHPP is that the distribution of the interarrival times of a NHPP not only depends on the length of the interval, but on the age of the process as well [6-7]. Another difference is that the NHPP has "minimal repairs", which means that when an event occurs the system restores itself to the same condition it had just before that event happened, whereas the HPP restores to the original condition.

2.5 Trend-renewal process

As the name of the process reveals, a trend-renewal process is a generalization of the renewal process that allows trends [1]. For the *trend-renewal process* (TRP) we need to specify both the intensity function $\lambda(t)$ and a distribution function F for the times between events. The expected value of F is usually assumed to be 1. The cumulative intensity function $\Lambda(t)$ for the TRP is defined in the same way as in equation (6). [7-8]

Observe the process we get by letting the first time point be $\Lambda(T_1)$, the second time point $\Lambda(T_2)$ and so on, i.e. the process $\Lambda(T_1), \Lambda(T_2), \ldots$ If this process has the distribution RP(F), then the process $T_1, T_2, \ldots$ which we are interested in has the distribution TRP(F, $\lambda(t)$). [7-8]

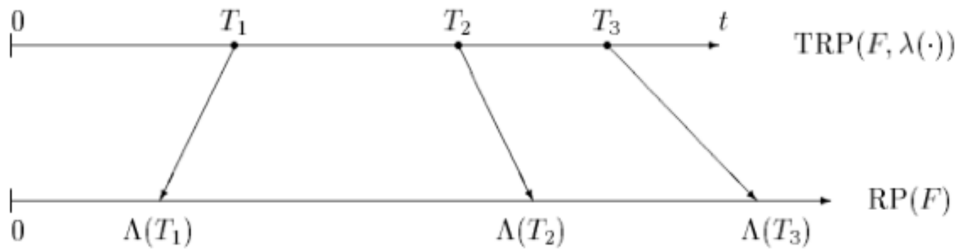

Figure 2: The character of the trend renewal process

This process contains both time trends and the behaviour of a renewal process [7]. The trend-renewal process is thus a model that allows an increase/decrease of the intensity over time, i.e. a trend, and that in the same time also allows drastic changes in the intensity after a repair [9]. Consequently, it is not hard to realize that all of RP, HPP and NHPP are special cases of the trend renewal process.

The NHPP follows when the interarrivals are exponential distributed with mean value 1. The renewal process follows when the intensity function is constant and the special case of a HPP follows when both the intensity function and the hazard rate of the distribution of the interarrivals are constant. For more information about the hazard rate, the reader might want to consult [8]. [1, 8]

This process is not discussed further in this paper but since both NHPP and RP are special cases of this process, it deserves to be at least mentioned.

3 Statistical hypothesis testing

Statistical tests are often used when it is of interest to draw conclusions about a statement regarding a population or a process. One uses sample data and test statistics based on the sample to either accept or reject a hypothesis concerning the population. Hypothesis testing can be applied to various problems in real life and is an essential part of statistics. There are lots of cases when one would like to use a statistical hypothesis test. Examples of hypotheses one might want to test are:

- The mean of the population has a certain value, say μ
- The standard deviation of the population has a certain value, say σ
- The process follows a specified distribution
- The process is a renewal process

3.1 Explanation of commonly used terms

The *null hypothesis*, H_0, is the hypothesis that is tested and is either accepted or reject. When H_0 is rejected, it is "proved" (there is always an error risk) that H_0 can not be the case but the contrary, that H_0 is true, can never be proved. To accept H_0 simply means that there is not evidence enough to believe it is false. There is always an *alternative hypothesis*, H_A, which is accepted if H_0 is rejected.

To test the null hypothesis a *test statistic* is used which is a function of the sample data and the result after putting in the sample data in the function is a numerical value. There are numerous well-known test statistics. Which tests to use depends on how the null hypothesis is phrased. The numerical value obtained from the test statistic is compared to an appropriate quantile of the distribution of the test statistic or to another suitable *critical value*. The critical value depends on the desired *significance level* of the test, α, which is the risk of rejecting H_0 although it is true. So if $\alpha = 0.05$ there is a 5% risk to rejected the null hypothesis although it is true. The most commonly used values of α are 0.1, 0.05 and 0.01.

There are tests that are *one-sided* and there are tests that are *two-sided*. When performing a one-sided test, H_0 is rejected either if the value of the test statistic is above the upper critical value or if it is below the lower critical value. If it is the higher or lower values that leads to a rejection of

H_0 depends on how the alternative hypothesis is expressed. Both values which are above the upper critical value and values which are below the lower critical value leads to a rejection of H_0 if the test is two-sided. When the null hypothesis is rejected because of a test, the test is said to be *statistically significant*.

3.2 Tests for detecting trends

There are different kinds of statistical tests. The tests that are of relevance for this thesis are those that investigate whether a stochastic process contains trends. A process is said to contain trend in the pattern of events if the marginal distributions of the interarrival times are not identically distributed [10]. The purpose of using trend tests is to detect if the pattern of events is significantly changing over time [11]. In this section some tests that can be used to detect trends are presented.

3.2.1 Laplace type tests

The Laplace test is used to test if a set of data follows a HPP [6]. As we noted in Section 2.2 a homogeneous Poisson process has constant intensity and thus does not have trends. The alternative to the null hypothesis is that the process is a NHPP with an increasing or decreasing trend [6]. This kind of trend, that is either increasing or decreasing, is usually called *monotonic trend*. The test statistic for this test is

$$L = \frac{\sum_{i=1}^{\hat{n}} T_i - \frac{1}{2}\hat{n}(b+a)}{\sqrt{\frac{\hat{n}(b-a)^2}{12}}}$$

where $(a, b]$ is the time interval in which the process is observed. For a time censored process $b = \tau$ and for a failure censored process $b = T_n$ [12].

Since L is asymptotically standard normally distributed under the null hypothesis [6], quantiles of a normal distribution may be used as critical values. Thus, at the 5 % significance level, one rejects the null hypothesis if L gives larger value than 1.96 or smaller value than -1.96.

Lindqvist [8] and Wang [11] emphasize that rejecting H_0 for this test does not necessarily imply that there exists a trend in the process, just that the process does not follow a HPP. As mentioned in Section 2.2 the HPP is a special case of a RP when times between events are exponentially distributed. Thus if times between events come from a different distribution than exponential, the process could still be a RP and hence lack trend [8].

A test statistic that is asymptotically equivalent to the Laplace test statistic [2] is presented below

$$B_1 = \frac{\sqrt{12}}{n\sqrt{n}} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left(\frac{X_j}{\overline{X}} - 1 \right) = - \frac{\sqrt{12}}{n\sqrt{n}\,\overline{X}} \sum_{i=1}^{n-1} \left(T_i - \frac{i}{n} T_n \right)$$

$$= - \sqrt{\frac{n-1}{n}}\, L$$

where $\overline{X} = T_n/n$. In contrast to the Laplace test, the alternative to H_0 is one-sided for B_1 [2]. The 5 % asymptotic critical value of this test is -1.65. For more information about this test the reader might want to read [2].

3.2.2 Anderson-Darling type tests

The Anderson-Darling test for trend was presented by Kvaløy and Lindqvist in [6] and is based on the well-known Anderson-Darling test statistic. The Anderson-Darling test statistic is used to find out whether a certain distribution or a certain family of distributions can describe the set of data being tested [13]. Since this test can be used to test for various distributions, there is not just one critical value. Which critical value that is appropriate to use depends on the distribution specified in the null hypothesis [13].

However, the Anderson-Darling test for trend is a test of HPP versus NHPP [1]. The test statistic, which may be found in [6, 12], is the following:

$$\boldsymbol{AD = -\hat{n} - \frac{1}{\hat{n}} \sum_{i=1}^{\hat{n}} (2i-1) \left(\ln\left(\frac{T_i}{b}\right) + \ln\left(1 - \frac{T_{\hat{n}+1-i}}{b}\right) \right)}.$$

This test is one-sided and the null hypothesis of a homogeneous Poisson process is rejected if the value obtained from AD is greater than the critical value. Critical values of the regular Anderson-Darling statistic may be used, since the limit distribution of AD is the same as the limit distribution of the Anderson-Darling statistic [14]. For some distributions critical values have been published and these can be found in [14-15]. The 5 % asymptotic critical value of interest for the Anderson-Darling test for trend is 2.49 and the 1 % asymptotic critical value is 3.86.

A different type of Anderson-Darling test statistic is presented in [2] and it is the following

$$B_2 = \frac{1}{\overline{X}^2} \sum_{i=1}^{n-1} \frac{1}{i(n-i)} \left(T_i - \frac{i}{n} T_n \right)^2$$

which is a two-sided test with the same critical values as the regular Anderson-Darling statistic. A problem with the discussed Anderson-Darling type tests and other tests with H_0 of a HPP is that rejecting the null hypothesis of a HPP does not necessarily mean that there is a trend in the process. Kvaløy and Lindqvist [1] therefore propose the following test statistic

$$GAD = \frac{(n-4)\overline{X}^2}{\hat{\sigma}^2} \sum_{i=1}^{n} \left({q_i}^2 \ln\left(\frac{i}{i-1}\right) + (\, q_i + \, r_i)^2 \ln\left(\frac{n-i+1}{n-i}\right) - \frac{{r_i}^2}{n} \right)$$

which is a test of the null hypothesis RP. In the formula presented above $q_i = (T_i - iX_i)/T_n$, $r_i = nX_i/T_n - 1$ and $\hat{\sigma}^2 = \frac{1}{2(n-1)} \sum_{i=1}^{n-1} (\, X_{i+1} - X_i)^2$. Several estimators of σ^2 may be used but Kvaløy [1] recommend using $\hat{\sigma}^2$ instead of $S^2 = \sum_{i=1}^{n} (X_i - \overline{X})^2 / (n-1)$ since $\hat{\sigma}^2$ is a better choice when there exists a trend in the process.

This statistic is called the generalized Anderson-Darling test for trend. As is visible in the expression of GAD, the generalized Anderson- Darling test for trend assumes a failure censored process. For time censored processes, Kvaløy and Lindqvist suggest [1] “conditioning on the observed number of failures, and treating the data as if they were data from a failure censored system”.

Note that when $i = 1$ and when $i = n$ we get division by zero in the expression of GAD. In the code of this statistic, I define those terms to be zero [12].

3.2.3 Lewis-Robinson type tests

As mentioned in Section 3.1 there is a risk of drawing the wrong conclusions when rejecting H_0 in the Laplace test. Instead of just ascertain that the process does not follow a HPP it is easy to think that the process contains trends since there are no trends in the HPP. The Lewis-Robinson test is a modification of the Laplace test that tests the null hypothesis of the process being a RP against the alternative of not being a RP [11]. The thought behind the test is to avoid making those wrong conclusions that is easy to make with the Laplace test and the difference is that we divide the Laplace test statistic with the estimated coefficient of variation of the interarrivals [8, 11].

As mentioned, the null hypothesis of this test is that the process is a RP, and this is a much more general assumption than the assumption that the process is a HPP (since a HPP is a special case of a RP). The proposed test statistic for this test is

$$LR = \frac{b/n}{S} \frac{\sum_{i=1}^{\hat{n}} T_i - \frac{\hat{n}}{2} b}{b\sqrt{\frac{\hat{n}}{12}}}$$

where b is the time point when we stop observing the process. Similarly as for the Anderson-Darling test for trend, S^2 works fine when there are no trends in the process but when there are, $\hat{\sigma}^2$ is a better estimation of the variance of X_i [1-2]. The modified Lewis-Robinson test statistic becomes

$$LR_2 = \frac{b/n}{\hat{\sigma}} \frac{\sum_{i=1}^{\hat{n}} T_i - \frac{\hat{n}}{2} b}{b\sqrt{\frac{\hat{n}}{12}}}.$$

Since the Lewis-Robinson test statistic is a modification of the Laplace statistic where we have divided by the estimated coefficient of variation of the $\boldsymbol{X_i}$ we can also write the expression for the statistic in this manner

$$LR = \frac{L}{\widehat{CV}(X)}.$$

For an exponential distribution, the coefficient of variation is equal to 1, such that there is equivalence between the Lewis-Robinson test statistic and the Laplace test statistic [8].

3.2.4 The Mann test

Yet another test for detecting if the process is a renewal process is the Mann test. One uses the test statistic

$$\widetilde{M} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} I(X_i < X_j)$$

where $I(X_i < X_j)$ is equal to 1 if $X_i < X_j$ and 0 otherwise. For this test the alternative to H_0 is that there exists a monotonic trend. If $n < 10$ then there are tables to consult, but for $n \geq 10$ there has to be some calculations done. [1-2]

For $n \geq 10$, $\widetilde{M}$ is rescaled to be standard normally distributed and then the obtained value is compared to an appropriate quantile of a normal distribution. This is easily done since $\widetilde{M}$ is approximately normally distributed with expectation $\mu = n(n-1)/4$ and variance $\sigma^2 = (2n^3 + 3n^2 - 5n)/72$ for $n \geq 10$ [1]. Hence the resulting pivot variable is

$$M = \frac{\widetilde{M} - \mu}{\sigma}.$$

3.2.5 Comparison between the tests

If one wishes to use only one of the presented tests, the generalized Anderson-Darling test for trend is recommended [1, 6]. A comparison between the Lewis-Robinson test, the Mann test and the generalized Anderson-Darling test for trend was made in [1]. The conclusion was that the generalized Anderson-Darling test for trend has clearly better power against bathtub shaped and increasing trends and that the other tests are somewhat better against decreasing trends. A different comparison was made in [6] where the Laplace test and the Anderson-Darling test for trend were considered. It turns out that the Laplace test is slightly better against increasing trends and that the Anderson-Darling test for trend is much more powerful against bathtub shaped trends.

4 Analysis of data

In the sequel, the tests from the previous section will be applied on some data with the aim to detect possible trends. The data considered here are the worst disasters between the years 1970 and 2000, in terms of costs and victims. The re/insurance company Swiss Re published these data in [16] 2001. The costs correspond to (about three-quarter of the) insured losses worldwide and when calculating the losses, the costs are converted into USD and the inflation is taken into account [16]. Victims refer to dead and missed people.

All of the tests presented in Section 3 can be performed in the statistical program R. There are of course several ways of implementing these tests. For those who are interested, my way of implementing the tests on this specific data is available in Appendix B. Furthermore are tables of the worst catastrophes in both cases available in Appendix A.

4.1 Swiss Re

Swiss Re was founded in 1863 to provide effective ways of handling risks in connection with mayor devastations. To estimate and price risks are things that the employers at Swiss Re do on a daily basis. Swiss Re is headquartered in Zürich, Switzerland, but are stationed in over 20 countries all over the world and represented in all continents. Consequently Swiss Re is one of the biggest and most widespread reinsurance companies in the world. [17]

This re/insurance company helps government managing the financial effects of catastrophic disasters such as for example hurricanes, floods and pandemics. Governments are thus able to transfer some of the risks involved with catastrophes to private reinsurers as Swiss Re. Swiss Re cooperates with governments and public sector divisions around the world (such as the World Bank) to develop procedures to prevent risks and financing regarding natural and catastrophic disasters. Because they offer solutions to mayor risks, they spend lots of resources on analysing developments that constitute the risks. Trends in the environment and trends in the market are such things that are analysed. Technological and socio-political developments may also constitute risks and are therefore investigated. [17]

Swiss Re publishes parts of their analyses in different kinds of publications on a regular basis. The publications emphasize materials that are of great importance for their costumers and stakeholders. Significant topics of these publications are longevities, climate changes and agricultural risks. Swiss Re distributes different kinds of publications, such as expertise publications and sigma reports. The sigma reports contain analyses of economic trends and exhaustive information on the international insurance markets. Both external and internal clients use the sigma reports. The external clients correspond to mostly insurance companies but also the media consults these reports. Other units than sigma, which lies within Swiss Re, uses the sigma report to explore business opportunities. The data, which will be examined in the following section, come from one of these sigma reports. [16, 17]

4.2 Results

As mentioned earlier, data regarding the worst catastrophes in 1970-2000 will be tested for trends. Two different cases will be considered:

1) Worst catastrophes in terms of costs
2) Worst catastrophes in terms of victims

Before performing the tests, some graphs describing the material will be presented. To begin with, the first case is examined.

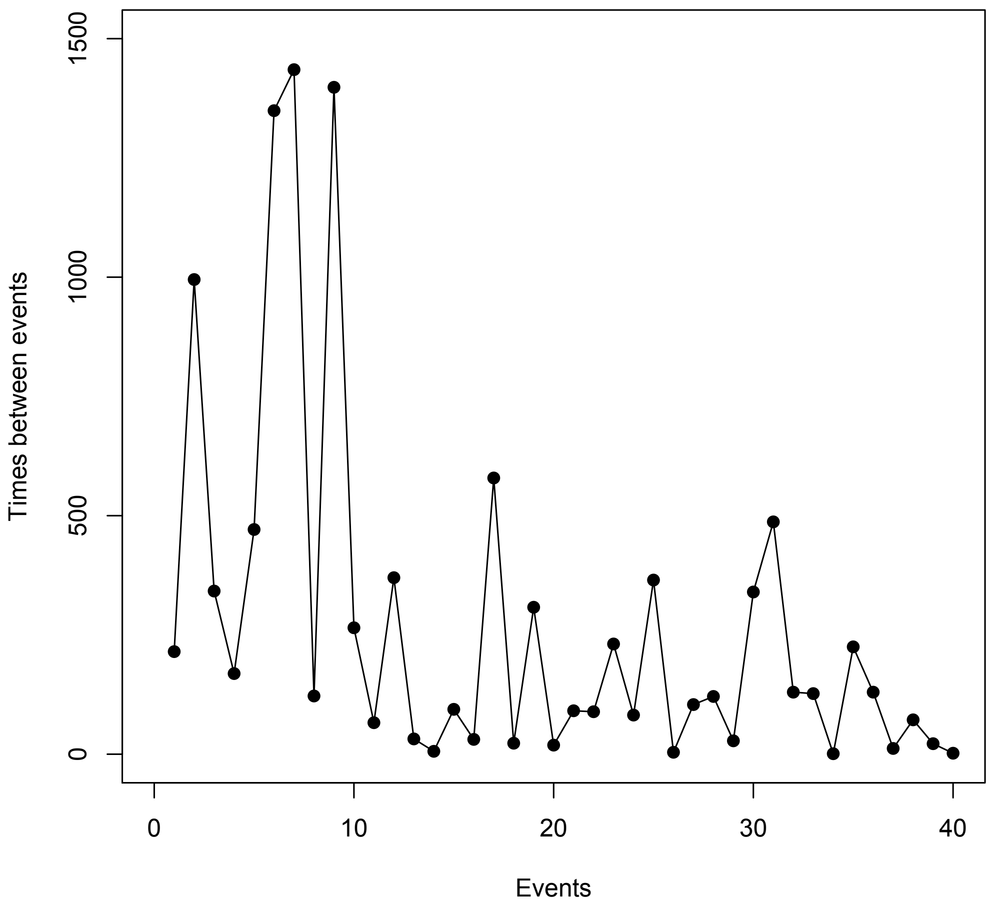

Figure 3: Times between catastrophes in the first case

From Figure 3 it seems like the times between costly catastrophes decrease around the 10^{th} event, i.e. that costly catastrophes appear more frequently after the 10^{th} event. Let us look at another graph, showing the corresponding counting process.

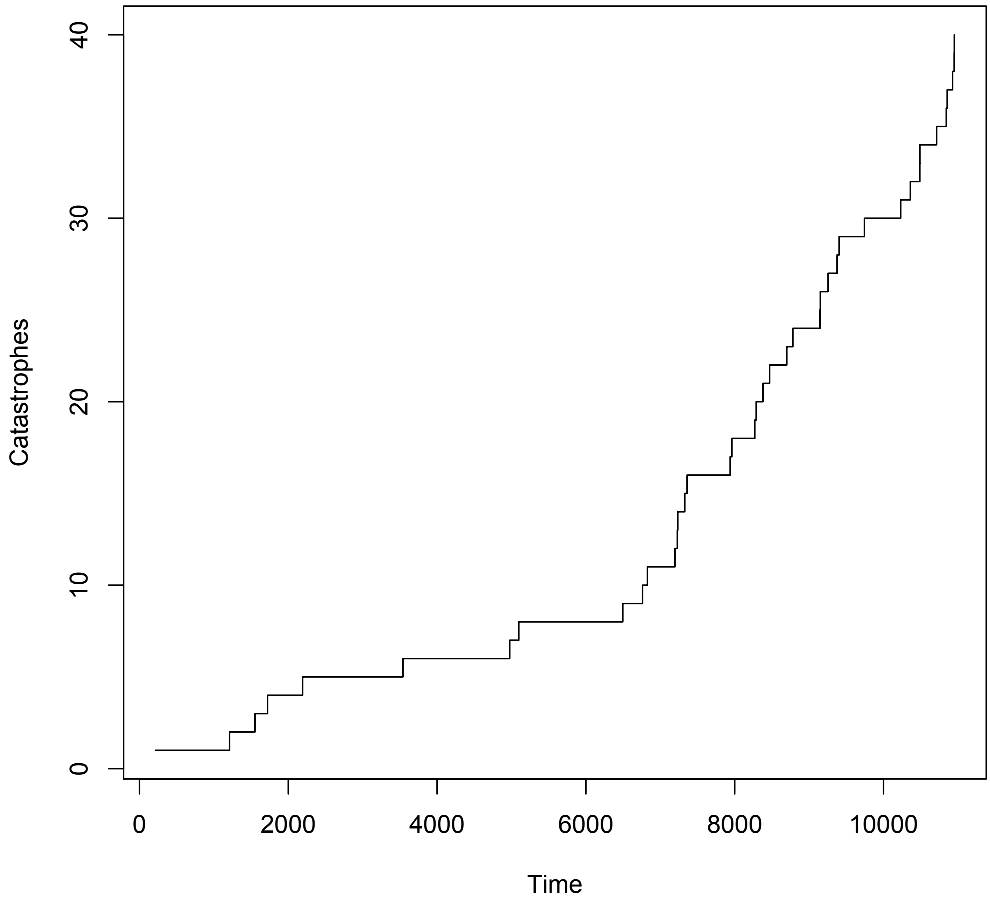

Figure 4: Corresponding counting process in the first case

From Figure 4 it appears as costly disasters occur more often after about 7000 days, i.e. times between the events become smaller around 7000 days. Could it bee that the 10^{th} event occurred around 7000 days? Then the two graphs would indicate the same thing, namely that there exists a monotonic trend. This will be investigated further by applying the tests presented in the previous section. The specifics of how the tests were implemented are available in Appendix B.

As mentioned in Section 2.1 a process can be either time censored or failure censored. Although the 40 worst catastrophes were given, still a larger number of catastrophes actually happened (but were not presented in data). In addition, the time frame in which we observe the process is fixed. We stop observing the process at the 31^{st} of December 1999 and not at the time point of the 40^{th} catastrophe. Hence, we consider data rather as time censored than failure censored. Some of the test statistics were applicable on systems of both types by just changing the values of $\hat{n}$ and b. However, a few of the statistics were based on a failure censored process. For those tests one could condition on the number of catastrophes and treat the process as if it was failure censored and thus stop observing the process at time T_n [1]. The results of the test are presented below.

Test Statistic	L	B_1	B_2	AD	GAD	LR	LR_2	M
Value	4.16	-4.11	9.43	9.99	6.00	3.00	3.51	-2.91
5 % asympt. crit. value	1.96	-1.65	2.49	2.49	2.49	1.65	1.65	1.96
1 % asympt. crit. value	2.58	-2.33	3.86	3.86	3.86	2.33	2.33	2.58

The test results when treating the process as failure censored

Test Statistic	L	B_1	B_2	AD	GAD	LR	LR_2	M
Value	4.37	-4.32	9.43	11.5	6.00	3.16	3.69	-2.91
5 % asympt. crit. value	1.96	-1.65	2.49	2.49	2.49	1.65	1.65	1.96
1 % asympt. crit. value	2.58	-2.33	3.86	3.86	3.86	2.33	2.33	2.58

The test results when treating the process as time censored

As visible, the results are significantly the same in both cases. All of the tests are statistically significant at the 5 % level, as the tables above show. All of the tests are significant at the 1 % level as well. Is this the case when disasters with the most victims are studied? This will soon be revealed, as the second case is about to be investigated in the same way as the first case.

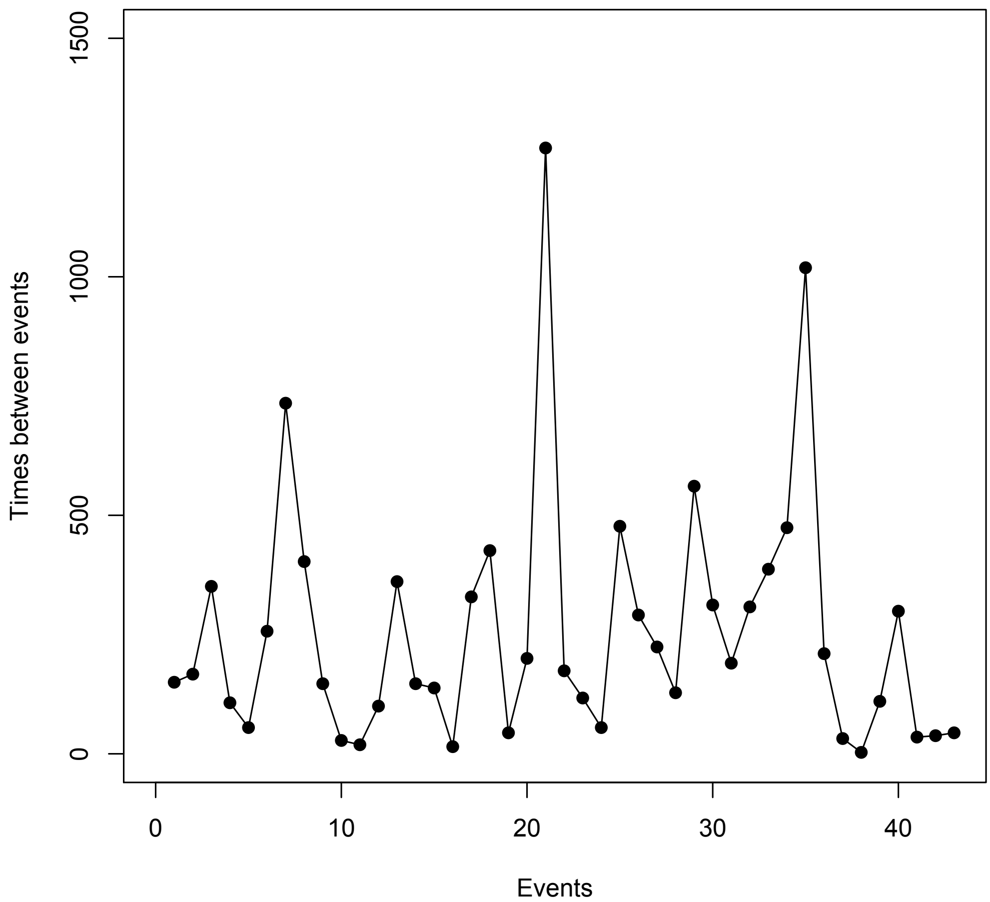

Figure 5: Times between catastrophes in the second case

Times between events in Figure 5 seems to be varying without any clear pattern, accept possibly from the peak in the middle, around the 20^{th} event, and the last three events. Thus, from this graph it is not clear whether or not catastrophes of type 2 contain trends. Let us see if the next figure is easier to interpret.

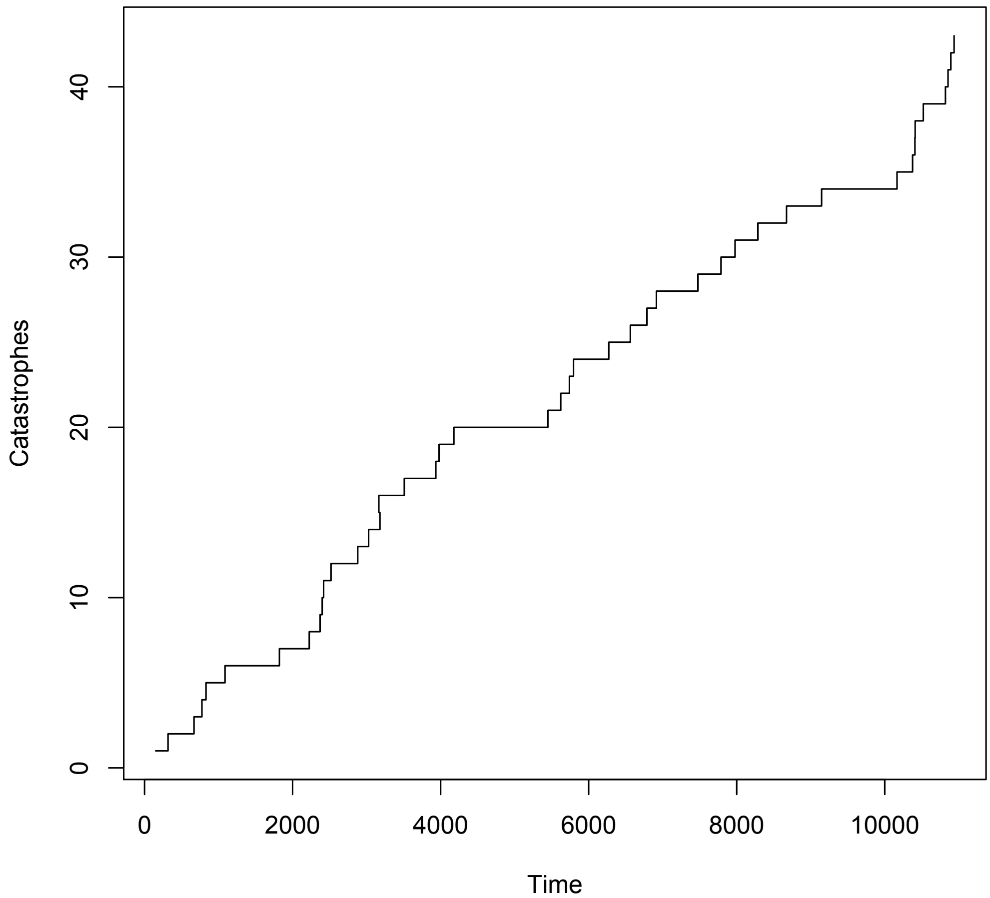

Figure 6: Corresponding counting process in the second case

Figure 6 tells us about the same thing as the previous figure, that there does not seem to be any particular pattern except for possibly at two spots. It is very obvious that there are two disasters somewhere between 4000 and 6000 days that have larger distance between them then any two of the other events in the graph. Similarly, at the end of the graph it looks like the last three catastrophes occur with smaller distance than the others. As for the first case, the tests are applied to investigate this hypothesis further. Even in this case the process is treated as both time censored and failure censored and the results are the following:

Test Statistic	L	B_1	B_2	AD	GAD	LR	LR_2	M
Value	-0.06	0.06	0.68	0.89	0.66	-0.06	-0.07	-0.26
5 % asympt. crit. value	1.96	-1.65	2.49	2.49	2.49	1.65	1.65	1.96
1 % asympt. crit. value	2.58	-2.33	3.86	3.86	3.86	2.33	2.33	2.58

The test results when treating the process as failure censored

Test Statistic	L	B_1	B_2	AD	GAD	LR	LR_2	M
Value	0.18	-0.18	0.68	1.27	0.66	0.18	0.19	-0.26
5 % asympt. crit. value	1.96	-1.65	2.49	2.49	2.49	1.65	1.65	1.96
1 % asympt. crit. value	2.58	-2.33	3.86	3.86	3.86	2.33	2.33	2.58

The test results when treating the process as time censored

As visible in the table, the results are significantly the same when treating the process as failure censored as when treating it like time censored. None of the tests are statistically significant at the 5 % level. None of the tests are statistically significant at the 1 % level either, which is expected since they were not significant at the 5 % level.

5 Conclusions

In Section 4 we have seen examples of how statistical trend tests can be applied to real life problems. The worst catastrophes 1970-2000 concerning costs and victims were examined and the two different cases result in completely distinct conclusions, which will be discussed in this section.

5.1 First case; Worst catastrophes in terms of costs

In the first case, all of the tests were statistically significant and thus the null hypothesis was rejected in each test. However, the fact the null hypothesis is rejected implicates slightly different things depending on which test that is used. The test statistics L, B_1, B_2 and AD all test the hypothesis that the underlying process is s homogeneous Poisson process. Thus, rejecting H_0 in any of these tests implicates that the process does not follows a HPP, but it could still be a renewal process as we learned in Section 3.2.1. Due to this fact, it is difficult to draw conclusions from these tests alone.

The Generalized Anderson Darling test for trend, the Lewis-Robinson test and the Mann test have the null hypothesis that the process follows a renewal process. Thus, the conclusions we can draw from these tests are different. Since the tests in question investigate whether the process is a renewal process, rejecting this null hypothesis implicates that the process contains trend. This together with the other test results strongly implicates that there is a trend when the worst catastrophes in terms of costs are considered.

That this is the right conclusion is actually obvious from Figure 3 and 4 which both displays an increasing trend. That there is an increasing trend in the most costly catastrophes is also visible in Table 3. It is noticeable from Table 3 that costly disasters occur more frequently after July 1988, which in fact is the time of the 10^{th} event.

It is likely that we had a more complex society in the later years of the study than we had in the earlier years and that this could partly be the explanation of the results. Another possible explanation is an increase in the total number of catastrophes.

5.2 Second case; Worst catastrophes in terms of victims

In contrary to the first case, none of the tests were statistically significant when catastrophes with the most victims were investigated. In other words, the null hypothesis of no trend is accepted in all tests. But as mentioned in Section 3.1, accepting the null hypothesis does not necessarily mean that it is true, although in this case it is reasonable to believe the null hypothesis of no trend. Reviewing Figure 5 and 6 probably will convince you that this is a reasonable conclusion. Particularly clear is Figure 6 since it is easy to note that the “measure” points almost lies on a straight line, which indicates a constant intensity of the underlying process.

It is also quite obvious from Table 4 that there is no particular pattern in catastrophes with the most victims as the size of times between events vary broadly throughout the whole table, in contrary to the first case when the corresponding table indicated an explicit trend.

A final conclusion that can be drawn from Table 1 and 2 is that there does not seem to be much correlation between the worst catastrophes in terms of costs and the worst catastrophes in terms of victims.

Appendix A: Tables of the worst catastrophes

Insured damage[22] (in USD m, at 2000 prices)	Victims[23]	Date/beginning	Event	Country
19 649	38	23.08.1992	Hurricane Andrew	United States, Bahamas
16 277	60	17.01.1994	Northridge earthquake	United States[24]
7 142	51	27.09.1991	Typhoon Mireille	Japan
6 053	95	25.01.1990	Winterstorm Daria	France, GB, B et al.
5 998	80	25.12.1999	Winterstorm Lothar over Western Europe	France, CH et al.
5 829	61	15.09.1989	Hurricane Hugo	Puerto Rico, United States et al.
4 550	22	15.10.1987	Storm and floods	Europe
4 206	64	25.02.1990	Winterstorm Vivian	Western/central Europe
4 178	26	22.09.1999	Typhoon Bart hits south of country	Japan
3 731	600	20.09.1998	Hurricane Georges	United States, Caribbean
2 913	167	06.07.1988	Explosion on platform Piper Alpha	Great Britain
2 795	6 425	17.01.1995	Great Hanshin earthquake in Kobe	Japan
2 482	45	27.12.1999	Winterstorm Martin	France, Spain, CH
2 441	70	10.09.1999	Hurricane Floyd; heavy downpours, flooding	United States et al.
2 374	59	01.10.1995	Hurricane Opal	United States, Mexico
2 087	246	10.03.1993	Blizzard, tornadoes	United States
1 965	4	11.09.1992	Hurricane Iniki	United States
1 841	23	23.10.1989	Explosion in a petrochemical plant	United States
1 785	-	12.09.1979	Hurricane Frederic	United States
1 758	39	05.09.1996	Hurricane Fran	United States
1 747	2 000	18.09.1974	Tropical Cyclone Fifi	Honduras
1 696	116	03.09.1995	Hurricane Luis	Caribbean
1 620	350	10.09.1988	Hurricane Gilbert	Jamaica et al.
1 551	20	03.12.1999	Winterstorm Anatol	Western/northern Europe
1 536	54	03.05.1999	Series of more than 70 tornadoes in the Midwest	United States
1 522	500	17.12.1983	Blizzards, cold wave	United States, Canada et al.
1 518	26	20.10.1991	Forest fires which spread to urban areas, drought	United States
1 505	350	02.04.1974	Tornadoes in 14 states	United States
1 435	-	25.04.1973	Flooding on the Mississippi	United States
1 422	-	15.05.1998	Wind, hail and tornadoes (MN, IA)	United States
1 390	63	17.10.1989	Loma Prieta earthquake	United States
1 376	31	04.08.1970	Hurricane Celia	United States, Cuba
1 349	12	19.09.1998	Typhoon Vicki	Japan, Philippines
1 301	46	05.01.1998	Cold spell with ice and snow	Canada, United States
1 283	21	05.05.1995	Wind, hail and flooding (TX, NM)	United States
1 228	12	11.12.1992	Rain, snowstorms, floods	United States
1 222	100	02.01.1976	Winterstorm Capella	Northwestern Europe
1 168	21	17.08.1983	Hurricane Alicia	United States
1 133	3	27.10.1993	Forest fires which spread to urban areas (CA)	United States
1 131	40	21.01.1995	Storms and flooding	Northern/central Europe

[22] excl. liability damage
[23] dead or missing
[24] figures for natural catastrophes in the United States with the kind permission of Property Claims Service (PCS)

Table 1: The 40 worst catastrophes in terms of costs 1970-2000

Victims[25]	Insured damage[26] (in USD m, at 2000 prices)	Date/beginning	Event	Country
300 000	–	14.11.1970	Storm and flood catastrophe	Bangladesh
250 000	–	28.07.1976	Earthquake in Tangshan (8.2 Richter scale)	China
138 000	3	29.04.1991	Tropical Cyclone Gorky	Bangladesh
60 000	–	31.05.1970	Earthquake (7.7 Richter scale)	Peru
50 000	414	12.12.1999	Floods, mudflows and landslides	Venezuela, Colombia
50 000	152	21.06.1990	Earthquake in Gilan	Iran
25 000	–	16.09.1978	Earthquake in Tabas	Iran
25 000	–	07.12.1988	Earthquake	Armenia, former USSR
23 000	–	13.11.1985	Volcanic eruption on Nevado del Ruiz	Colombia
22 000	227	04.02.1976	Earthquake (7.4 Richter scale)	Guatemala
19 118	1 034	17.08.1999	Earthquake in Izmit	Turkey
15 000	103	29.10.1999	Cyclone 05B devastates Orissa state	India, Bangladesh
15 000	–	01.09.1978	Flooding following monsoon rains in northern parts	India
15 000	516	19.09.1985	Earthquake (8.1 Richter scale)	Mexico
15 000	–	11.08.1979	Dyke burst in Morvi	India
10 800	–	31.10.1971	Flooding in Bay of Bengal and Orissa state	India
10 000	–	25.05.1985	Tropical cyclone in Bay of Bengal	Bangladesh
10 000	–	20.11.1977	Tropical cyclone in Andrah Pradesh, Bay of Bengal	India
9 500	–	30.09.1993	Earthquake (6.4 Richter scale) in Maharashtra	India
9 000	529	22.10.1998	Hurricane Mitch in Central America	Honduras, Nicar. et al.
8 000	–	16.08.1976	Earthquake on Mindanao	Philippines
6 425	2 795	17.01.1995	Great Hanshin earthquake in Kobe	Japan
6 304	–	05.11.1991	Typhoons Thelma and Uring	Philippines
5 300	–	28.12.1974	Earthquake (6.3 Richter scale)	Pakistan
5 000	1 016	05.03.1987	Earthquake	Ecuador
5 000	–	10.04.1972	Earthquake in Fars	Iran
5 000	415	23.12.1972	Earthquake in Managua	Nicaragua
5 000	–	30.06.1976	Earthquake in West Irian	Indonesia
4 500	–	10.10.1980	Earthquake in El Asnam	Algeria
4 375	–	21.12.1987	Ferry Dona Paz collides with oil tanker Victor	Philippines
4 000	–	30.05.1998	Earthquake in Takhar	Afghanistan
4 000	–	24.11.1976	Earthquake in Van	Turkey
4 000	–	15.02.1972	Storms and snow in Ardekan	Iran
3 840	5	01.11.1997	Typhoon Linda	Vietnam et al.
3 800	–	08.09.1992	Flooding in Punjab	India, Pakistan
3 656	318	01.07.1998	Flooding along Yangtze River	China
3 400	1 034	21.09.1999	Earthquake in Nantou	Taiwan
3 200	–	16.04.1978	Tropical cyclone	Reunion
3 100	84	23.11.1980	Earthquake (7.2 Richter scale) in Campagna and Basilicata	Italy
3 000	–	04.07.1998	Flooding and landslides	India, Bangladesh et al.
3 000	–	11.06.1981	Earthquake	Iran
3 000	–	02.12.1984	Accident in chemical plant in Bhopal	India
3 000	–	01.08.1988	Rain, floods	Bangladesh

[25] dead or missing
[26] excl. liability damage

Table 2: The 43 worst catastrophes in terms of victims 1970-2000

Dates	Times	Times between
1970-01-01	0	Starting time
1970-08-04	215	215
1973-04-25	1210	995
1974-04-02	1552	342
1974-09-18	1721	169
1976-01-02	2192	471
1979-09-12	3541	1349
1983-08-17	4976	1435
1983-12-17	5098	122
1987-10-15	6496	1398
1988-07-06	6761	265
1988-09-10	6827	66
1989-09-15	7197	370
1989-10-17	7229	32
1989-10-23	7235	6
1990-01-25	7329	94
1990-02-25	7360	31
1991-09-27	7939	579
1991-10-20	7962	23
1992-08-23	8270	308
1992-09-11	8289	19
1992-12-11	8380	91
1993-03-10	8469	89
1993-10-27	8700	231
1994-01-17	8782	82
1995-01-17	9147	365
1995-01-21	9151	4
1995-05-05	9255	104
1995-09-03	9376	121
1995-10-01	9404	28
1996-09-05	9744	340
1998-01-05	10231	487
1998-05-15	10361	130
1998-09-19	10488	127
1998-09-20	10489	1
1999-05-03	10714	225
1999-09-10	10844	130
1999-09-22	10856	12
1999-12-03	10928	72
1999-12-25	10950	22
1999-12-27	10952	2
1999-12-31	10956	End time

Table 3: Times of the worst catastrophes and times between them when costs are considered

Dates	Times	Times between
1970-01-01	0	Starting time
1970-05-31	150	150
1970-11-14	317	167
1971-10-31	668	351
1972-02-15	775	107
1972-04-10	830	55
1972-12-23	1087	257
1974-12-28	1822	735
1976-02-04	2225	403
1976-06-30	2372	147
1976-07-28	2400	28
1976-08-16	2419	19
1976-11-24	2519	100
1977-11-20	2880	361
1978-04-16	3027	147
1978-09-01	3165	138
1978-09-16	3180	15
1979-08-11	3509	329
1980-10-10	3935	426
1980-11-23	3979	44
1981-06-11	4179	200
1984-12-02	5449	1270
1985-05-25	5623	174
1985-09-19	5740	117
1985-11-13	5795	55
1987-03-05	6272	477
1987-12-21	6563	291
1988-08-01	6787	224
1988-12-07	6915	128
1990-06-21	7476	561
1991-04-29	7788	312
1991-11-05	7978	190
1992-09-08	8286	308
1993-09-30	8673	387
1995-01-17	9147	474
1997-11-01	10166	1019
1998-05-30	10376	210
1998-07-01	10408	32
1998-07-04	10411	3
1998-10-22	10521	110
1999-08-17	10820	299
1999-09-21	10855	35
1999-10-29	10893	38
1999-12-12	10937	44
1999-12-31	10956	End time

Table 4: Times of the worst catastrophes and times between them when victims are considered

Appendix B: R code

B.1 Code for tests applied on the first case

```
n <- 40; Tn <- 10952; tau <- 10956;
# n is the number of events and Tn is the time when the last
 observed event occurred and tau is the “stop time” in the
 time censored process

T <- c(215, 1210, 1552, 1721, 2192, 3541, 4976, 5098, 6496,
 6761, 6827, 7197, 7229, 7235, 7329, 7360, 7939, 7962, 8270,
 8289, 8380, 8469, 8700, 8782, 9147, 9151, 9255, 9376, 9404,
 9744, 10231, 10361, 10488, 10489, 10714, 10844, 10856,
 10928, 10950, 10952);
# Creating a vector T containing the times of the events

X <- c(215, 995, 342, 169, 471, 1349, 1435, 122, 1398, 265,
 66, 370, 32, 6, 94, 31, 579, 23, 308, 19, 91, 89, 231, 82,
 365, 4, 104, 121, 28, 340, 487, 130, 127, 1, 225, 130, 12,
 72, 22, 2);
# Creating a vector X containing the times between the events

sumTi <- sum(T)-10952;
# Add together times of events, except for the time of the
 last event, for the failure censored process. For the time
 censored sumTi <- sum(T).


L <- ( sumTi - (n-1)*Tn/2 ) / ( Tn*sqrt((n-1)/12) );
# The Laplace test statistic. For a time censored process n-1
 is exchanged to n and Tn to tau.


B1 <- -( ( sqrt((n-1)/n) ) * L);
# The test statistic B1


sumB2 <- 0;
# sumB2 is the sum in the test statistic B2
for( i in 1:(n-1) ){
 sumB2 <- sumB2 + ( 1/(i*(n-i)) )*( T[i] - i/n*Tn)^2;
}
B2 <- ( 1/(Tn/n)^2 )*sumB2;
# The test statistic B2
```

```
sumAD <- 0;
# sumAD is the sum in the Anderson Darling test for trend
 statistic
for( i in 1:(n-1) ){
 sumAD <- sumAD + (2*i-1)*( log(T[i]/Tn) + log(1 - (T[n-
 i]/Tn)) );
}
AD <- -(n-1) - (1/(n-1))*sumAD;
# The Anderson Darling test for trend statistic. For the time
censored process n-1 is exchanged to n and Tn to tau.


sumS <- 0;
# sumS is the sum in the estimated variance of the
 interarrival times, S2
for( i in 1:n ){
 sumS <- sumS + (X[i] - Tn/n)^2 ;
}
S2 <- sumS/(n-1);
 # S2 is the estimated variance of the interarrival times
S <- sqrt(S2);
# S is the estimated standard deviation of the interarrival
 times
LR <- ( (Tn/n)/S ) * ( (sumTi - ((n-1)/2) * Tn ) / ( Tn *
 sqrt((n-1)/12) ) );
# The Lewis Robinson test statistic. For the time censored
 process n-1 is exchanged to n and Tn to tau.


sumSigma <- 0;
# sumSigma is the sum in the estimated variance of the
 interarrival times
for( i in 1:(n-1) ){
 sumSigma <- sumSigma + (X[i+1] - X[i])^2 ;
}
Sigma2 <- sumSigma / (2*(n-1));
# Sigma2 is the estimated variance of the interarrival times
Sigma <- sqrt(Sigma2);
# Sigma is the estimated standard deviation of the
 interarrival times
LR2 <- ( (Tn/n)/Sigma ) * ( ( sumTi - ((n-1)/2) * Tn ) / ( Tn
 * sqrt((n-1)/12) ) );
# The modified Lewis Robinson test statistic LR2 . For the
 time censored process n-1 is exchanged to n and Tn to tau.
```

```
q <- NULL; r <- NULL;
# q and r are a part of the GAD test statistic, see Section
 3.2.2
 for( i in 1:n){
     q[i] <- (T[i] - i*X[i]) / Tn;
 }
for( i in 1:n ){
     r[i] <- n * X[i]/Tn - 1;
 }
sumGAD <- 0;
# sumGAD is the sum in the Generalized Anderson Darling test
 statistic
q1 <- 0;
# q1 is the part of sumGAD when i=1
q1 <- ( (q[1] + r[1])^2 ) * log(n/(n-1)) - (r[1]^2)/n
qn <- 0;
# qn is the part of sumGAD when i=n
qn <- (q[n]^2) * log(n/(n-1)) - (r[n]^2)/n
sumqr <- 0;
# sumqr is the rest of sumGAD, i.e. all terms except the
 first and the last
for( i in 2:(n-1) ){
     sumqr <- sumqr + (q[i]^2) * log(i/(i-1)) + ((q[i] +
 r[i])^2) * log( (n-i+1)/(n-i) ) - (r[i]^2)/n;
}
sumGAD <- q1 + qn + sumqr;
# Adding the components of sumGAD to get the result of sumGAD
GAD <- (n-4) / Sigma2 * ((Tn/n)^2) * sumGAD;
# The Generalized Anderson Darling test statistic

Mt <- 0;
for( i in 1:(n-1) ){
 for( j in (i+1):n ){
     if(X[i] < X[j]){
          Mt <- Mt + 1
     }
 }
}
# Mt is the Mann test statistic
mu <- n*(n-1)/4;
s2 <- (2*n^3 + 3*n^2 - 5*n)/72;
M <- (Mt - mu)/sqrt(s2);
# Mt is rescaled to be standard normally distributed
```

B.2 Code for tests applied on the second case

```
n <- 43; Tn <- 10937; tau <- 10956;
# n is the number of events and Tn is the time when the last
 observed event occurred and tau is the “stop time” in the
 time censored process

T <- c(150, 317, 668, 775, 830, 1087, 1822, 2225, 2372, 2400,
 2419, 2519, 2880, 3027, 3180, 3165, 3509, 3935, 3979, 4179,
 5449, 5623, 5740, 5795, 6272, 6563, 6787, 6915, 7476, 7788,
 7978, 8286, 8673, 9147, 10166, 10376, 10408, 10411, 10521,
 10820, 10855, 10893, 10937);
# Creating a vector T containing the times of the events

X <- c(150, 167, 351, 107, 55, 257, 735, 403, 147, 28, 19,
 100, 361, 147, 138, 15, 329, 426, 44, 200, 1270, 174, 117,
 55, 477, 291, 224, 128, 561, 312, 190, 308, 387, 474, 1019,
 210, 32, 3, 110, 299, 35, 38, 44);
# Creating a vector X containing the times between the events

sumTi <- sum(T)-10937;
# Add together times of events, except for the time of the
 last event, for the failure censored process. For the time
 censored sumTi <- sum(T).


L <- ( sumTi - (n-1)*Tn/2 ) / ( Tn*sqrt((n-1)/12) );
# The Laplace test statistic


B1 <- -( ( sqrt((n-1)/n) ) * L );
# The test statistic B1


sumB2 <- 0;
# sumB2 is the sum in the test statistic B2
for(i in 1:(n-1)){
 sumB2 <- sumB2 + ( 1/(i*(n-i)) )*( T[i] - i/n*Tn)^2;
}
B2 <- ( 1/(Tn/n)^2 )*sumB2;
# The test statistic B2


sumAD <- 0;
# sumAD is the sum in the Anderson Darling test for trend
```

```
statistic
for( i in 1:(n-1) ){
sumAD <- sumAD + (2*i-1)*( log(T[i]/Tn) + log(1 - (T[n-
i]/Tn)) );
}
AD <- -(n-1) - (1/(n-1))*sumAD;
# The Anderson Darling test for trend statistic. For the time
censored process n-1 is exchanged to n and Tn to tau.


sumS <- 0;
# sumS is the sum in the estimated variance of the
interarrival times, S2
for( i in 1:n ){
sumS <- sumS + (X[i] - Tn/n)^2 ;
}
S2 <- sumS/(n-1);
# S2 is the estimated variance of the interarrival times
S <- sqrt(S2);
# S is the estimated standard deviation of the interarrival
times
LR <- ( (Tn/n)/S ) * ( ( sumTi - ((n-1)/2) * Tn ) / ( Tn *
sqrt((n-1)/12) ) );
# The Lewis Robinson test statistic. For the time censored
process n-1 is exchanged to n and Tn to tau.


sumSigma <- 0;
# sumSigma is the sum in the estimated variance of the
interarrival times
for( i in 1:(n-1) ){
sumSigma <- sumSigma + (X[i+1] - X[i])^2 ;
}
Sigma2 <- sumSigma / (2*(n-1));
# Sigma2 is the estimated variance of the interarrival times
Sigma <- sqrt(Sigma2);
# Sigma is the estimated standard deviation of the
interarrival times
LR2 <- ( (Tn/n)/Sigma ) * ( ( sumTi - ((n-1)/2) * Tn ) / ( Tn
* sqrt((n-1)/12) ) );
# The modified Lewis Robinson test statistic LR_2 . For the
time censored process n-1 is exchanged to n and Tn to tau.


q <- NULL; r <- NULL;
# q and r are a part of the GAD test statistic, see Section
```

3.2.2

```
for( i in 1:n ){
     q[i] <- (T[i] - i*X[i]) / Tn;
 }
for( i in 1:n ){
     r[i] <- n * X[i]/Tn - 1;
 }
sumGAD <- 0;
# sumGAD is the sum in the Generalized Anderson Darling test
 statistic
q1 <- 0;
# q1 is the part of sumGAD when i=1
q1 <- ( (q[1] + r[1])^2 ) * log(n/(n-1)) - (r[1]^2)/n;
qn <- 0;
# qn is the part of sumGAD when i=n
qn <- (q[n]^2) * log(n/(n-1)) - (r[n]^2)/n;
sumqr <- 0;
# sumqr is the rest of sumGAD, i.e. all terms except the
 first and the last
for( i in 2:(n-1) ){
     sumqr <- sumqr + (q[i]^2) * log(i/(i-1)) + ((q[i] +
 r[i])^2) * log( (n-i+1)/(n-i) ) - (r[i]^2)/n;
}
sumGAD <- q1 + qn + sumqr;
# Adding the components of sumGAD to get the result of sumGAD
GAD <- (n-4) / Sigma2 * ((Tn/n)^2) * sumGAD;
# The Generalized Anderson Darling test statistic


Mt <- 0;
# Mt is the Mann test statistic
for( i in 1:(n-1) ){
 for( j in (i+1):n ){
     if(X[i] < X[j]){
          Mt <- Mt + 1
     }
 }
}
mu <- n*(n-1)/4;
s2 <- (2*n^3 + 3*n^2 - 5*n)/72;
M <- (Mt - mu)/sqrt(s2);
# Mt is rescaled to be standard normally distributed
```

References

[1] Kvaløy, Jan Terje & Lindqvist, Bo Henry & Malmedal, Håkon: *A statistical test for monotonic and non-monotonic trend in repairable systems*. Conference Proceedings ESREL 2001, Torino, 16-20 September, pp. 1563-1570

[2] Antoch, Jaromír & Jarusková, Daniela: *Testing a homogeneity of stochastic processes*. Kybernetika. 2007; 43(4), pp 415-430

[3] Gallager, Robert G: *Renewal processes* (Cited 2011 November 3) Available from: http://www.rle.mit.edu/rgallager/documents/6.262chap3Vy.pdf

[4] Chen, Kani: *Renewal Phenomena* (Cited 2011 November 3) Available from: http://www.math.ust.hk/~makchen/Math341/Chap7part1.pdf

[5] Alm, Sven Erick & Britton, Tom: *Stokastik Sannolikhetsteori och statistikteori med tillämpningar*. Stockholm: Liber AB. 2008, pp. 196-204

[6] Kvaløy, Jan Terje & Lindqvist, Bo Henry: *TTT-based tests for trend in repairable systems data*. Reliability Engineering and System Safety. 1998; 60, pp. 13-28

[7] Elvebakk, Georg & Lindqvist, Bo Henry & Heggland, Knut: *The trend-renewal process for statistical analysis of repairable systems*. Technometrics. 2003; 45(1), pp. 31-44

[8] Lindqvist, Bo Henry: *Statistical Modeling and Analysis of Repairable Systems*. Invited lecture given at 1st International Conference on Mathematical Methods in Reliability, Bucharest, Romania, September 16-19. 1997

[9] Lindqvist, Bo Henry & Heggland, Knut: *A nonparametric monotone maximum likelihood estimator of time trend for repairable systems data*, Reliability Engineering and System Safety. Special issue on Stochastic Processes in Reliability. 2007; 92,pp. 575–584

[10] Kvaløy, Jan Terje & Lindqvist, Bo Henry: *A class of tests for renewal process versus monotonic and nonmonotonic trend in repairable data*. Mathematical and Statistical Methods in Reliability. 2003; 7, pp. 401-414

[11] Wang, Peng & Coit, David W: *Repairable Systems Reliability Trend Tests and Evaluation*. Annual Reliability and Maintainability Symposium, 2005 Proceedings. 2005, pp. 416-421

[12] Malmedal, Håkon: *Trendtesting for repeterte hendelser*. Diploma Thesis, (In Norwegian), Department of Mathematical Sciences, Norwegian University of Science and Technology. 2001

[13] Wadagale, Atul Viraj & Thatkar, P.V. & Dase, R.K. & Tandale, D.V: *Modified Anderson Darling Test for Wind Speed Data*. International Journal of Computer Science & Emerging Technologies (E-ISSN: 2044-6004). 2011; 2(2), pp. 249-251

[14] Anderson, T. W. & Darling, D. A: *A test of goodness of fit*, Journal of American Statistical Association. 1954; 49(268), pp. 765– 769

[15] Chen, Colin: *Test of fit for the three-parameter lognormal distribution*. Computational Statistics & Data Analysis. 2006; 50(6), pp. 1418-1440

[16] Sigma No. 2/2001: *Natural catastrophes and man-made disasters in 2000: fewer insured losses despite huge floods*. 2001 (Cited 2011 November 3) Available from: http://reliefweb.int/sites/reliefweb.int/files/resources/A60EE9AB30BC3B5BC1256C1E0038949F-swissre-natdis2000-26jan.pdf

[17] Swiss Re. (Cited 2011 November 3) Available from: http://www.swissre.com/

Sources of figures and tables

Figure 1-2: [8]

Table 1-2: [16]