



University of  
**Nottingham**

UK | CHINA | MALAYSIA

School of  
Mathematical  
Sciences



## Mathematics News from the University of Nottingham

### Volume 2, Number 2 (March 2024)

In this issue of the University of Nottingham Mathematics Newsletter we have:

- **[Nottingham's Data Science Apprenticeship Programme](#)**: Ria Symonds, Associate Professor in the School of Mathematical Sciences, explains the university's Data Science Apprenticeship Programme
- **[Puzzle 2.2](#)**: Find the counterfeit!
- **[Solution to Puzzle 2.1](#)**: The solution to last issue's puzzle
- **[Interesting Mathematical Facts](#)**: An article about Benford's Law and detecting fraud
- **[Hints for Puzzle 2.2](#)**: Full details next issue
- **[Useful Links](#)**: Links to useful resources
- **[Back Issues](#)**: Links to all issues of this newsletter so far

We welcome feedback, comments and suggestions. Please let us know what you found most interesting, what else you would like to see, and any other comments you have by filling in the short feedback form at <https://tinyurl.com/uonmathsnewsfeedback>. Alternatively, you can contact us by email at [james.walton@nottingham.ac.uk](mailto:james.walton@nottingham.ac.uk)

Editors: [Jamie Walton](#)



# Nottingham's Data Science Apprenticeship Programme

[Back to contents](#)

In this article, Dr Ria Symonds explains Degree apprenticeships, and in particular the Data Science Apprenticeship programme that was developed by the School of Mathematical Sciences at the University of Nottingham.

## Introduction

Degree apprenticeships (DAs) form an extension to higher education and are a type of apprenticeship programme that combines on-the-job training with academic study at the university level. This apprenticeship model allows individuals to work and earn a full-time salary while also working towards a bachelor's or master's degree (predominantly taught remotely). At the University of Nottingham we currently offer four degree apprenticeship programmes, one of which is in the area of Data Science.

The Data Science degree apprenticeship was launched in 2021 at the University of Nottingham. It is a 3.5 year course and we currently have apprentices from organisations such as Experian, National Grid, Rolls Royce, GlaxoSmithKline, Toyota, QinetiQ, Pfizer, E.on (amongst others). An apprentice will spend most of their time working within their relevant organisation with at least 20% of their time being dedicated to “off the job learning”. Degree apprenticeships typically have a structured curriculum that integrates both theoretical and practical aspects of the chosen field so that any knowledge and skills acquired through the learning is then transferred to the workplace.



*Apprentices working on Python code.  
Photography throughout this article by Alex Wilkinson Media.*

## What does the academic programme look like?

An apprentice will study aspects of mathematics, statistics, programming, software development, machine learning and AI.

- In **Year 1** emphasis is placed on understanding the data science pipeline and securing foundational skills in programming (Python), mathematics, statistics and probability.
- In **Year 2** apprentices will add depth and breadth to the knowledge and skills achieved in Year 1. They will develop a thorough grounding in techniques needed for the analysis of probabilistic and statistical models and will be introduced to software development practices and AI/Machine Learning methods.
- In **Year 3**, apprentices will broaden their knowledge and experience of using big data and will take part in a substantial work-based project as they prepare for work as a professional data scientist.
- The **final 6 months** of the apprenticeship is the end-point assessment. This requires apprentices to demonstrate that their learning can be applied in the real world via a Knowledge Test, a Project Report and a Professional Discussion.

## How is the academic content delivered?

The programme splits teaching and assessment into separate blocks to allow assessment to be driven by work-based context. Content is predominantly delivered remotely using a variety of resources such as bitesize videos, online interactive notes, quizzes, written exercises etc., accessed via the university learning environment Moodle so that an apprentice can learn at their own pace. The remote learning content is supplemented by weekly 1-hour online workshops with the teaching team and face-to-face “block release” days which are held on the University campus (approximately 10 days over a one-year period).



*Apprentices on campus for face-to-face tuition.*

## How is an apprentice assessed?

Assessment on the programme does not follow the “traditional” coursework/exam structure that are typical of many traditional university degrees. Instead, the assessment is “authentic” to provide the opportunity to incorporate work-based examples. Some examples of assessment used on programme are:

- Written reports (Statistical report, business case report etc)
- Presentations/ Posters
- Professional Discussions
- Portfolios
- Work-based projects



*A poster presentation for the Data Science Apprenticeship Programme.*

## Feedback from apprentices and employers

Current employers value the skills and knowledge apprentices gain on the programme which can be directly transferred to the workplace.

*Our apprentice understands our business and data science, and can act as the link between the two. (Employer, Toyota)”*

Our apprentices thoroughly enjoy engaging with academic and practical side of the course. They can see the real benefit of the taught content since they are able to apply it to practical examples within their own organisations.

*You’re not just taught the skills that are available now, you’re taught how to develop the skills that will be needed in the future. You’re taught how to learn what will be the next big thing in data science. (Apprentice, Experian) ”*

## Is a degree apprenticeship for me?

Degree apprenticeships are ideal if you know what line of work you want to go into, but also want to study the theory behind what you do from a more academic point-of-view. This way, you'll get the best of both worlds, gaining a deep understanding of your subject as well as the ability to apply it in the workplace. We also have a flexible admissions policy which means there are various routes to enter the programme to consider differing backgrounds and academic experience. Found out more here:

<https://www.nottingham.ac.uk/mathematics/business/data-scientist-degree-apprenticeship.aspx>

## About the author

Dr Ria Symonds is associate professor at the University of Nottingham. By background, Ria is a specialist in Mathematics Education and, although a mathematician by trade, she is primarily interested in supporting the teaching and learning of mathematics to non-specialist students. Ria was an Area Coordinator for the Advanced Mathematics Support Programme (AMSP) for 11 years whereby she was involved with teaching, professional development and organisation of large enrichment events for KS4 and KS5 Mathematics. More recently, in 2021, Ria was awarded a team Lord Dearing award for her contribution to teaching and learning with the development and roll out of a successful MOOC (massive online open course) to help students prepare for university mathematics. Ria has further developed her skills and knowledge in curriculum planning and development during the 2017 redesign of the A-level curriculum (via AMSP involvement) and, more currently, with the design and implementation of the Data Science Degree Apprenticeship. Ria is an advocate for creating authentic learning and assessment to encourage learner autonomy and, thus, ensuring that the skills and knowledge attained is directly transferable to the workplace.



*Dr Ria Symonds*

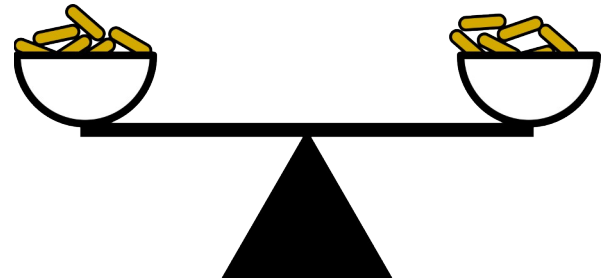


## Puzzle 2.2

[Back to contents](#)

### Find the counterfeit

The following is a classic balance puzzle. Suppose you have some number of coins — let's say 8 coins for now — but one is a counterfeit. We must find it! It is known that all genuine coins are of equal weight whilst the counterfeit is lighter than the others. We have access to a balance scale. A 'weighing' involves choosing coins to go on each of the two plates and the scale will always accurately tell us which of the two piles is heavier, or will perfectly balance if both piles are of identical weight.



*A set of balance scales*

We would like to find the counterfeit coin but using the scales is time-consuming, so we want to find it with as few weighings as possible. Here are the questions (as usual, some [hints](#) are given later):

- (a) Find a strategy for efficiently finding the counterfeit coin in a pile of 8 total coins. It may surprise you that we can guarantee that we find it in just two weighings!
- (b) Now suppose that we have 9 coins instead, how many weighings are necessary this time?
- (c) Now suppose we use 27 coins. How many weighings are needed?
- (d) And what about 81 coins?
- (e) Hopefully you have started to see a pattern. So suppose you are given any number  $n$  of coins. Explain a strategy that finds the coin in a relatively small number of weighings. Try to find a strategy that is optimal (that is, it guarantees finding the coin and that no other strategy exists that uses fewer weighings) and, as an extra tricky challenge, do your best to give a rigorous argument as to why it is optimal.
- (f) Let's change the game a bit: suppose now that all coins are the same weight, except for a single coin. Unlike before, however, we now do not know if the defective coin is lighter or heavier than the others. How many weighings are needed now for, say, for 8 coins, 9 coins, 27 coins, 81 coins, .... or any number of coins?

## Solution to Puzzle 2.1

[Back to contents](#)

### Stern's diatomic sequence

In the last newsletter we discussed the following intriguing infinite sequence: 1, 1, 2, 1, 3, 2, 3, ..., called Stern's (diatomic) sequence. We recall that the  $n$ th term is denoted  $a(n)$  and three rules generate the sequence: firstly, we start with  $a(1) = 1$ , secondly we may find the value of even terms from previous terms by the rule  $a(2n) = a(n)$  and finally we can find odd terms from previous terms by the rule  $a(2n + 1) = a(n) + a(n + 1)$ . Here are the solutions:

(a) The first part of the question was just to get more familiarity with the sequence by writing out the first 20 terms. The easiest way of doing this is to write out a table for quick reference between a term and its index in the sequence:

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	...
a(n)	1	1	2	1	3	2	3	1	4	3	5	2	5	3	4	1	5	4	7	3	...

(b) We now recall the operation  $F(i, j)$  on input pairs  $(i, j)$  of numbers, defined by replacing the larger of the two numbers with the difference between it and the other number. We needed to show that repeated application always ends in the pair  $(g, g)$ , where  $g$  is the greatest common divisor of  $i$  and  $j$ .

To see why, suppose an integer  $q$  divides (that is, with no remainder) both  $i$  and  $j$ . Then  $q$  also divides the sum and difference of them. For example,  $q$  dividing  $i$  and  $j$  means there are integers  $m$  and  $n$  with  $i = qm$  and  $j = qn$ . Then  $i + j = q(m + n)$  where  $m + n$  is an integer, so  $q$  divides  $i + j$ . Analogously,  $q$  divides  $i - j = q(m - n)$ . Thus,  $q$  divides both terms of  $F(i, j)$ , which must be one of  $i, j, i - j$  or  $j - i$ .

Conversely, suppose that  $q$  divides both elements of  $F(i, j)$ , let's say  $i > j$  so that  $F(i, j) = (i - j, j)$ . Then  $q$  divides the sum  $(i - j) + j = i$ , so  $q$  divides both terms of  $(i, j)$ . The case of  $i < j$  is analogous. We conclude that  $q$  divides both elements of a pair  $(i, j)$  if and only if it divides both elements of the transformed pair  $F(i, j)$ . In particular, the greatest common divisor of the pair does not change under application of  $F$ . Since one element of the pair always gets smaller under application of  $F$ , we must eventually hit a pair of two equal numbers, say  $(q, q)$ . But the greatest common divisor of a number  $q$  and itself is  $q$ . By the above this must be equal to the greatest common divisor  $g$  of the initial  $i$  and  $j$ , so we conclude that repeated application of  $F$  eventually ends on the pair  $(g, g)$ .

(c) Take any pair  $(a(n), a(n + 1))$  of consecutive terms of Stern's sequence. We show that the **only** pairs  $(i, j)$  with  $F(i, j) = (a(n), a(n + 1))$  are  $((a(2n), a(2n + 1))$  and  $(a(2n + 1), a(2n + 2))$  i.e., they are also consecutive terms from the sequence. Let us note that such consecutive terms are never equal, so we may always apply  $F$ , except for the initial  $a(1) = a(2) = 1$ . Indeed, if  $n = 2k + 1 > 1$  is odd then the consecutive pair is  $(a(n), a(n + 1)) = (a(2k + 1), a(2k + 2)) = (a(k) + a(k + 1), a(k + 1))$ , so  $a(n) > a(n + 1)$  as, by construction of the sequence,  $a(k) > 0$ . Similarly, if  $n = 2k$  is even, then  $(a(n), a(n + 1)) = (a(2k), a(2k + 1)) = (a(k), a(k) + a(k + 1))$ , and thus  $a(n) < a(n + 1)$  since  $a(k + 1) > 0$ . Note that in both cases the odd index term is larger than its neighbouring even index term (you can also see this in the above table).

Now, suppose that  $F(i, j) = (a(n), a(n + 1))$  and consider the case of  $i > j$ . Then  $F(i, j) = (i - j, j) = (a(n), a(n + 1))$  so, from the right-hand term,  $j = a(n + 1) = a(2(n + 1)) = a(2n + 2)$ . From the left-hand term we see that  $i - j = a(n)$ . Since  $i - j = i - a(n + 1) = a(n)$ , we have  $i = a(n) + a(n + 1) = a(2n + 1)$ . Thus,  $(i, j) = (a(2n + 1), a(2n + 2))$ , as required. The case for  $i < j$  is similar: we then have  $F(i, j) = (i, j - i) = (a(n), a(n + 1))$  so that  $i = a(n) = a(2n)$  and  $j - i = j - a(n) = a(n + 1)$  and

hence  $j = a(n) + a(n+1) = a(2n+1)$  so that  $(i, j) = ((a(2n), a(2n+1)))$ , again a consecutive pair of terms from the sequence.

We can also show that, conversely,  $F(a(2n), a(2n+1)) = F(a(2n+1), a(2n+2)) = (a(n), a(n+1))$ . This is just applying the rules from Stern's sequence (and using that the odd index term is always larger) and we omit the details. But we conclude that these are precisely the two pairs leading to  $(a(n), a(n+1))$  after applying  $F$ .

**(d)** We now show that each consecutive pair of terms  $(a(n), a(n+1))$  are relatively prime, that is, the greatest common divisor is 1. We iteratively apply  $F$  to this pair. By part (c), this always maps to another such consecutive pair from the sequence, at a lower index, and as explained we are allowed to apply  $F$  since the pair is non-equal, until we reach  $(a(1), a(2)) = (1, 1)$ . By part (b), the terminating pair  $(1, 1)$  determines the greatest common divisor of the original pair to be 1, that is, they must be relatively prime.

Conversely, suppose that  $(i, j)$  is a pair of relatively prime numbers. We wish to show they occur as a consecutive pair of terms from Stern's sequence. We may iteratively apply  $F$  until we get an equal pair. By part (b), we must always end at the pair  $(1, 1) = (a(1), a(2))$ , which is a consecutive pair of terms from the sequence. And by part (c), the only pair of numbers that can map to consecutive pairs of terms from Stern's sequence must themselves be consecutive pairs of terms from Stern's sequence. By applying this iteratively back up to our original pair, we see that  $(i, j)$  must be of the form  $(a(n), a(n+1))$  for some  $n$ .

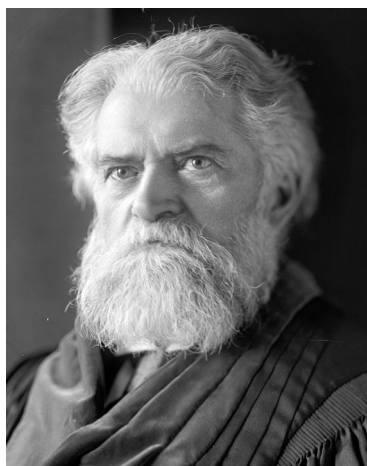
**(e)** We finally show that no consecutive pair from the sequence is ever repeated. So suppose that  $(a(n), a(n+1)) = (a(m), a(m+1))$ . Since these are equal,  $F$  acts on them identically under iteration. The action of  $F$  depends on which of  $a(n)$  or  $a(n+1)$  is bigger (equivalently  $a(m)$  versus  $a(m+1)$ ) which, by comments in part (c), mean that  $n$  and  $m$  are both even or both odd (that is, they have the same "parity"). Going back to part (c) in further detail and "working backwards", at each step we know that  $F$  replaces a pair  $(a(i), a(i+1))$  with either  $(a(i/2), a((i+1)/2))$  (when  $a(i) < a(i+1)$ , that is when  $i$  is even) or  $(a((i-1)/2), a(i/2))$  (when  $a(i) > a(i+1)$ , that is when  $i$  is odd).

Now suppose that  $n$  is not equal to  $m$ , without loss of generality  $n > m$ . Keep applying  $F$  iteratively using the rules above to the pairs  $(a(n), a(n+1)) = (a(m), a(m+1))$  to reach  $(1, 1)$ , let's say ending with  $(a(u), a(u+1)) = (1, 1)$  and  $(a(v), a(v+1)) = (1, 1)$ , respectively. Reading "backwards" in part (c), applying  $F$  has the effect of replacing the indices of  $(a(i), a(i+1))$  to either  $(a(i/2), a((i+1)/2))$  in the first case (when  $i$  even), or  $(a((i-1)/2), a(i/2))$  in the second case (when  $i$  is odd). Since we start with the same pairs, the same operation is done at each step and we deduce that the ending indices have  $u > v$  and, in particular, we find  $(a(u), a(u+1)) = (1, 1)$  even though  $u \neq 1$  (since  $u > v$  but we need  $v > 0$ ). But then we have found two consecutive terms  $a(u) = a(u+1)$  with  $u > 1$ . However, we have already seen this is impossible, since the difference between these two consecutive terms is another element the sequence, which is necessarily non-zero. We are done!

Recall something amazing about the above: it shows that listing the sequence of reciprocals  $\frac{1}{1}, \frac{2}{1}, \frac{1}{2}, \frac{3}{1}, \frac{2}{3}, \frac{3}{2}, \frac{1}{3}, \frac{4}{1}, \frac{3}{4}, \frac{5}{3}, \dots$  of consecutive pairs of terms from the sequence actually lists **every** positive rational number exactly once! There are no repeats, because being in reduced form is equivalent to the numerator and denominator being relatively prime and each relatively prime pair occurs exactly once as a consecutive pair in Stern's sequence. This shows that there are "just as many" positive integers as there are rational positive numbers, because we can pair up each positive integer  $n$  with the  $n$ th term of the above sequence of rationals. This may at first seem quite unintuitive, as it may appear that there are "more" positive rationals than positive integers. However, the fact that these sets have the same cardinality becomes second nature when one starts studying such topics in more detail.

## Detecting Fraud with Benford's Law

[Back to contents](#)



[Simon Newcomb](#), discoverer of Benford's Law. Photo from 1905, public domain

Here is an interesting way of multiplying two large numbers  $a$  and  $b$ , or two numbers with many decimal places, that can be very helpful in the absence of calculators. Suppose you know their (base 10) logarithms  $\log(a)$  and  $\log(b)$ . Then the logarithm of their product is  $\log(ab) = \log(a) + \log(b)$ . It is much easier to add numbers than multiply them. We may then calculate the product of the original numbers by taking the “antilogarithm” i.e., raising 10 to the power of the above number:  $ab = 10^{\log(a) + \log(b)}$ . The catch, of course, is: how do we compute the (anti)logarithm of a (large or many decimal places) number without a calculator? Nowadays many of us have calculators on phones in our pockets, but before computers it was common to use tables of logarithms to multiply large numbers in just this way. One looked up the logarithms of numbers in the table, took their sum then the answer is the antilogarithm, found in a different table in the book. This method is also useful for taking powers of numbers.

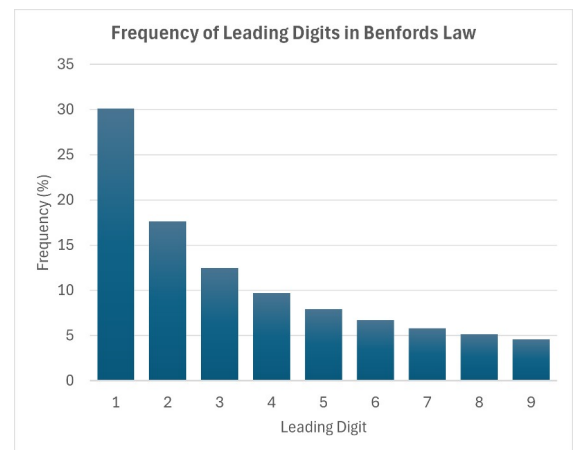
This process is a lot more efficient than listing all possible pairs of numbers you might want to multiply since we only need one logarithm (and antilogarithm) for each number, rather than one entry for each pair of numbers we might want to multiply. Another efficiency is that the integer part of the logarithm is easily determined from the leading power of 10 defining the number. For example, we would write the number 253100, in scientific notation, as  $2.531 \times 10^5$  and  $\log(253100) = 5.38578\dots$ , where the integer part of 5 can be read off from the power of 10. As another example,  $0.0023 = 2.3 \times 10^{-3}$  and  $\log(0.0023) = -2.63827\dots$  where the integer part of  $-2$  can be read off from the power of 10. This rule again follows from laws of logarithms:  $\log(10^n \times m) = \log(10^n) + \log(m) = n \log(10) + \log(m) = n + \log(m)$ , where  $\log(m)$  is the fractional part of the logarithm,  $0 \leq \log(m) < 1$ . So tables of logarithms only needed to list the logarithms, sometimes called “mantissas”, of numbers  $\log(m)$  with  $1 \leq m < 10$ ; the full logarithm can be found by adding the appropriate “characteristic” integer  $n$  in the power of 10.

In the 1880s [Simon Newcomb](#), a Canadian–American astronomer and mathematician, noticed something strange. The tables of logarithms he had been using were more worn out on the earlier pages of lower numbers than later pages. That is, numbers with smaller mantissas, such as  $1.53 \times 10^5$  or  $1.22 \times 10^{-4}$ , were on more heavily used pages than those containing numbers such as  $9.31 \times 10^7$  or  $7.34 \times 10^{-12}$ . Numbers with smaller mantissas are those with small leading digits (and the power of 10 does not matter). So Newcomb had made the startling discovery that numbers with a small leading digit seem to occur more frequently in practise than those with large leading digit. It is interesting to wonder if this discovery would have ever happened, or happened much later, if we had electronic calculators back in Newcomb's Day!

Newcomb proposed that a number having leading digit equal to  $d$  had probability  $\log(d+1) - \log(d)$ . For example, he conjectured that we should see numbers with leading digit 1 around 30.10% of the time, since  $\log(2) - \log(1) = (0.3010\dots) - 0 \approx 0.3010$ , whereas numbers with leading digit 9 should be much rarer, seen only around 4.58% of the time, as  $\log(10) - \log(9) = 1 - 0.9542\dots \approx 0.0458$ . See the graph on the next page for the probability distribution.

Collections of numbers which have a reasonably good fit with this distribution are said to satisfy Benford's Law, named after the physicist [Frank Benford](#), who found this relationship again in the late 1930s. It seems a bit unfair that Newcomb is not credited in the name of this law, although it is surprisingly common that results in mathematics and other sciences often fail to credit the original discoverer! So it is also sometimes called the [Newcomb–Benford Law](#). However, Benford did play an important role in bringing attention to Newcomb's discovery and demonstrated that the phenomenon occurs in a surprising variety of data sets,

such as street addresses, weights of molecules, river lengths, population sizes and various mathematical sequences, such as the Fibonacci sequence. The same behaviour was apparent in many disparate places, confirming the prevalence of smaller leading digits. You can try this for yourself: pick up a newspaper and tally how many numbers in it start with a 1 digit and how many do not. If there is enough data in the newspaper spread over an appropriate range of magnitudes (and no other particular reasons to break the rule), then you should find that roughly 30% of the numbers start with a 1, which is a lot higher than the  $1/9 \approx 11.11\%$  you would expect if each of the 9 possible starting digits were equally likely.



Not all data follows Benford's Law. In particular, it only tends to work when the data is spread over several orders of magnitude. But why does it sometimes work? This is a difficult question and there are various interesting explanations (some more relevant in particular contexts than others). One is that there are many instances where the numbers are, at some level, a product of other sufficiently random quantities. For example, the number of units of a particular item a company sells is the product of the number of its customers and how many items each customer buys on average. The number of customers itself could be estimated as a product, such as (if the company is local) how many people live in the area where the company operates times the fraction of people who tend to need that product (which itself might break roughly into a product). Quantities that are defined as a product of sufficiently many sufficiently random variables with enough spread will obey Benford's Law.

Another interesting explanation is that quantities spanning several orders of magnitude often arise from growth processes. For example, an investment or town/city population might be expected to roughly grow some fixed percentage each year on average. In that case, the number tends to move faster through the higher digits than the lower ones, so we spend more time at leading digit 1, say, than leading digit 9. More precisely, suppose that our quantity grows like  $cr^t$  where  $c$  is a starting value,  $r$  is a fixed growth rate (for example,  $r = 1.05$  if our quantity grows at 5% per time unit) and  $t$  is time. If we start, say, at  $c = 10^n$  at  $t = 0$  then we reach the next order of magnitude when  $10^n r^t = 10^{n+1}$ , which we may easily solve as  $t = 1/\log(r)$ . In between, to move from digit  $d$  to digit  $d+1$ , we take time  $t$  satisfying  $(d \times 10^n)r^t = (d+1) \times 10^n$ , which is  $t = (\log(d+1) - \log(d))/\log(r)$ . So the proportion of time spent at digit  $d$  is  $\log(d+1) - \log(d)$ , which is exactly Benford's Law! Other explanations exist and some of the deeper mathematics is explained, for instance, in the book [Mil].

This week's puzzle was on finding a counterfeit coin, so it seemed fitting to discuss Benford's Law as it turns out it can be useful in detecting fraud! It is important to be able to tell when data has been tampered with, for example in accounting or socio-economic data when public planning decisions are made. This data can be checked against what might be expected from Benford's Law (at least when we expect the data to Benford's Law, although the mathematics of when it should and when it should not can get complicated). When there is not agreement, this might indicate that data has been falsified. Benford's Law is a legally accepted method by which the IRS (who collect US taxes) flags potential fraud in organisations' finances and has even been used in criminal trials! For other interesting examples, see the Wikipedia page [Wik] on this fascinating topic.

## References

- [Mil] Usteven J. Miller, **Benford's Law: Theory and Applications**, Princeton University Press (2015).
- [Wik] Wikipedia article on Newcomb–Benford's Law, [https://en.wikipedia.org/wiki/Benford%27s\\_law](https://en.wikipedia.org/wiki/Benford%27s_law)



## Hints of Puzzle 2.2

[Back to contents](#)

[Back to Puzzle 2.2...](#)

**(a)** We do not know how much the counterfeit coin weighs relative to the others, so it will be hard to make a deduction unless we put the same number of coins on each plate for each weighing.

So we should start with either 1, 2, 3, or 4 coins on each plate to start off with. Try any of these and see what options we'd have left after the first weighing. If the counterfeit coin is in neither of the piles we start with, what happens? What if it is in one of the initial piles, what should we do next?

**(b)** If you solved part (a), try running a similar strategy with 9 coins. It turns out that you can still find the counterfeit in just two weighings with 9 coins, with an almost identical strategy!

**(c)** If you've solved parts (a) and (b) perhaps you noticed that it's a good strategy to use about a third of the pile on each plate at each weighing. Then try to find a way of finding the counterfeit in a pile of 27 coins with just three weighings! You should be able to reduce to the case of 9 coins in one step...

**(d)** You should see the pattern from the above case:  $81 = 27 \times 3$  and you will need just one more step. You can find the counterfeit now in four weighings.

**(e)** If you did parts (a-d) you should see that powers of three are special, and we can solve the case of  $3^n$  coins with just  $n$  weighings. Can you show that you cannot do it in fewer? If there are  $n$  coins with  $3^m < n < 3^{m+1}$  it can be shown that you only need  $m + 1$  weighings but that you cannot do it in fewer.

**(f)** This part is more an exploration of similar ideas. However, let us point out that you will generally need more weighings when you do not know if the counterfeit is heavier or lighter than the others!

## Useful Links

[Back to contents](#)

Here are some links to useful resources.

### Study with us!

Full details of our mathematics courses and our entry requirements

<https://tinyurl.com/mathscourseuon>



### Careers

What do our mathematics graduates do?

<https://tinyurl.com/uonmathscareers>



### Mathematics Taster Sessions

Taster lectures and popular mathematics talks

<https://tinyurl.com/uonmathstaster>



### Open Day mathematics talks

Video on demand

<https://tinyurl.com/uonmathsvod>



### Our YouTube channel

More of our mathematics videos

<https://tinyurl.com/uonmathsyt>



### Subscribe to this e-newsletter

If you haven't already signed up for this Mathematics Newsletter, you can do so at

<https://tinyurl.com/uonmathsnewsform>



## Back Issues

[Back to contents](#)

Here is a list of our issues so far, with QR codes and a brief summary of the contents of each issue.

[Volume 1, Number 1](#)



Information on the careers of University of Nottingham mathematics graduates, Puzzle 1.1 (about prime numbers), a short article about Edmund Landau and Landau's problems, links to useful resources

[Volume 1, Number 2](#)



Information about our three-year single-honours mathematics BSc degree, Puzzle 1.2 (about discs), the solution to Puzzle 1.1, an article about Alice Roth and Swiss cheeses, links to useful resources, back issues

[Volume 1, Number 3](#)



Maths graduation celebration and prizegiving, spotlight on Bindi Brook, Puzzle 1.3 (about peg solitaire), the solution to Puzzle 1.2, an article about John Horton Conway, links to useful resources, back issues

[Volume 1, Number 4](#)



Information about our four-year MMath degree, Puzzle 1.4 (on Santa's logistics), the solution to Puzzle 1.3, an article about Leonhard Euler, links to useful resources, back issues

[Volume 2, Number 1](#)



Recent Maths Outreach events at the University of Nottingham, Puzzle 2.1 (Stern's Diatomic Sequence), an article about Georg Cantor, links to useful resources, back issues

[Volume 2, Number 2](#)



Nottingham's Data Science Apprenticeship Programme, Puzzle 2.2 (finding a counterfeit coin), an article about Newcomb–Benford's Law and detecting fraud, links to useful resources, back issues