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RELIABILITY ANALYSIS FOR COMPLEX, REPAIRABLE SYSTEMS

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LARRY H. CROW

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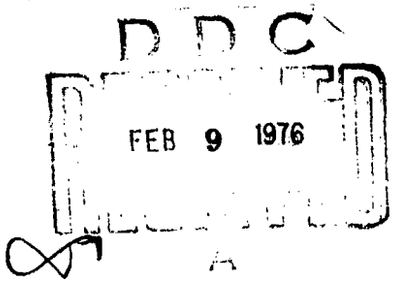

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RELIABILITY ANALYSIS FOR COMPLEX, REPAIRABLE SYSTEMS

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Reliability Analysis For Complex, Repairable Systems

Larry H. Crow*

Abstract. The reliability of a complex system that is repaired (but not replaced) upon failure will often depend on the system chronological age. If only minimal repair is made so that the intensity (instantaneous rate) of system failure is not disturbed, then a nonhomogeneous Poisson process may be used to model this age-dependent reliability. This paper considers the theoretical and practical implications of the nonhomogeneous Poisson process model for reliability, and gives estimation, hypotheses testing, comparison and goodness of fit procedures when the process has a Weibull intensity function. Applications of the Weibull model in the field of reliability and in other areas are discussed.

1. Introduction. Many systems can be categorized into two basic types; one-time or nonrepairable systems, and reusable or repairable systems. The term "system" is used in a broad sense in this paper and may, for example, simply mean a component. If continuous operation of the system is desired, then in the former case the system would be replaced by a new system upon failure. An example would be the replacement of a failed light bulb by a new bulb. The component or

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system may, of course, be part of a larger system. For example, the water pump of a vehicle may be considered a one-time or nonrepairable system. If failure data are available for a nonrepairable system, then, since the failure times are independent and identically distributed, the analyses may involve the estimation of the corresponding life distribution. In the latter case, under continuous operation, the system is repaired, but not replaced, after each failure. For example, if the system is a vehicle and the water pump fails, then the water pump is replaced and, hence, the vehicle is repaired.

For a repairable system, one is rarely interested primarily in time to first failure. Rather, interest generally centers around the probability of system failure as a function of system age. Exact reliability analyses for complex, repairable systems are often difficult because of the complicated failure process that may result from the replacement or repair policy. A common procedure in practice is to approximate the complicated stochastic process by a simpler stochastic process, which although not exact, still yields useful practical results. One such mathematical idealization assumes that the failure times of the complex repairable system follow a (non) homogeneous Poisson process.

In the next section, this paper discusses the homogeneous and nonhomogeneous Poisson processes with respect to the theoretical and practical implications of these models for the reliability theory of complex, repairable systems. In Section 3 we give estimation,

hypothesis testing, comparison and goodness of fit procedures for a nonhomogeneous Poisson process with mean value function given by a Weibull intensity function. In Section 4 application of these procedures in reliability and other areas is discussed.

2. The Poisson Processes. In this section, we discuss the theoretical and practical implications of the Poisson processes in terms of the reliability of a complex, repairable system. We begin with the homogeneous Poisson process and later generalize to the process of particular concern in this paper, the nonhomogeneous Poisson process.

Let λ denote the intensity of a homogeneous Poisson process. If Δt is infinitesimally small, then $\lambda\Delta t$ is approximately the probability of an event occurring in any interval of length Δt, regardless of the time t at the beginning of the interval. In terms of a repairable system, this implies that the system is not improving nor wearing out with age, but rather is maintaining a constant intensity of failure.

For various repairable systems, particularly of the complex electronic type, a constant intensity of failure has been observed to be closely representative, after perhaps an initial burn-in period. The period in which a system exhibits a constant intensity of failure is often called the system "useful life." If a complex system consists of a large number of components, each acting independently, if the failure of a component results in a failure of the system, and if each component is replaced upon failure, then under fairly general conditions, the occurrence of failures of the system will approach a constant

intensity as component number and operating time become large. (See Barlow and Proschan [1965], pp. 18-21.) To benefit from this result in practice may require an extensive amount of system operating time. Also, many complex repairable systems, for example vehicles, generally experience a wear-out phase which eventually makes them economically impractical or too unreliable to continue in service without perhaps undergoing overhaul. Therefore, these systems generally will never achieve the equilibrium state of a homogeneous Poisson process.

The homogeneous Poisson process cannot describe the occurrence of failures for many systems over their entire life cycle but may often be suitable as a model over some portion of their life. For studies involving the consideration of mission reliability, reliability growth, maintenance policies, overhaul and trade-in times, etc., it is important that realistic models be applied.

A generalization of the homogeneous Poisson process which allows for changes or trend in the intensity of system failures is the nonhomogeneous Poisson process with intensity function $u(t)$. Analogous to $\lambda\Delta t$ in the homogeneous Poisson process, $u(t)\Delta t$ is approximately the probability that an event will occur in the interval $(t, t + \Delta t)$. Note that in the special case when the intensity function $u(t)$ is constant for all t, the nonhomogeneous Poisson process reduces to the homogeneous Poisson process. Therefore, any results or considerations related to the nonhomogeneous Poisson process are also valid for the homogeneous Poisson process.

Unlike the homogeneous Poisson process failure probability, the intensity, $u(t)\Delta t$, may depend on the age t of the system. During debugging, u(t) would be decreasing, u(t) would be constant over the system useful life, and would be increasing during the wear-out phase of the system.

If the failures of a repairable system follow a nonhomogeneous Poisson process, then the number of failures and the types of repair actions taken during a period [0,t] do not affect the probability of failure during $(t, t + \Delta t)$. In particular, suppose that the system fails at time t and is subsequently repaired and put back into service. (Repair time is assumed to be negligible.) According to the model, the probability of a system failure during $(t, t + \Delta t)$ is $u(t)\Delta t$, and would equal this value even if the system had not failed at time t. In practice, however, if repair is competent, then one would expect a decrease in failure probability after repair from its value at the time of failure. If the system is complex, consisting of many components, then the replacement of a single component may not decrease this probability significantly. For example, the replacement of a failed water pump in a vehicle would generally not improve vehicle reliability greatly immediately after replacement from what it was immediately before failure. The nonhomogeneous Poisson process assumes idealistically that the reliability does not change at all.

A basic property of the nonhomogeneous Poisson process will be given next.

Theorem 2.1. Let $\{N(t), t \geq 0\}$ be a nonhomogeneous Poisson process, let $X_0 = 0$, and $X_1 < X_2 < X_3 \ldots$ be successive times of occurrence of events. Let $Y_i = X_i - X_{i-1}$, $i = 1,2,\ldots$, be the times between successive events. Then the cumulative distribution function (c.d.f.) F_i of Y_i, given that the (i-1)st event occurred at time X_{i-1}, is

$$F_i(y) = \frac{F(X_{i-1} + y) - F(X_{i-1})}{1 - F(X_{i-1})},$$

$y > 0$, $i = 1,2,\ldots$, where $F(x) = 1 - e^{-U(x)}$ and $U(x) = \int_0^x u(z)dz$.

Hence, in general, the times between successive events are not independent and identically distributed. However, when $u(t) \equiv \lambda$ (the homogeneous Poisson process), then these times are independent and identically distributed according to the exponential distribution with mean $1/\lambda$, since $F_i(y) = 1 - e^{-\lambda y}$, $y > 0$, $i = 1,2,\ldots$. Furthermore, observe that in the nonhomogeneous case the c.d.f. F_1 of the time to first event is identically equal to $F(y) = 1 - e^{-U(y)}$. The failure rate of F is defined by $r(y) = f(y)/[1-F(y)]$ for $F(y) < 1$, and $r(y) = \infty$ for $F(y) = 1$, where f denotes the density of F. Hence, the failure rate for F is $r(y) \equiv u(y)$, $y > 0$. That is, the intensity function $u(\cdot)$ of the nonhomogeneous Poisson process is equivalent to the failure rate of F. The physical interpretation of the failure rate is that for an infinitesimally small Δy, $r(y)\Delta y$ is approximately the probability that the

first event occurs in $(y,y+\Delta y)$, given no event to time y.

For the homogeneous Poisson process, F is the exponential distribution with constant failure rate. A popular alternative to the exponential distribution is the Weibull distribution $H(y) = 1 - e^{-\lambda y^{\beta}}$, $y > 0$, $\lambda > 0$, $\beta > 0$, with failure rate $r(y) = \lambda\beta y^{\beta-1}$. When $\beta = 1$, the Weibull reduces to the exponential. This suggests that a possibly useful extension of the homogeneous Poisson process with exponential times between failures is the nonhomogeneous Poisson process with Weibull time to first failure. The intensity function for this nonhomogeneous Poisson process is $u(t) = \lambda\beta t^{\beta-1}$, $t > 0$. If the failures of a repairable system follow this process, then for $\beta > 1$ ($\beta < 1$), $u(t)$ is increasing (decreasing) and, hence, the system is wearing out (improving) with age. When $\beta = 1$, the process reduces to the homogeneous case with intensity λ.

The nonhomogeneous Poisson process with Weibull intensity function will be considered further in the remainder of this paper.

3. The Weibull Intensity Function. In practice one may not know various parameters of a particular model that is to be applied. Consequently, the application of the model may depend on which statistical procedures regarding these unknown parameters are available.

In this section it is assumed that events are occurring according to a nonhomogeneous Poisson process with Weibull intensity function

$$u(t) = \lambda\beta t^{\beta-1} \tag{3.1}$$

$\lambda > 0$, $\beta > 0$, $t > 0$. Estimation, hypotheses testing, comparison and goodness of fit procedures are given when data consist of the times of the successive events occurring during the period of study. First, we need the following preliminaries.

In reliability terminology, suppose that the number of systems under study is K and the q-th system is observed continuously from time S_q to time T_q, $q = 1,\ldots,K$. During the period $[S_q, T_q]$, let N_q be the number of failures experienced by the q-th system and let X_{iq} be the age of this system at the i-th occurrence of failure, $i = 1,\ldots,N_q$, $q = 1,\ldots,K$. The times S_q, T_q, $q = 1,\ldots,K$, may possibly be observed failure times for the q-th system. If $X_{N_q,q} = T_q$, then the data on the q-th system are said to be failure truncated, and T_q is a random variable with N_q fixed. If $X_{N_q,q} < T_q$, then the data on the q-th system are said to be time truncated with N_q a random variable. Note that when S_q is not a failure time and data are time truncated, then N_q is a Poisson random variable with mean $U(T_q) - U(S_q) = \int_{S_q}^{T_q} u(t)dt$.

3.1. Maximum Likelihood Estimates of λ and β. Suppose the q-th system is observed continuously from time S_q to time T_q, $q = 1,\ldots,K$. Then the maximum likelihood (ML) estimates of λ and β are values $\hat{\lambda}$ and $\hat{\beta}$ satisfying the equations

$$\hat{\lambda} = \frac{\sum_{q=1}^{K} N_q}{\sum_{q=1}^{K} (T_q^{\hat{\beta}} - S_q^{\hat{\beta}})}, \tag{3.2}$$

$$\hat{\beta} = \frac{\sum_{q=1}^{K} N_q}{\hat{\lambda} \sum_{q=1}^{K} (T_q^{\hat{\beta}} \log T_q - S_q^{\hat{\beta}} \log S_q) - \sum_{q=1}^{K} \sum_{i=1}^{N_q} \log X_{iq}}, \tag{3.3}$$

where $0 \cdot \log 0$ is taken to be 0. In general, these equations cannot be solved explicitly for $\hat{\lambda}$ and $\hat{\beta}$, but must be solved by iterative procedures. Obtaining $\hat{\lambda}$ and $\hat{\beta}$, one may then estimate the intensity function u(t) by

$$\hat{u}(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1}, \tag{3.4}$$

$t > 0$.

When $S_q = 0$, and data are time truncated at $T_q = T$, $q = 1,\ldots,K$, then the ML estimates $\hat{\lambda}$ and $\hat{\beta}$ are in closed form. Specifically,

$$\hat{\lambda} = \frac{\sum_{q=1}^{K} N_q}{KT^{\hat{\beta}}}, \tag{3.5}$$

$$\hat{\beta} = \frac{\sum_{q=1}^{K} N_q}{\sum_{q=1}^{K} \sum_{i=1}^{N_q} \log(\frac{T}{X_{iq}})}. \tag{3.6}$$

When $K = 1$, $S_1 = 0$ and data are failure truncated (i.e., $X_{N_1,1} = T_1$), then $\hat{\lambda}$ and $\hat{\beta}$ are also in closed form. The estimates are

$$\hat{\lambda} = \frac{N_1}{T_1^{\hat{\beta}}} \tag{3.7}$$

$$\hat{\beta} = \frac{N_1}{\sum_{i=1}^{N_1-1} \log\left(\frac{T_1}{X_{i1}}\right)}. \tag{3.8}$$

Example. Suppose $K = 3$ systems are observed during $[0, T]$, $T = 200$. That is, the data are time truncated with $T_q = 200$, $q = 1,2,3$. This experiment was simulated on a computer with $\lambda = 0.6$ and $\beta = 0.5$. These results are given in Table 1. Since the T_q's are equal, the ML estimates of λ and β are calculated from the closed form expressions (3.5) and (3.6). From the simulated data, the ML estimate of λ is $\hat{\lambda} = 0.461$, and the ML estimate of β is $\hat{\beta} = 0.615$.

If it is assumed a priori that systems 1, 2, and 3 are observed only to the 10th, 15th and 11th failure respectively, then the data are failure truncated. Thus, the ML estimates of λ and β are calculated from expressions (3.2) and (3.3) using iterative procedures. From the simulated data $T_1 = 197.2$, $T_2 = 190.8$, $T_3 = 195.8$, and the ML estimates of λ and β are $\hat{\lambda} = 0.443$, $\hat{\beta} = 0.626$.

TABLE 1

Simulated Data for K=3 Systems Operated for Time T=200 when $\lambda = 0.6$ and $\beta = 0.5$

System 1	System 2	System 3
X_{i1}	X_{i2}	X_{i3}
4.3	0.1	8.4
4.4	5.6	32.5
10.2	18.6	44.7
23.5	19.5	48.4
23.8	24.2	50.6
26.4	26.7	73.6
74.0	45.1	98.7
77.1	45.8	112.2
92.1	75.7	129.8
197.2	79.7	136.0
	98.6	195.8
	120.1	
	161.8	
	180.6	
	190.8	
$N_1 = 10$	$N_2 = 15$	$N_3 = 11$
$\sum_{i=1}^{10} \log\left(\frac{T}{X_{i1}}\right)$	$\sum_{i=1}^{15} \log\left(\frac{T}{X_{i2}}\right)$	$\sum_{i=1}^{11} \log\left(\frac{T}{X_{i3}}\right)$
= 19.661	= 26.434	= 12.398

$$N = N_1 + N_2 + N_3 = 36$$

$$\sum_{q=1}^{3} \sum_{i=1}^{N_q} \log\left(\frac{T}{X_{iq}}\right) = 58.493$$

$$\hat{\beta} = \frac{N}{\sum_{q=1}^{3} \sum_{i=1}^{N_q} \log\left(\frac{T}{X_{iq}}\right)} = 0.615 \qquad \bar{\beta} = \frac{N-1}{N}\hat{\beta} = 0.598$$

$$\hat{\lambda} = \frac{N}{KT\hat{\beta}} = 0.461$$

3.2. Conditional Maximum Likelihood Estimates of β.

Estimates of β which are always in closed form w'enever $S_q = 0$, $q = 1,\ldots,K$, may be obtained by considering the conditional ML estimates. If the intensity function for the q-th system is

$$u_q(t) = \lambda_q \beta t^{\beta-1}, \tag{3.9}$$

$\lambda_q > 0$, $\beta > 0$, $t > 0$, these conditional ML estimates are the same whether the λ_q's are specified to be equal or different.

Suppose that data on the q-th system are failure truncated (i.e., $X_{N_q,q} = T_q$) and $S_q = 0$, $q = 1,\ldots,K$. Then, conditioned on the failure times $T_1,\ldots,T_K$, the ML estimate of β is

$$\tilde{\beta} = \frac{\sum_{q=1}^{K} (N_q - 1)}{\sum_{q=1}^{K} \sum_{i=1}^{N_q - 1} \log\left(\frac{T_q}{X_{iq}}\right)}. \tag{3.10}$$

If the data on the q-th system are time truncated and $S_q = 0$, $q = 1,\ldots,K$, then N_q is the random number of failures experienced by this system during $[0, T_q]$, $q = 1,\ldots,K$. Conditioned on $N_1,\ldots,N_K$, the ML estimate of β is

$$\tilde{\beta} = \frac{\sum_{q=1}^{K} N_q}{\sum_{q=1}^{K} \sum_{i=1}^{N_q} \log\left(\frac{T_q}{X_{iq}}\right)}. \tag{3.11}$$

In general, let

$$M_q = \begin{cases} N_q & \text{if data on the q-th system are time truncated} \\ N_q - 1 & \text{if data on the q-th system are failure truncated,} \end{cases} \tag{3.12}$$

$q = 1,\ldots,K$. Then, conditioned on N_q (time truncated) or $X_{N_q,q}$ (failure truncated), $q = 1,\ldots,K$, the ML estimate of β is

$$\tilde{\beta} = \frac{\sum_{q=1}^{K} M_q}{\sum_{q=1}^{K} \sum_{i=1}^{M_q} \log\left(\frac{T_q}{X_{iq}}\right)}. \tag{3.13}$$

Further, if

$$M = \sum_{q=1}^{K} M_q, \tag{3.14}$$

then

$$\bar{\beta} = \frac{M-1}{M} \cdot \tilde{\beta} \tag{3.15}$$

is an unbiased estimate of β.

In the special case when the data are time truncated at $T_q = T$ and $S_q = 0$, $q = 1,\ldots,K$, observe that the ML estimate $\hat{\beta}$ given by (3.6) and the conditional ML estimate $\tilde{\beta}$ given by (3.13) are equal.

Example. Consider the simulated data in Table 1. Since the data for each of the 3 systems are time truncated at $T = 200$, then $\hat{\beta} = \tilde{\beta} = 0.615$. The unbiased estimate of β is $\bar{\beta} = (35/36)\tilde{\beta} = 0.598$.

Suppose that it is assumed a priori that systems 1, 2 and 3

are observed only to the 10th, 15th and 11th failure, respectively. In this case the data are failure truncated respectively at 197.2, 190.8 and 195.8 for systems 1, 2 and 3. The conditional ML estimate of β is therefore

$$\tilde{\beta} = \frac{M}{\sum_{q=1}^{3} \sum_{i=1}^{M_q} \log\left(\frac{T_q}{X_{iq}}\right)} = 0.575,$$

where $M = M_1 + M_2 + M_3$, $M_1 = 9$, $M_2 = 14$, $M_3 = 10$, $T_1 = 197.2$, $T_2 = 190.8$, and $T_3 = 195.8$. The unbiased estimate of β is $\bar{\beta} = (32/33)\tilde{\beta} = 0.557$.

3.3. Hypotheses Tests and Confidence Bounds on β. We may use the conditional ML estimate of β, $\tilde{\beta}$ given by (3.13) to test hypotheses and construct conditional confidence bounds on the true value of β. To do this we use the result that

$$\chi^2 = \frac{2M\beta}{\tilde{\beta}} \tag{3.16}$$

is distributed as a Chi-Square random variable with 2M degrees of freedom. This statistic may therefore be used to test hypotheses on β. When M is moderate, the statistic

$$\omega = \sqrt{M}\left(\frac{\beta}{\tilde{\beta}} - 1\right) \tag{3.17}$$

is distributed approximately normal with mean 0 and variance 1. Consequently, one may also use this statistic to test hypotheses on β for moderate M.

To construct exact, conditional confidence bounds on β we use the χ^2 statistic given by (3.16). The exact $(1-\alpha)\cdot 100$ percent lower and upper confidence bounds are

$$\beta_{lb} = \tilde{\beta}\frac{\chi^2(\frac{\alpha}{2},2M)}{2M}, \tag{3.18}$$

$$\beta_{ub} = \tilde{\beta}\frac{\chi^2(1-\frac{\alpha}{2},2M)}{2M}, \tag{3.19}$$

respectively, where $\chi^2(\frac{\alpha}{2},2M)$ $[\chi^2(1-\frac{\alpha}{2},2M)]$ is the $\frac{\alpha}{2}$-th $[(1-\frac{\alpha}{2})$-th] percentile for the Chi-Square distribution with 2M degrees of freedom.

When M is moderate however, the statistic ω, which is approximately normal, may be used to construct approximate confidence bounds. This approximation yields $(1-\alpha)\cdot 100$ percent lower and upper confidence bounds

$$\beta_{lb} = \tilde{\beta}[1 - \frac{P_\pi}{\sqrt{M}}], \tag{3.20}$$

$$\beta_{ub} = \tilde{\beta}[1 + \frac{P_\pi}{\sqrt{M}}], \tag{3.21}$$

respectively, where P_π is the π-th percentile for the normal distribution with mean 0 and variance 1, $\pi = 1 - \alpha/2$

Example. Consider again the simulated results presented in Table 1. The conditional ML estimate of β was computed to be $\tilde{\beta} = 0.615$, with M = 36. Using the normal approximation, 90 percent conditional confidence bounds on β are

$$\beta_{1b} = \tilde{\beta}[1 - \frac{1.645}{6}] = 0.446,$$

$$\beta_{ub} = \tilde{\beta}[1 + \frac{1.645}{6}] = 0.784.$$

3.4. Hypotheses Tests and Confidence Bounds on λ (β known).

Suppose that observations begin at time 0 on each of the K systems under study. That is, $S_q = 0$, $q = 1,\ldots,K$. Under time truncated testing on these K systems, the random total number of failures $N = N_1 + \cdots + N_K$ has the Poisson distribution with mean $\theta = \lambda \sum_{q=1}^{K} T_q^{\beta}$, where T_q is the truncation time for the q-th system $q = 1,\ldots,K$. Also, under failure truncated testing on the K systems, the statistic $V = 2\lambda \sum_{q=1}^{K} X_{N_q,q}^{\beta}$ has the Chi-Square distribution with $2(N_1 + \cdots + N_K)$ degrees of freedom, where $X_{N_q,q}$ is the age of the q-th system at the N_q-th failure, $q = 1,\ldots,K$.

The statistics N and V may be used in the usual manner to test hypotheses and construct confidence bounds on λ when β is known. For time truncated testing, the $(1-\gamma)\cdot 100$ percent lower and upper confidence bounds on λ are

$$\lambda_{1b}(\beta) = \frac{\chi^2(\frac{\gamma}{2}, 2N)}{2\sum_{q=1}^{K} T_q^{\beta}}, \tag{3.22}$$

$$\lambda_{ub}(\beta) = \frac{\chi^2(1 - \frac{\gamma}{2}, 2N+2)}{2\sum_{q=1}^{K} T_q^{\beta}}), \tag{3.23}$$

respectively. For failure truncated testing, the $(1-\gamma)\cdot 100$ percent lower and upper confidence bounds on λ are

$$\lambda_{lb}(\beta) = \frac{\chi^2(\frac{\gamma}{2}, 2N)}{2\sum_{q=1}^{K} x_{N_q,q}^{\beta}}, \tag{3.24}$$

$$\lambda_{ub}(\beta) = \frac{\chi^2(1 - \frac{\gamma}{2}, 2N)}{2\sum_{q=1}^{K} x_{N_q,q}^{\beta}}, \tag{3.25}$$

respectively.

Example. Consider again the simulated data in Table 1. These data on K = 3 systems are time truncated at $T_1 = T_2 = T_3 = 200$. For N = 36, $\gamma = 0.05$, we obtain $\chi^2(\frac{\gamma}{2}, 2N) = 50.2$, $\chi^2(1 - \frac{\gamma}{2}, 2N+2) = 99.6$. Consequently, from (3.22) and (3.23), 95 percent lower and upper confidence bounds on λ are

$$\lambda_{lb}(\beta) = \frac{50.2}{2KT^{\beta}} = 0.592,$$

$$\lambda_{ub}(\beta) = \frac{99.6}{2KT^{\beta}} = 1.17,$$

where $\beta = 0.5$.

3.5. Simultaneous Confidence Bounds on λ and β. Recall from (3.18) and (3.19) that β_{lb} and β_{ub} denote respectively the $(1-\alpha)\cdot 100$ percent lower and upper conditional confidence bounds on β. Also, recall from (3.22) and (3.23) [(3.24) and (3.25)] that $\lambda_{lb}(\beta)$ and

$\lambda_{ub}(\beta)$ denote respectively the $(1-\gamma)\cdot 100$ percent lower and upper confidence bounds on λ when data are time truncated [failure truncated] on the K systems and β is known.

Suppose that data on the K systems are time truncated. Then $(1-\alpha)(1-\gamma)\cdot 100$ percent conservative* simultaneous confidence bounds on λ and β may be constructed in the following way. Given $(1-\alpha)\cdot 100$ percent conditional confidence bounds, β_{lb}, β_{ub}, on β, let

$$\lambda_{lb}(\beta_{ub}) = \frac{\chi^2(\frac{\gamma}{2}, 2N)}{2\sum_{q=1}^{K} T_q^{\beta_{ub}}}, \tag{3.26}$$

$$\lambda_{ub}(\beta_{lb}) = \frac{\chi^2(1-\frac{\gamma}{2}, 2N+2)}{2\sum_{q=1}^{K} T_q^{\beta_{lb}}}, \tag{3.27}$$

using (3.22) and (3.23). Then $(1-\alpha)(1-\gamma)\cdot 100$ percent conservative simultaneous confidence bounds on λ and β are $\{\lambda_{lb}(\beta_{ub}), \lambda_{ub}(\beta_{lb}); \beta_{lb}, \beta_{ub}\}$ when data on the K systems are time truncated.

When data on the K systems are failure truncated, then similar bounds are constructed using (3.24) and (3.25) to compute $\lambda_{lb}(\beta_{ub})$ and $\lambda_{ub}(\beta_{lb})$.

*That is, our assurance is at least, instead of exactly equal to, a specified value that the parameters will lie within the stated bounds.

Example. From the data in Table 1 and (3.18) - (3.19), we calculate that exact 90 percent conditional confidence bounds on β are $\beta_{1b} = 0.459$ and $\beta_{ub} = 0.793$. These data on the K = 3 systems are time truncated at $T_1 = T_2 = T_3 = 200$, with N = 36 failures. For $\gamma = 0.10$, we have $\chi^2(\frac{\gamma}{2},2N) = 53.5$, $\chi^2(1 - \frac{\gamma}{2},2N+2) = 95.1$. Also, $(200)^{\beta_{1b}} = 11.4$, $(200)^{\beta_{ub}} = 66.8$. Therefore, from (3.26) and (3.27),

$$\lambda_{1b}(\beta_{ub}) = \frac{53.5}{2\cdot 3\cdot (66.8)} = 0.133,$$

$$\lambda_{ub}(\beta_{1b}) = \frac{95.1}{2\cdot 3\cdot (11.4)} = 1.39.$$

Hence, $(.90)(.90)\cdot 100$ (= 81) percent conservative simultaneous confidence bounds on λ and β are $\{0.133, 1.39; 0.459, 0.793\}$.

3.6. Comparisons for β_q. Suppose that $K \geq 2$ systems are under study and the intensity function for the q-th system is

$$u_q(t) = \lambda_q \beta_q t^{\beta_q - 1} \tag{3.28}$$

$\lambda_q > 0$, $\beta_q > 0$, $t > 0$, $q = 1,\ldots,K$. Regarding the λ_q's as nuisance parameters, one may wish to compare the intensity functions $u_q(\cdot)$, $q = 1,\ldots,K$, by comparing the β_q's. Procedures for testing the hypothesis

$$H_o: \beta_1 = \cdots = \beta_K \tag{3.29}$$

will be considered next.

Let $\tilde{\beta}_q$ denote the conditional ML estimate of β_q given by

(3.13) calculated from data on the q-th system, q = 1,...,K. Then from (3.16),

$$\chi_q^2 = \frac{2M_q\beta_q}{\tilde{\beta}_q} \tag{3.30}$$

(q = 1,...,K) are conditionally distributed as independent Chi-Square random variables with respective degrees of freedom $2M_q$, where M_q is defined by (3.12).

When K = 2, we may test H_o using the statistic

$$F = \frac{\chi_1^2/2M_1}{\chi_2^2/2M_2}. \tag{3.31}$$

If H_o is true, then F equals $\tilde{\beta}_2/\tilde{\beta}_1$ and has, conditionally, the F distribution with $(2M_1, 2M_2)$ degrees of freedom. Consequently, one may calculate this statistic and refer to tables for the F distribution to determine the appropriate critical points to test H_o when K = 2.

For K > 2, to test the hypothesis $\beta_1 = \cdots = \beta_K$, the likelihood ratio procedure (see Lindgren [1962], p. 253) yields the statistic

$$L = \sum_{q=1}^{K} M_q \log(\tilde{\beta}_q) - M\log(\beta^*), \tag{3.32}$$

where $M = \sum_{q=1}^{K} M_q$ and $(\beta^*)^{-1} = \sum_{q=1}^{K} M_q \tilde{\beta}_q^{-1}/M$. Let

$$a = 1 + \frac{1}{6(K-1)}\left[\sum_{q=1}^{K}\frac{1}{M_q} - \frac{1}{M}\right].$$

Then, if H_o is true, the statistic

$$D = \frac{2L}{a} \tag{3.33}$$

is approximately distributed as a Chi-Square random variable with (K - 1) degrees of freedom, using (3.30) and the results of Bartlett (1937). One may therefore calculate the statistic D to test H_o when K > 2, and refer to the Chi-Square tables with (K - 1) degrees of freedom to determine the appropriate critical points.

Example. For the simulated data in Table 1 suppose that the intensity function for the q-th system is $u_q(t) = \lambda_q \beta_q t^{\beta_q - 1}$, q = 1,2,3, and we wish to test the hypothesis that $\beta_1 = \beta_2$. From the data on these systems we calculate that $\tilde{\beta}_1 = 0.509$ and $\tilde{\beta}_2 = 0.567$. Therefore, $F = (\tilde{\beta}_2 / \tilde{\beta}_1) = 1.11$. The 95-th percentile for the F distribution with (20, 30) degrees of freedom is 1.93. Since 1.11 < 1.93, we accept the hypothesis that $\beta_1 = \beta_2$ at the 5 percent significance level.

Suppose instead that we had wished to test the hypothesis that $\beta_1 = \beta_2 = \beta_3$. The appropriate statistic is D given by (3.33). From the data we find that D = 1.84. From the Chi-Square tables with K - 1 = 2 degrees of freedom, we find that the 2.5 and 97.5 percentiles are respectively 0.0506 and 7.38. Since 0.0506 < D < 7.38, we accept the hypothesis $\beta_1 = \beta_2 = \beta_3$ at the 5 percent significance level.

3.7. Goodness of Fit Test. In practice we may not be willing to assume that the failure times of the repairable systems under study

follow a nonhomogeneous Poisson process with Weibull intensity function, but want to test this hypothesis by statistical means. One possible method (see Parzen [1962], p. 143) is to use the Cramér-Von Mises goodness of fit statistic.

To illustrate the application of this statistic, suppose that K like systems are under study and we wish to test the hypothesis H_1 that their failure times follow a nonhomogeneous Poisson process with intensity function

(3.34) $$u(t) = \lambda\beta t^{\beta-1},$$

$t > 0$, $\lambda > 0$, and

(3.35) $$\beta = \beta_o,$$

$\beta_o > 0$, a fixed value. Assume also that the q-th system is observed during the period $[0, T_q]$, $q = 1,\ldots,K$. From (3.12) we have that if $X_{N_q,q} = T_q$ (failure truncation), then $M_q = N_q - 1$, and if $X_{N_q,q} < T_q$ (time truncation), then $M_q = N_q$, $q = 1,\ldots,K$. Also, from (3.1), recall that $M = \sum_{q=1}^{K} M_q$.

To compute the Cramér-Von Mises statistic W_M^2, we need only consider the M transformed failure times

(3.36) $$X_{iq}^* = \frac{X_{iq}}{T_q},$$

$i = 1,\ldots,M_q$, $q = 1,\ldots,K$. Treat all the M X_{iq}^*'s as one group and order them from smallest to largest. Call these ordered values

$Z_1, Z_2, \ldots, Z_M$. That is, Z_1 is the smallest X^*_{iq}, Z_2 is the next smallest $X^*_{iq}, \ldots, Z_M$ is the largest X^*_{iq}, $i = 1, \ldots, M_q$, $q = 1, \ldots, K$. The Cramér-Von Mises statistic W^2_M is given by

$$W^2_M = \frac{1}{12M} + \sum_{j=1}^{M} \left(Z_j^{\beta_o} - \frac{2j-1}{2M} \right)^2 . \tag{3.37}$$

The asymptotic significance points of W^2_M when the hypothesis H_1 is true may be found in Anderson and Darling (1952) and used when M is only moderately large.

Observe, in particular, that to test the hypothesis that the failure times of the K systems follow a homogeneous Poisson process is equivalent to testing H_1 for $\beta_o = 1$.

Generally, we will not have a fixed value of β in mind for the hypothesis H_1. That is, we will usually only want to test the hypothesis H_2 that the failure times follow a nonhomogeneous Poisson process with intensity function (3.34), β unspecified. If this hypothesis is accepted, then λ and β may be estimated from the data.

A reasonable approach to test H_2 is to use the W^2_M statistic in (3.37), but to replace β_o by an estimate of β derived from the data. In general (see Darling [1955]), this modified W^2_M statistic will not have the same distribution, even asymptotically, as the W^2_M statistic. However, Darling (1955) showed that when the proper estimate of β is used, then the modified W^2_M statistic C^2_M is parameter-free (independent of the true value of β) for any sample size M. Moreover, the

distribution of C_M^2 converges asymptotically to a distribution with mean 0.09259 and variance 0.00435 as $M \to \infty$ when the hypothesis H_2 is true.

The proper estimate of β to use in calculating C_M^2 is $\bar{\beta}$, the unbiased estimate given by (3.15). That is,

$$C_M^2 = \frac{1}{12M} + \sum_{j=1}^{M} \left(Z_j^{\bar{\beta}} - \frac{2j-1}{2M} \right)^2, \tag{3.38}$$

where

$$\bar{\beta} = \frac{M-1}{\sum_{q=1}^{K} \sum_{i=1}^{M_q} \log\left(\frac{T_q}{X_{iq}}\right)}.$$

Critical values of the C_M^2 statistic for M = 2 thru 60 have been determined at the U. S. Army Materiel Systems Analysis Activity from Monte Carlo simulation, using 15,000 samples for each value of M. Various critical values of C_M^2 are given in Table 2, rounded to the nearest integer. All values in the tables are, of course, subject to sampling error.

If the statistic C_M^2 is greater than the selected critical value, then the hypothesis H_2 that the failure times for the K systems follow a nonhomogeneous Poisson process with Weibull intensity function, is rejected at the designated significance level. If C_M^2 is less than this value, then the hypothesis H_2 is accepted.

Example. Consider again the data in Table 1 for K = 3 systems tested for time $T_1 = T_2 = T_3 = 200$. The unbiased estimate of β is

Table 2. Critical Values of C_M^2

SAMPLE SIZE M	Level of Significance				
	.20	.15	.10	.05	.01
2	.139	.150	.161	.175	.186
3	.121	.135	.154	.183	.231
4	.121	.136	.156	.195	.278
5	.123	.138	.160	.202	.305
6	.123	.139	.163	.206	.315
7	.124	.141	.166	.207	.305
8	.124	.141	.165	.209	.312
9	.124	.141	.167	.212	.324
10	.124	.142	.169	.213	.321
11	.124	.142	.166	.216	.324
12	.125	.143	.170	.213	.323
13	.126	.143	.168	.218	.337
14	.126	.142	.169	.213	.331
15	.125	.144	.169	.215	.335
16	.125	.143	.169	.214	.329
17	.126	.143	.169	.216	.334
18	.126	.143	.170	.216	.339
19	.126	.143	.169	.214	.336
20	.127	.145	.169	.217	.342
21	.126	.145	.170	.216	.332
22	.126	.144	.171	.216	.337
23	.127	.144	.169	.217	.343
24	.126	.143	.169	.216	.339
25	.127	.145	.170	.216	.342
26	.127	.145	.171	.215	.333
27	.127	.144	.170	.215	.335
28	.127	.145	.170	.218	.334
29	.127	.146	.171	.217	.334
30	.127	.145	.172	.218	.328
31	.127	.145	.170	.215	.328
32	.127	.145	.169	.214	.330
33	.127	.144	.169	.215	.337
34	.126	.143	.171	.213	.334
35	.127	.144	.170	.215	.326
36	.126	.144	.169	.213	.331
37	.127	.145	.170	.215	.339
38	.127	.145	.170	.217	.331
39	.127	.145	.173	.218	.334
40	.128	.146	.172	.220	.335
41	.128	.146	.173	.218	.335
42	.128	.146	.172	.217	.333
43	.127	.146	.172	.217	.334
44	.128	.147	.173	.218	.341
45	.128	.146	.172	.217	.342
46	.129	.146	.172	.216	.346
47	.128	.147	.173	.216	.343
48	.128	.145	.172	.219	.343
49	.127	.145	.171	.218	.335
50	.127	.145	.172	.219	.345
51	.128	.146	.173	.220	.344
52	.127	.146	.172	.216	.346
53	.127	.146	.172	.218	.348
54	.127	.146	.172	.219	.351
55	.127	.145	.173	.219	.356
56	.127	.145	.172	.221	.355
57	.127	.145	.171	.218	.352
58	.127	.145	.171	.221	.353
59	.128	.146	.171	.222	.350
60	.127	.146	.172	.219	.352

computed to be $\bar{\beta} = 0.598$. We next order the M transformed failure times $(X_{iq}/200)$, $i = 1,\ldots,M_q$, $q = 1,2,3$, where $M_1 = 10$, $M_2 = 15$, $M_3 = 11$, $M = 36$. This gives $Z_1 = 0.1/200$, $Z_2 = 4.3/200$, $Z_3 = 4.4/200$, $Z_4 = 5.6/200,\ldots,Z_{35} = 195.8/200$, $Z_{36} = 197.2/200$. Using these order-transformed failure times and $\bar{\beta}$, we calculate from (3.38), $C^2_{36} = 0.069$.

For a hypothesis test at the 0.05 significance level, we find in Table 2 that the corresponding critical value for $M = 36$ is 0.213. Since 0.069 is less than 0.213, we accept the hypothesis H_2 at this significance level.

4. Applications. In this section we will discuss some possible applications of the Weibull model to reliability, the study of industrial accidents and medicine. Observe that the Weibull intensity function $u(t) = \lambda\beta t^{\beta-1}$ is strictly monotonic for $\beta \neq 1$, and, of course, constant for $\beta = 1$. Therefore, any applications of the Weibull intensity function should be made only over regions of t where, it is felt, the intensity function of the nonhomogeneous Poisson process is monotone.

4.1. Reliability Growth. In 1962 J. T. Duane of General Electric Company's Motor and Generator Department (see Duane [1964]) published a report in which he presents his observations on failure data for five divergent types of systems during their development programs at G.E. These systems included complex hydromechanical devices, complex types of aircraft generators and an aircraft jet engine. The study of the failure data was conducted in an effort to determine if

any systematic changes in reliability occurred during the development programs for these systems. His analysis revealed that for these systems, the observed cumulative failure rate versus cumulative operating hours fell close to a straight line when plotted on log-log paper. Similar plots have been noted in industry for other types of systems, and by the U. S. Army for various military weapon systems during development.

Suppose that the development program (or test phase) for a system is conducted in a "find and fix" manner. That is, the system is tested until a failure occurs. Design and/or engineering modifications are then made as attempts to eliminate the failure mode(s) and the system is tested again. This process is continued for a fixed time period or until the desired reliability is attained. If the cumulative failure rate (expected number of failures at time t divided by t) versus test time is linear on log-log scale, then the system failure times follow a nonhomogeneous Poisson process with Weibull intensity function $u(t) = \lambda\beta t^{\beta-1}$. If the system reliability is improving, then u(t) is decreasing; i.e., $0 < \beta < 1$.

At time t_o the Weibull intensity function is $u(t_o) = \lambda\beta t_o^{\beta-1}$. If no further system improvements are made after time t_o, then it is reasonable to assume that the intensity function would remain constant at the value $u(t_o)$ if testing were continued. In particular, if the system were put into production with the configuration fixed as it was at time t_o, then the life distribution of the systems produced

would be exponential with mean

$$\rho(t_o) = [u(t_o)]^{-1} = \frac{t_o^{1-\beta}}{\lambda\beta}. \tag{4.1}$$

From (4.1) the mean $\rho(t)$ increases as the development testing time t increases (since $\beta < 1$), and is proportional to $t^{1-\beta}$. Hence, β is a growth parameter reflecting the rate at which reliability, or $\rho(t)$, increases with development testing time.

4.2. Mission Reliability. The probability R(t) that a system of age t will successfully complete a mission of fixed duration $d > 0$ is called "mission reliabiiity." If the system is repairable and mission aborting failures follow a nonhomogeneous Poisson process with Weibull intensity function, then

$$\begin{aligned} R(t) &= \text{Prob[system of age t will not fail in (t,t+d)]} \\ &= e^{-[\lambda(t+d)^{\beta}-\lambda(t)^{\beta}]}. \end{aligned}$$

Note that if $\beta > 1$ (wear-out), then R(t) is decreasing with age. If $\beta < 1$ (improvement), then R(t) is increasing with age. When $\beta = 1$ (constant intensity of failures), then $R(t) = e^{-\lambda d}$, a constant.

4.3. Maintenance Policies. Barlow and Hunter (1960) considered optimum replacement or overhaul policies for a system whose failure times follow a nonhomogeneous Poisson process. The optimum replacement or overhaul times for the system were derived so as to maximize the expected fractional amount of system uptime over an infinite period of time. Furthermore, one can also use the same approach to determine optimum replacement or overhaul times which will minimize the

expected maintenance cost over an infinite time span. When the system has a Weibull intensity function, with $\beta > 1$, the time which minimizes expected maintenance cost is given in Barlow and Proschan (1965), page 98.

4.4. Industrial Accidents. The control of industrial accidents generally requires, from time to time, new safety equipment, safety regulations, improved machinery, etc. Hence, one may expect that the occurrence of accidents would tend to decrease with time. Because of serious injuries or, perhaps, deaths that may occur as a result of an industrial accident, it is usually important to know whether or not the safety actions are resulting in a significant decrease of accidents. The nonhomogeneous Poisson process with Weibull intensity function may possibly be useful in measuring this decrease, as demonstrated by the following example.

The data in Table 1 of Maguire, Pearson and Wynn (1952) represents days between explosions in mines in Great Britain involving more than 10 men killed. The data covers the period from December 6, 1875 to May 29, 1951. As noted by Barnard (1953) and also by Maguire, Pearson and Wynn (1953), there is convincing statistical evidence that the data in this table are not consistent with the hypothesis of a homogeneous Poisson process. That is, the occurrence of these mine accidents departs statistically from a constant intensity. Barnard suggested that the mine accidents may follow a nonhomogeneous Poisson process with decreasing intensity function. The following analysis was

conducted on these accident data using the Weibull intensity model.

Regarding December 6, 1875 as time 0, the occurrence times of the 109 successive mine accidents were measured from this date in days, with the last accident occurring on May 29, 1951, or at T = 26,263 days. The data, therefore, were failure truncated. Secondly, the ML estimates of λ and β were computed, giving $\hat{\lambda} = 0.7626$, $\hat{\beta} = 0.7139$. Since the data were failure truncated, with N = 109, we have M = N - 1 = 108. From (3.8) and (3.10) $\tilde{\beta} = \frac{M}{N}\hat{\beta} = 0.7074$, and the unbiased estimate of β is, from (3.15), $\bar{\beta} = \frac{M-1}{M}\tilde{\beta} = 0.7008$.

Using $\bar{\beta}$, the goodness of fit statistic C_M^2 was calculated, giving $C_{108}^2 = 0.1817$. If we assume that the 5 percent critical value for M = 108 is reasonably close to the 5 percent critical value 0.219 for M = 60, then C_{108}^2 is not significant at this level. That is, we accept at the 5 percent level that the mine accidents follow a non-homogeneous Poisson process with Weibull intensity function.

A 95 percent upper confidence bound on β is $\beta_{ub} = \tilde{\beta}(1 + \frac{1.64}{\sqrt{108}}) = 0.8190$ and the ML estimate of the intensity of mine accidents is $\hat{u}(t) = \hat{\lambda}\hat{\beta}t^{\hat{\beta}-1}$, $t > 0$.

4.5. Medical. Various illnesses in humans are of a recurring nature. For example, in underdeveloped countries certain gastrointestinal ailments are generally prevalent in infants from shortly after birth to about age one year. Empirical studies indicate that many environmental type ailments in infants usually have an increasing rate

of occurrence over a certain period of time after birth. The rate then begins to decrease as the infant develops immunities to combat the environmental conditions.

The Johns Hopkins University's Department of International Health, assisted by the U. S. Army Materiel Systems Analysis Activity, is currently conducting studies on medical data to determine the adequacy of the nonhomogeneous Poisson process for representing the occurrence of various gastrointestinal ailments in humans. The ailments selected are those of relatively short duration since the nonhomogeneous Poisson process assumes an instantaneous time of occurrence. If the Weibull model is found to provide an adequate fit for an ailment, then the effects of different treatments would include comparisons of the respective Weibull intensity functions.

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