M E M O R A N D U M

Client Case: Muddy River Power Plant

Pittsburgh
Prepared by:
Chaitanya Patel, Divya Kasturirangan, Jiawen Zhang, Jingyi Guo

Re: Analysis on Real Option Valuation Analysis
Date: October 16th, 2017

Executive summary:

This memorandum describes a potential approach to value the real option that is involved as part of a tolling agreement with Bobba Fett. The shareholder suit are concerned about the initial payment. We have retained WrightConsultants to carry out a real option valuation analysis about the whole real option and then report the results.

Agreement Structure:

Muddy River Power Plant: pay 90% of the on-peak hours net operating earnings for a period from January 2015 to December 2016; In exchange, they receive an up-front payment of $24 million
Bobba Fett partnership: pay $24 million when the agreement initializes; In exchange, receive 90% of the on-peak hours net operating earnings from January 2015 to December 2016

Valuation methodology:

The risk-neutral methodology was adopted to perform the simulations.

Based on the underlying assets price process, we simulated 10,000 paths to generate the weekly power and gas spot price processes under 10,000. We cannot use physical discount factors when pricing because under the actual measure, money market account and various assets generate different rates of return and different people have different views of discount factors themselves. However, if we use risk-neutral measure, which is equivalent to actual measure, all assets generate the same mean rate of return, so we could directly use that risk-free rate as the discount rate of the asset that we are pricing.

Gas spot price paths:

Gas spot price paths were simulated using the Hull-White model:

$$dG_t = \alpha[\theta(t) - G_t]dt + \sigma G_t dW_t$$

First of all, we convert the un-annualized sample SD of weekly gas price changes to an annualized value and use it as σ.

$\sigma = \sigma_{un-annualized} * \sqrt{48}$ since we assume 48 weeks in one year

Here, for CC_t, the volatility is as follows:

Volatility (Continuous Component of Power)											
1	2	3	4	5	6	7	8	9	10	11	12
0.7503244	0.7503244	0.7503244	0.7503244	2.0001723	2.0001723	2.0001723	2.0001723	0.7503244	0.7503244	0.7503244	0.7503244

For gas process, the volatility is 0.5002163.

1. The central tendency function $\theta(t)$ was calibrated in order to match the historical market forward price data for gas and the spot price of natural gas.

Use Euler discretization, we have:

$$G_{t+1} = G_t + dt * \alpha[\theta(t) - G_t] + \sqrt{dt} * \sigma G_t * Z_1$$

where Z_1 is subject to $N(0, 1)$.

As we know that the spot gas prices are mean-reverting with an annualized speed of 3, this indicates that α=3.

We calibrate $\theta(t)$ for each time period using:

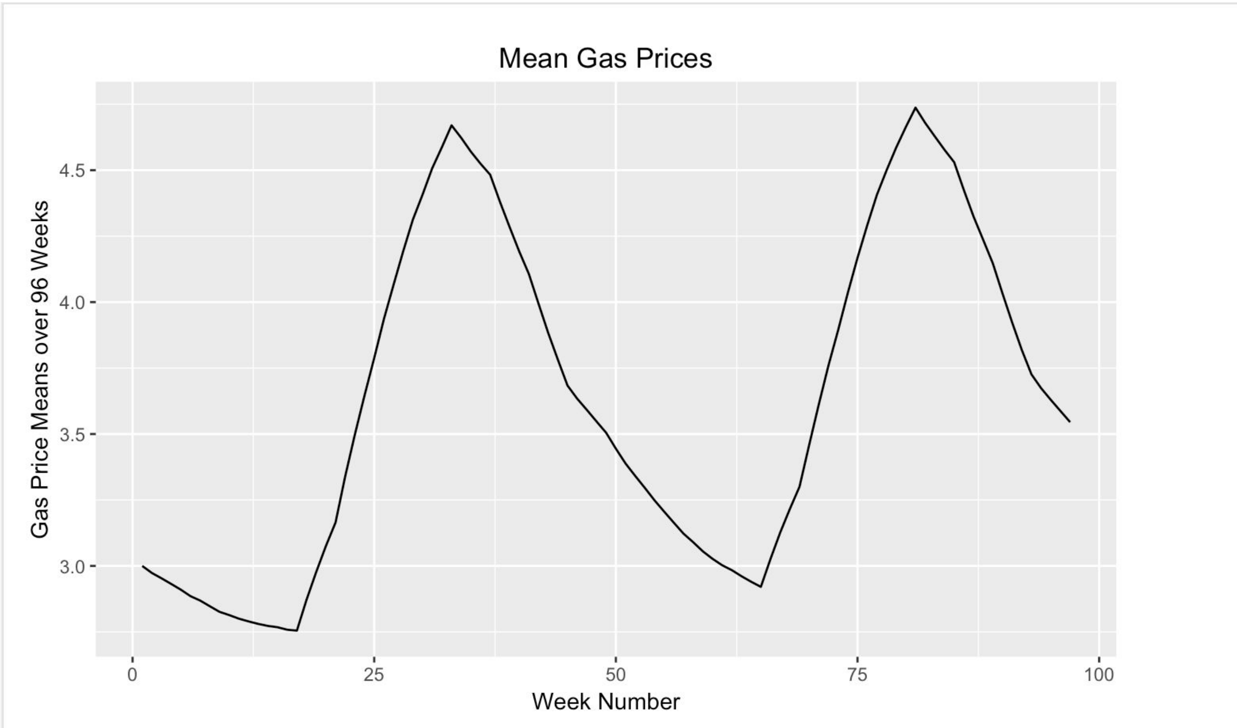
$$\theta(t) = \frac{G_{t+1} - G_t + G_t * dt * \alpha * G_t - \sqrt{dt} * \sigma G_t * Z_1}{dt * \alpha}$$

The simulation results are as follows:

Theta_Gas (Monthly)											
1	2	3	4	5	6	7	8	9	10	11	12
2.603041	2.51145	2.598096	2.65643	4.596959	5.923208	6.143552	5.851684	3.82693	2.855127	2.287769	2.928062

2. The spot prices of natural gas were then simulated for each path using the calibrated $\theta(t)$ and Euler discretization again.

The simulation results are as follows:



Weekly RN power price paths:

The RN power prices comprise two parts.

$$PP_t = CC_t + JC_t$$

1. For the continuous component CC_t, use the following dynamics:

$$dCC_t = \alpha_{CC}(\theta(t) - CC_t)dt + v(t)CC_t dW_t$$

First, we convert the vector of un-annualized sample SD of weekly non-spike power price changes to annualized values and use it as $v(t)$.

$$v(t) = v(t)_{un-annualized} * \sqrt{48}/100$$

Then, similar as what we did for gas prices, we calibrated $\theta(t)$ in order to match the historical market forward price data for electricity and the spot price of electricity in last week of December 2014.

Use Euler discretization, we have:

$$CC_{t+1} = CC_t + dt * \alpha_{CC}[\theta(t) - CC_t] + \sqrt{dt} * v(t)CC_t * Z_2$$

Since we know that the correlation between the gas prices and the power prices after purging power spike dynamics is equal to 0.3, we generate $Z_2 = 0.3Z_1 + \sqrt{1-0.3^2}X$, where X is subject to $N(0,1)$, and Z_1, X are independent with each other.

We calibrate $\theta(t)$ for each time period using:

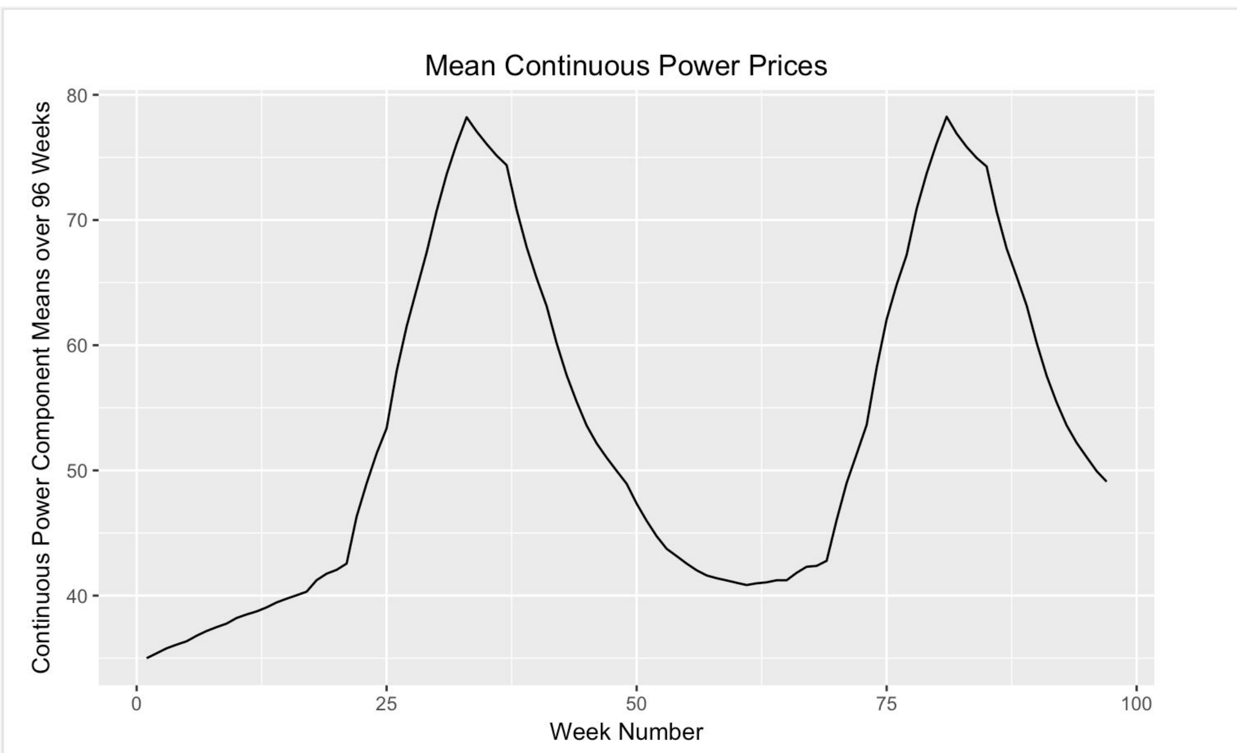
$$\theta(t) = \frac{CC_{t+1} - CC_t + CC_t * dt * \alpha_{CC} - \sqrt{dt} * v(t) CC_t * Z_2}{dt * \alpha_{CC}}$$

The calibrated $\theta(t)$ results for CC_t process are shown below:

Theta_CC (Monthly)											
1	2	3	4	5	6	7	8	9	10	11	12
37.66171	39.41738	40.20098	41.55901	45.19792	66.28668	83.82334	90.51659	69.88805	50.35804	42.77347	44.0146

Lastly, we utilized the calibrated $\theta(t)$ and Euler discretization to simulate continuous component of RN power price paths.

The simulation results are as follows:



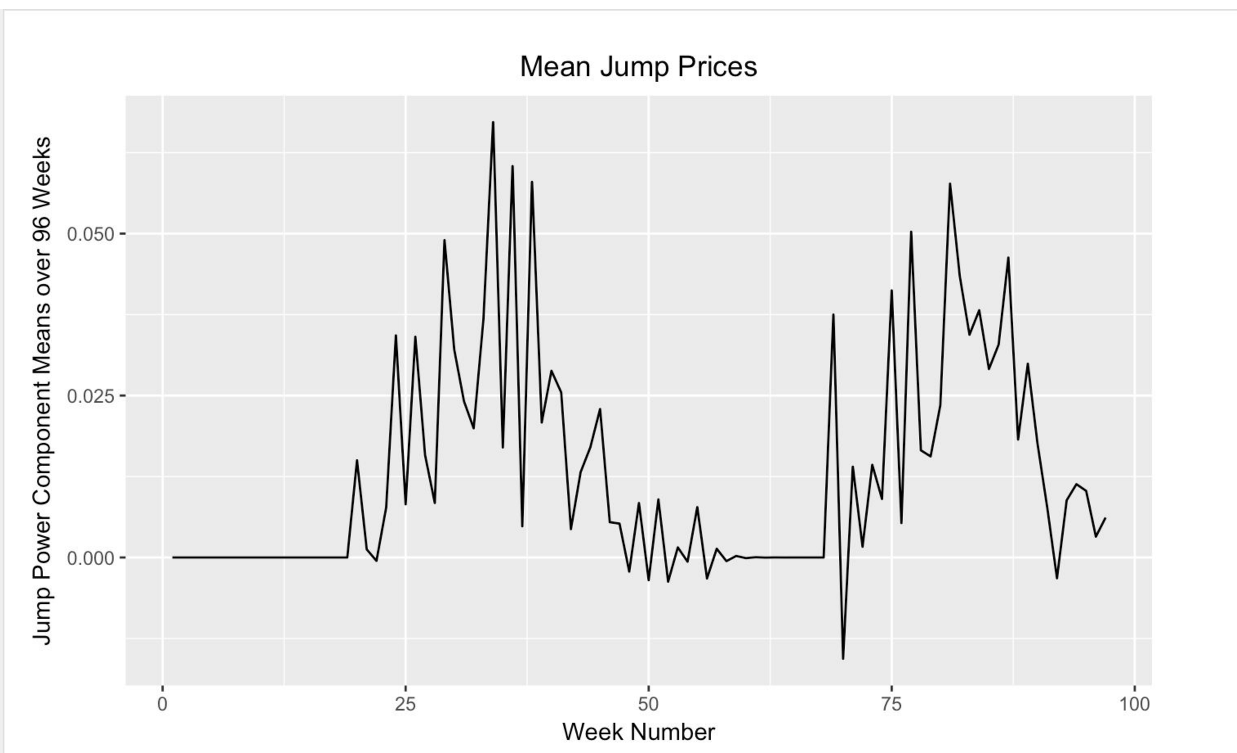
2. For the jump process component JC_t, use the following dynamics:

$$dJC_t = \alpha_{JC}(0 - JC_t)dt + m\ dq_t$$

When continuous component CC_t is larger or equal to $75/Mwh, price spikes might occur.
As we know there is an 8.3 percent weekly probability of a spike of m=$75/Mwh on average, we model jumps dq_t as a Poisson process with $\lambda = 8.3\%$. The speed of mean-reversion for jumps back to 0 is 20, indicating that $\alpha_{JC} = 20$.

Given the above dynamics, we simulate power price spikes as follows:
$$JC_{t+1} = I_{[CC_t>=75]}[\alpha_{JC}(0 - JC_t)dt + m\ *\ dq_t]$$
Where dq_t is 1 with probability 8.3% and 0 otherwise, and we simulated dq_t by generating a uniform random variable between 0 and 1 and checking whether it's greater than 8.3%.

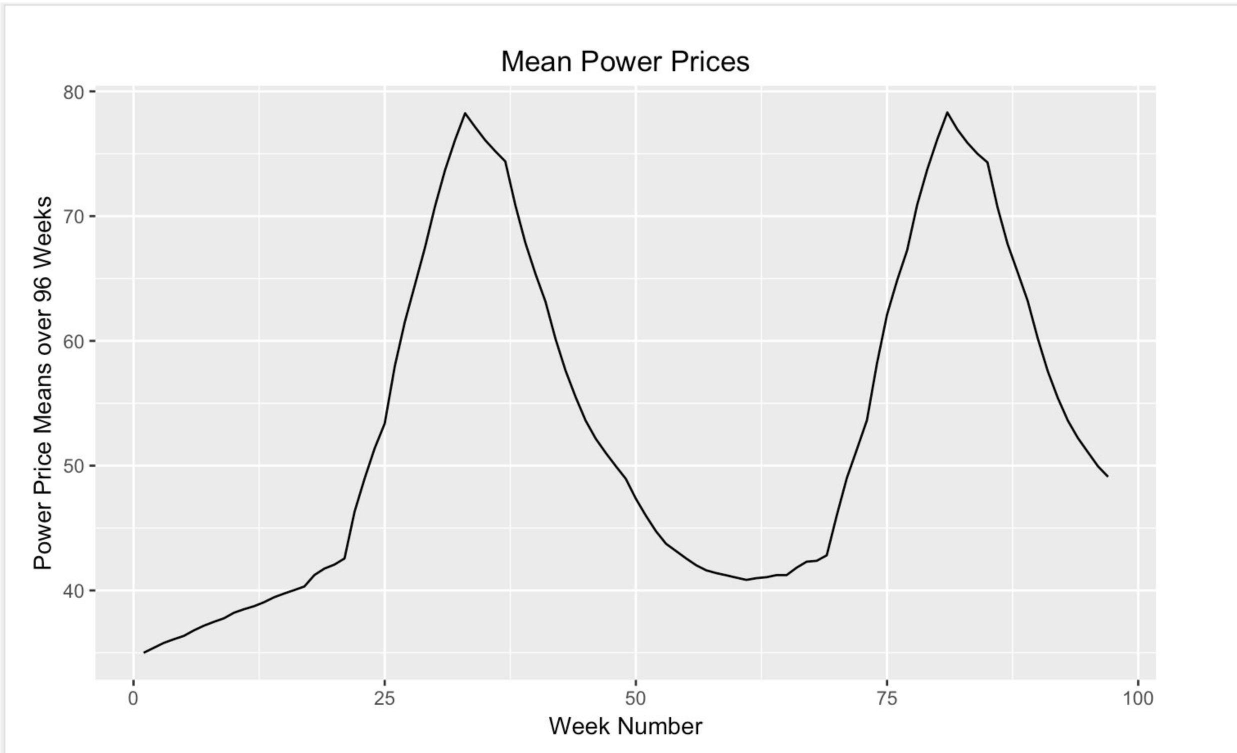
The simulation results are as follows:



3. Now that we already simulated the continuous components and the jump processes, we sum those two processes together to come to the RN power prices:

$$PP_t = CC_t + JC_t$$

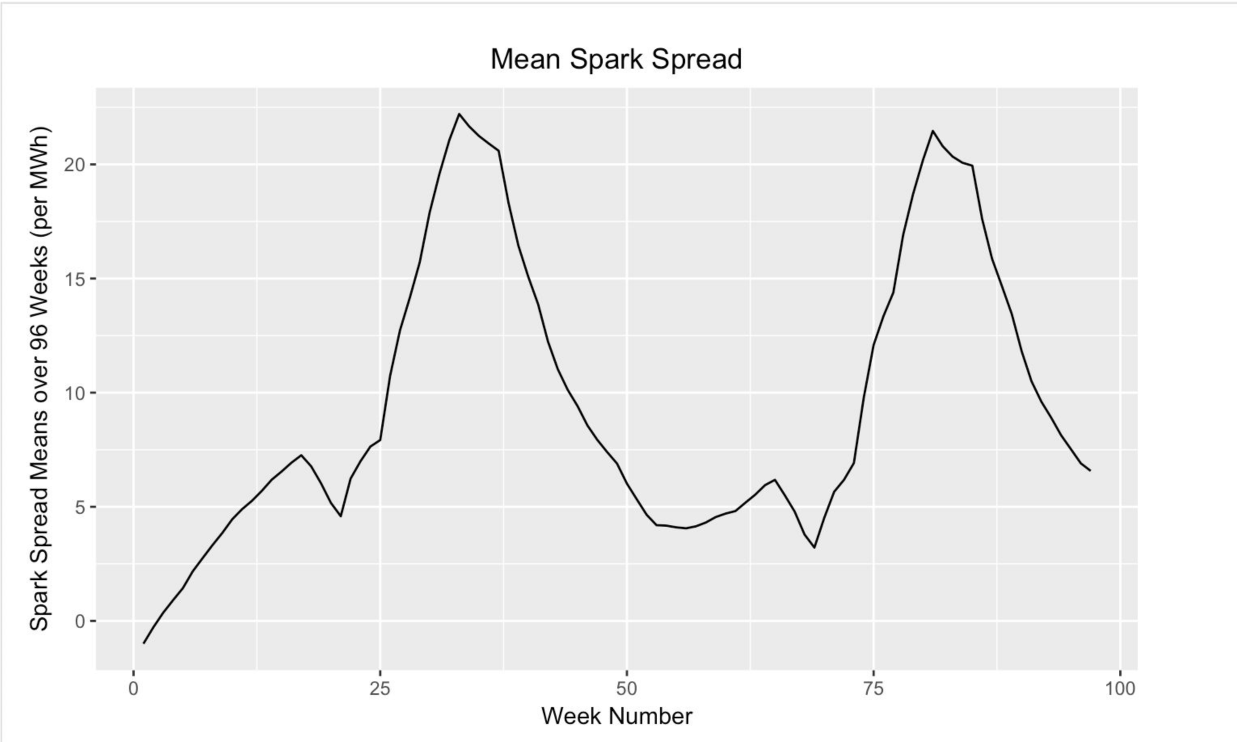
We plotted the power prices based on the simulation results as follows:



Now that we have simulated paths for weekly gas and power prices, we can calculate spark spreads as follows:

$$spark\ spreads_t = PP_t + G_t$$

We plotted the spark spreads based on the simulation results as follows:

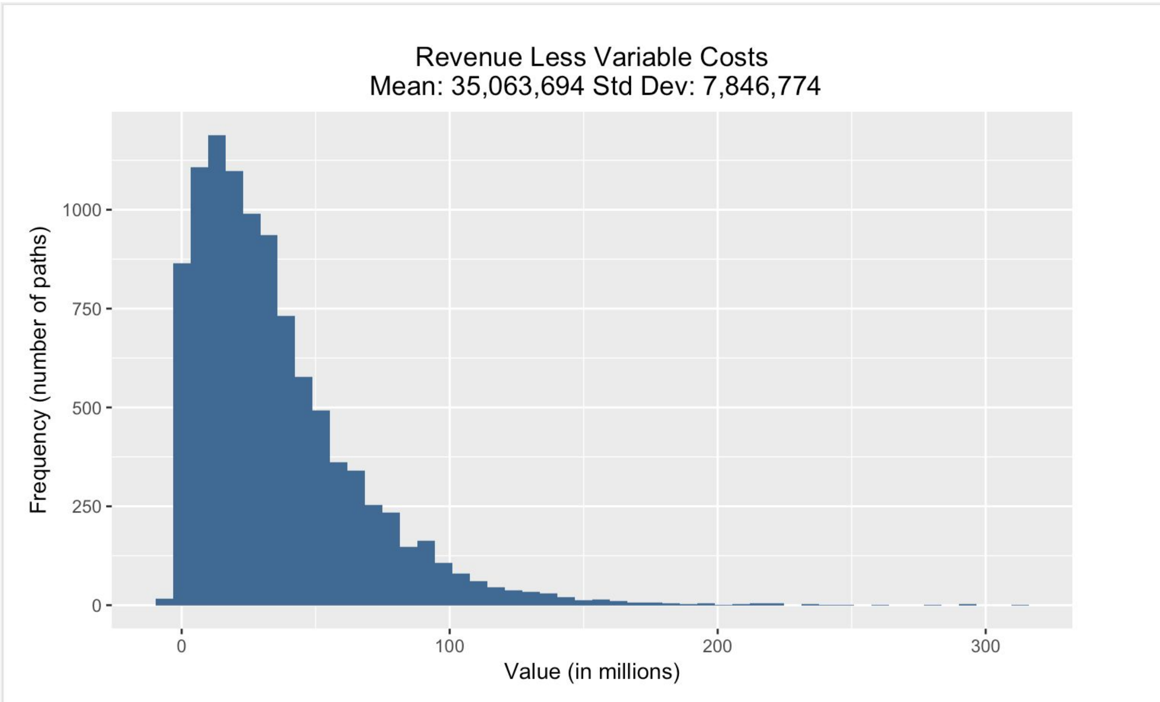


Operating Policy:

For the operating policy, we decided to take two threshold values for switching the power plants on and off. We thought it would be reasonable to switch on the power plant once the spark spread crosses 10 since we have a non-fuel cost of $5 per MWh when the power plant is on. When the power plant is on, we switch it off if the spark spread goes below 3. Although, we start making a loss when the spark is below 5, it may be temporary and we may be losing profits if we turned off the power plant since it would take 2 weeks to turn it back on. With the above thoughts in mind and choosing several values across different runs (since we should not overfit to a single set of simulation), we decided that these values would be reasonable.

Revenues less variable costs (both fuel and non-fuel):

Once the operating policy was decided, we ran a 10,000 path simulation to get an estimate of the revenue. The plot below shows the results.



We see that the mean revenue is about $35 million with a standard deviation of about $8 million. That being said, the agreement value would be approximately 90% of the mean value. This comes out to be $31,557,324. We should note that the standard deviations of many of the parameter values could swing this value however, it wouldn't shift by up to $11 million and therefore, our analysis says that the exchange value was understated and the correct value would be around $32 million.