Multi-scale Quantum Harmonic Oscillator Behaved Algorithm with Three-stage Perturbation for High-dimensional Expensive Problems (Supplementary)

Xinggui Ye, *Senior Member, IEEE*, Jianping Li and Peng Wang

APPENDIX I
TEST BENCHMARK FUNCTIONS

Benchmark functions are shown in Table S1.

APPENDIX II
PARAMETER SETTINGS

Parameters in each of the selected algorithms are defined as in Table S2.

TABLE S2 Parameter settings.

Algorithm	Parameter settings
BOA [1]	NP=50, switch probability p=0.8, power exponent e=0.1, sensory modality m=0.1
QQLMPA [2]	NP=25, Q-table size is 3× 3, FADs=0.2, P=0.8, γ=0.8
DEPSO [3]	the agent number of DE and PSO is set to 5
CCLNNA [4]	NP=50, initial weight w=0.5, β=1
MQE [5]	group number N_g=4, individual number in each group N_i=5
TSMQHOA [6]	NP=20, the truncated probability p_t=0.1, The contraction coefficient $\lambda = 2.0$
AMQHOA-ES [7]	NP=20, the scale factors in M process k_1=1.2 and k_2=1.6
MQHOA-TSP	NP=20, the scale factors in M process k_1=1.2 and k_2=1.6

The maximal evaluation (run) is set to G_{max}=$1000 * D$. The search domain of each benchmark function is defined as in Table S1. The computational error is $\varepsilon = 1.00E-06$ ($D < 200$) and $\varepsilon = 1.00E-03$ ($D \geq 200$). The stopping criteria for all of the algorithms are uniformly defined as: the computation error $\varepsilon \leq 1.00E-06$ or the evolution runs $nfe \geq maxFE$. All of the algorithms are coded in Matlab R2016a, executed on the same PC with an Intel core(TM) i5-1135G7@2.4GHz and Windows 11 operating system.

REFERENCES

[1] S. Arora and S. Singh, "Butterfly optimization algorithm: a novel approach for global optimization," *Soft Computing*, vol. 23, no. 3, pp. 715–734, 2019.

[2] S. Zhao, Y. Wu, S. Tan, J. Wu, Z. Cui, and Y.-G. Wang, "QQLMPA: A quasi-opposition learning and q-learning based marine predators algorithm," *Expert Systems with Applications*, vol. 213, p. 119246, 2023.

[3] S. H. Wang, Y. Z. Li, and H. Y. Yang, "Self-adaptive mutation differential evolution algorithm based on particle swarm optimization," *Applied Soft Computing*, vol. 81, pp. 1–22, 2019.

[4] Y. Zhang, "Chaotic neural network algorithm with competitive learning for global optimization," *Knowledge-Based Systems*, vol. 231, p. 107405, 2021.

[5] D. Tang, Z. Liu, J. Zhao, S. Dong, and Y. Cai, "Memetic quantum evolution algorithm for global optimization," *Neural Computing and Applications*, vol. 32, no. 13, pp. 9299–9329, 2020.

[6] X. Ye, P. Wang, G. Xin, J. Jin, and Y. Huang, "Multi-scale quantum harmonic oscillator algorithm with truncated mean stabilization strategy for global numerical optimization problems," *IEEE Access*, vol. 7, pp. 18 926–18 939, 2019.

[7] X. Ye and P. Wang, "Adaptive multi-scale quantum harmonic oscillator algorithm based on evolutionary strategy," in *2020 IEEE Congress on Evolutionary Computation (CEC)*, 2020, pp. 1–8.

TABLE S1 Benchmark functions.

Problem	Benchmark Function	D	Range	Optimum
Sphere	$f_1 = \sum_{i=1}^{n} x_i^2$	n	[-5.12,5.12]	$f(0,...,0) = 0$
Sum Squares	$f_2 = \sum_{i=0}^{n-1} ix_i^2$	n	[-10,10]	$f(0,...,0) = 0$
Rotated Hyper-Ellipsoid	$f_3 = \sum_{i=1}^{n}(\sum_{j=1}^{i} x_i)^2$	n	[-65.54,65.54]	$f(0,...,0) = 0$
Ellipsoidal	$f_4 = \sum_{i=1}^{n}(x_i - i)^2$	n	[-100,100]	$f(1,2,...n) = 0$
Bent Cigar	$f_5 = x_i^2 + 10^6 \sum_{i=2}^{n} x_i^2$	n	[-10,10]	$f(0,...,0) = 0$
High Conditioned Elliptic	$f_6 = \sum_{i=1}^{D}(10^6)^{\frac{i-1}{D-1}} x_i^2$	n	[-100,100]	$f(0,....0) = 0$
Ackley	$f_7 = -20exp(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}) - exp(\frac{1}{n}\sum_{i=1}^{n} cos(2\pi x_i)) + 20 + e$	n	$[-32.77, 32.77]$	$f(0,...,0) = 0$
Griewank	$f_8 = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	n	[-100,100]	$f(0,...,0) = 0$
Levy	$f_9 = sin^2(\pi\omega_1) + \sum_{i=1}^{n-1}(\omega_i - 1)^2[1 + 10sin^2(\pi\omega_i + 1)] + (\omega_n - 1)^2[1 + sin^2(2\pi\omega_n)]$,where $\omega_i = 1 + \frac{x_i - 1}{4}$,for all $i = 1, ..., n$	n	[-10,10]	$f(1,...,1) = 0$
Rastrigin	$f_{10} = 10n + \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i)]$	n	[-5.12,5.12]	$f(0,...,0) = 0$
Schwefel	$f_{11} = 418.9829d - \sum_{i=1}^{n} x_i sin(\sqrt{\lvert x_i \rvert})$	n	[-500,500]	f(420.9687, ..., 420.9687) $= 0$
Modified Schwefel	$f_{12} = 418.9829 \times D - \sum_{i=1}^{n} g(z_i) \quad z_i = x_i + 420.9687462275036, where$ $g(z_i) = \begin{cases} z_i sin(\lvert z_i \rvert^{1/2}) & if \quad \lvert z_i \rvert \leq 500 \\ (500 - mod(z_i, 500))sin(\sqrt{(\lvert 500 - mod(z_i, 500)\rvert)} - \frac{(z_i - 500)^2}{10000n} & if \quad z_i > 500 \\ (mod(\lvert z_i \rvert, 500) - 500)sin\sqrt{(\lvert mod(\lvert z_i \rvert, 500) - 500 \rvert)} - \frac{(z_i + 500)^2}{10000n} & if \quad z_i < -500 \end{cases}$	n	[-5.12,5.12]	$f(0,...,0)$ =0.000012727*D