M E M O R A N D U M

Main Case: Tillsonburgh

Pittsburgh

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Re: Analysis on Real Option Valuation Analysis
Date: September 27, 2017

Executive summary:

This memorandum describes a potential approach to securitize the leasing receivables of Tillsonburgh Corp.'s flagship computerized phoropter, EyeCheck™. Tillsonburgh is currently constrained by its over-levered balance sheet and has a large value of current leasing assets (and associated debt financing) relative to its equity. They are currently looking for a solution through asset-backed securitization to securitize part of their current leasing receivables in order to provide more financing for current receivables and allow for continued growth in their leasing activities. They have previously reached out to a competitor and are looking for a more flexible structure that raises more funds rather than the plain vanilla structure proposed to them.

Deal Structure:

The client is looking to securitize 750 lease receivables into an Asset-Backed Security. Each lease allows the lessee to rent Tillsonburgh Corp.'s flagship computerized phoropter, EyeCheck™ for a period of 12 years. A new phoropter costs $435,000 and each lease consists of 12 semi-annual payments of $41,318 such that the initial principal amortizes down to a notional residual value of $150,000 at the end of the lease. The lessee then has an option to either return the phoropter or pay a flat sum of $150,000 and buy the used phoropter.

Taking into account the targeted loss provided by the credit agencies, an ABS structure involving tranches of Class AAA, AA, A, BBB, BB and B has been made. This however does not account for the residual phoropter values at the end of the lease and only takes into account the lease payments made by the obligors. At the end of 6 years, there is a remaining "equity" claim on a credit enhancement reserve account. Also, the residual values of the phoropters has not been securitized as part of the ABS structure since they are an unknown value. By simulating the Green-book (market value) prices of the phoropters, and taking into account the default rates based on the historical data provided by Tillsonburgh, the equity claim and the residual purchase option has been valued.

The qualifying lease receivables – the semi-annual payments – will be sold into a trust where they are will collateralize a series of AAA rated tight-window sequential-pay tranches with maturities of 0.5 years, 1 year, 1.5 years, etc. up to six years. Underneath the AAA tranches come subordinated bond tranches of AA, A, BBB, BB and B rating. The coupon rates for the ABS has been based on the current market rates.

The table below shows the split of Principal in the various tranches in the ABS structure.

Maturity			Rating AAA	AA	A	BBB	BB	B
	31-Mar-18	Principal	12,262,300	130,450	130,450	130,450	130,450	65,225
		IR Spread (bps)	25	33	40	48	55	63
	30-Sep-18	Principal	12,110,560	275,240	275,240	275,240	275,240	137,620
		IR Spread (bps)	85	93	102	110	118	126
	31-Mar-19	Principal	11,905,580	435,570	435,570	435,570	435,570	217,785
		IR Spread (bps)	90	99	108	117	126	134
	30-Sep-19	Principal	11,641,680	612,720	612,720	612,720	612,720	306,360
		IR Spread (bps)	95	105	114	124	133	143
	31-Mar-20	Principal	11,312,000	808,000	808,000	808,000	808,000	404,000
		IR Spread (bps)	100	110	120	130	141	151
	30-Sep-20	Principal	10,911,360	1,022,940	1,022,940	1,022,940	1,022,940	511,470
		IR Spread (bps)	105	116	127	137	148	159
	31-Mar-21	Principal	10,432,460	1,259,090	1,259,090	1,259,090	1,259,090	629,545
		IR Spread (bps)	108	119	130	142	153	164
	30-Sep-21	Principal	9,867,520	1,518,080	1,518,080	1,518,080	1,518,080	759,040
		IR Spread (bps)	110	122	134	146	158	170
	31-Mar-22	Principal	9,209,200	1,801,800	1,801,800	1,801,800	1,801,800	900,900
		IR Spread (bps)	118	130	143	155	168	181
	30-Sep-22	Principal	8,448,400	2,112,100	2,112,100	2,112,100	2,112,100	1,056,050
		IR Spread (bps)	125	138	152	165	178	191
	31-Mar-23	Principal	7,576,220	2,451,130	2,451,130	2,451,130	2,451,130	1,225,565
		IR Spread (bps)	130	144	158	172	186	199
	30-Sep-23	Principal	6,582,240	2,820,960	2,820,960	2,820,960	2,820,960	1,410,480
		IR Spread (bps)	135	150	164	179	193	208
		Total Principal	122,259,520	15,248,080	15,248,080	15,248,080	15,248,080	7,624,040

Total Principal Securitized: $190,875,900

Commission rate: 1.3% (of their corresponding transaction value from both Tillsonburgh and Kitzstein Partners L.P.)

Valuation methodology:

The risk-neutral methodology was adopted to perform the simulations.

Gas spot price paths:

Gas spot price paths were simulated using the Hull-White model:

$$dG_t = \alpha[\theta(t) - G_t]dt + \sigma G_t dW_t$$

1. The central tendency function $\theta(t)$ was calibrated in order to match the historical market forward price data for gas and the spot price of natural gas.

Use Euler discretization, we have:

$$G_{t+1} = G_t + dt * \alpha[\theta(t) - G_t] + \sqrt{dt} * \sigma G_t * Z_2$$

where Z_2 is subject to $N(0, 1)$.

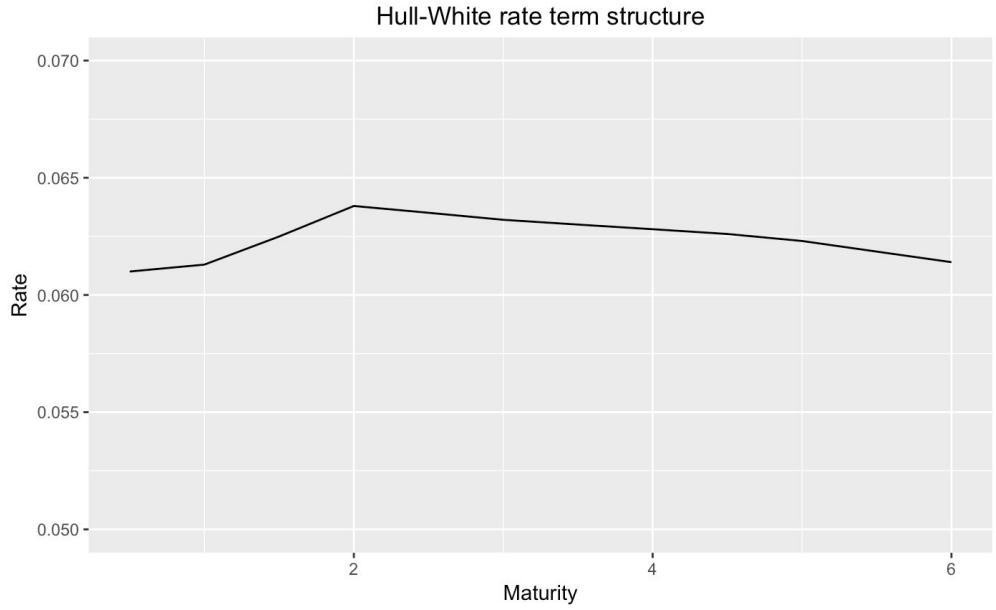
We calibrate $\theta(t)$ for each time period using:

$$\theta(t) = \frac{G_{t+1} - G_t + G_t * dt * \alpha * G_t - \sqrt{dt} * \sigma G_t * Z_2}{dt * \alpha}$$

2. The spot prices of natural gas were then simulated for each path using the calibrated $\theta(t)$ and Euler discretization again.

Based on the two components, we conducted valuation for the equity reserve account and the residual purchase option separately.

The graph below shows the average rate value of 1,000,000 simulations across the different maturities.



Equity Reserve Account:

For the equity reserve account, the standard ABS structure including multiple tranches and multiple maturities was used. This model is different from the previously suggested competitor model as it included all Tranches AAA, AA, A, BBB, BB and B instead of the proposed AAA and BB tranches alone. The advantages of such a model include more higher rated tranches resulting in lower interest payments, so higher value of equity reserve account and also, there is more principal being securitized in this structure. Based on the Rating Agency Credit Enhancement Requirements:

$$Percent\ credit\ enhancement = loss\ multiple * weighted\ average\ life * targeted\ loss$$

Based on the formula, the Percent Credit Enhancement was calculated for every single maturity based on their Weighted Average Life, their loss multiple and the targeted loss of 0.02 provided by the rating agencies to allocate securitized principal and Dollar Credit Enhancement. Below is our tranche structure:

Credit Enhancement calculated for each Tranche (In thousands of dollars):

	AAA	AA	A	BBB	BB	B
0.5	783	652	522	391	261	196
1	1,651	1,376	1,101	826	550	413
1.5	2,613	2,178	1,742	1,307	871	653
2	3,676	3,064	2,451	1,838	1,225	919
2.5	4,848	4,040	3,232	2,424	1,616	1,212
3	6,138	5,115	4,092	3,069	2,046	1,534
3.5	7,555	6,295	5,036	3,777	2,518	1,889
4	9,108	7,590	6,072	4,554	3,036	2,277
4.5	10,811	9,009	7,207	5,405	3,604	2,703
5	12,673	10,561	8,448	6,336	4,224	3,168
5.5	14,707	12,256	9,805	7,353	4,902	3,677
6	16,926	14,105	11,284	8,463	5,642	4,231
Total:	**91,488**	**76,240**	**60,992**	**45,744**	**30,496**	**22,872**

Total Allowable ABS Bond Principal Levels (Amount Securitized in thousands of dollars):

	AAA	AA	A	BBB	BB	B	Totals
0.5	12,262	130	130	130	130	65	12,849
1	12,111	275	275	275	275	138	13,349
1.5	11,906	436	436	436	436	218	13,866
2	11,642	613	613	613	613	306	14,399
2.5	11,312	808	808	808	808	404	14,948
3	10,911	1,023	1,023	1,023	1,023	511	15,515
3.5	10,432	1,259	1,259	1,259	1,259	630	16,098
4	9,868	1,518	1,518	1,518	1,518	759	16,699
4.5	9,209	1,802	1,802	1,802	1,802	901	17,317
5	8,448	2,112	2,112	2,112	2,112	1,056	17,953
5.5	7,576	2,451	2,451	2,451	2,451	1,226	18,606
6	6,582	2,821	2,821	2,821	2,821	1,410	19,277
Total:	**122,260**	**15,248**	**15,248**	**15,248**	**15,248**	**7,624**	**190,876**

Next, we calculated the scheduled payments for bondholders and potential collected payments from leases (taking into account possible defaults over the course of the lease) separately and compare them to do a cumulative calculation for the equity reserve account.

On the one hand, for the scheduled payments, we calculated the amount of money we need to pay out for every period. The interest rate we used comes from Exhibit 6, where we have the current ZCB term structure as well as the ABS pricing spreads. For simplicity, we used linear interpolation to fill the gaps in the spreads and got the following Maturity versus Spread to Maturity:

	AAA	AA	A	BBB	BB	B
0.5	25	32.5	40	47.5	55	62.5
1	85	93.25	101.5	109.75	118	126.25
1.5	90	98.875	107.75	116.625	125.5	134.375
2	95	104.5	114	123.5	133	142.5
2.5	100	110.125	120.25	130.375	140.5	150.625
3	105	115.75	126.5	137.25	148	158.75
3.5	107.5	118.875	130.25	141.625	153	164.375
4	110	122	134	146	158	170
4.5	117.5	130.125	142.75	155.375	168	180.625
5	125	138.25	151.5	164.75	178	191.25
5.5	130	143.875	157.75	171.625	185.5	199.375
6	135	149.5	164	178.5	193	207.5

The scheduled payments (in thousands of dollars) for each period, each tranche is listed below:

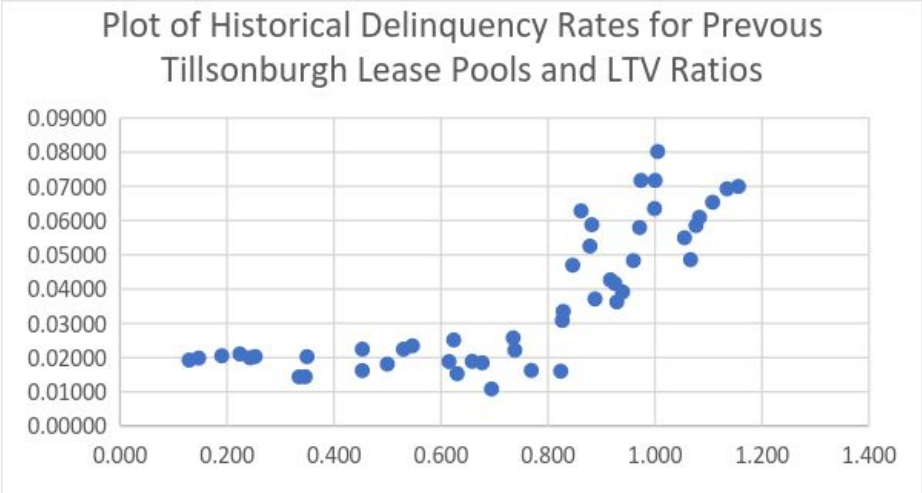
	AAA	AA	A	BBB	BB	B	Totals
0.5	12,664	135	135	135	135	67	**13,271**
1	12,986	295	294	296	296	148	**14,315**
1.5	13,218	484	482	485	486	243	**15,399**
2	13,379	705	703	707	709	355	**16,558**
2.5	13,433	962	958	965	967	485	**17,770**
3	13,376	1,257	1,253	1,263	1,266	635	**19,050**
3.5	13,188	1,596	1,592	1,605	1,610	807	**20,400**
4	12,852	1,984	1,980	1,997	2,003	1,005	**21,821**
4.5	12,353	2,426	2,422	2,444	2,453	1,231	**23,329**
5	11,662	2,928	2,923	2,952	2,964	1,488	**24,918**
5.5	10,753	3,495	3,491	3,526	3,543	1,780	**26,589**
6	9,598	4,135	4,131	4,175	4,196	2,109	**28,345**
Total:	**149,464**	**20,403**	**20,366**	**20,550**	**20,629**	**10,354**	**241,766**

Here, we only focus on the total value because if there is failure to pay either principal or interest payments, this leads to a default.

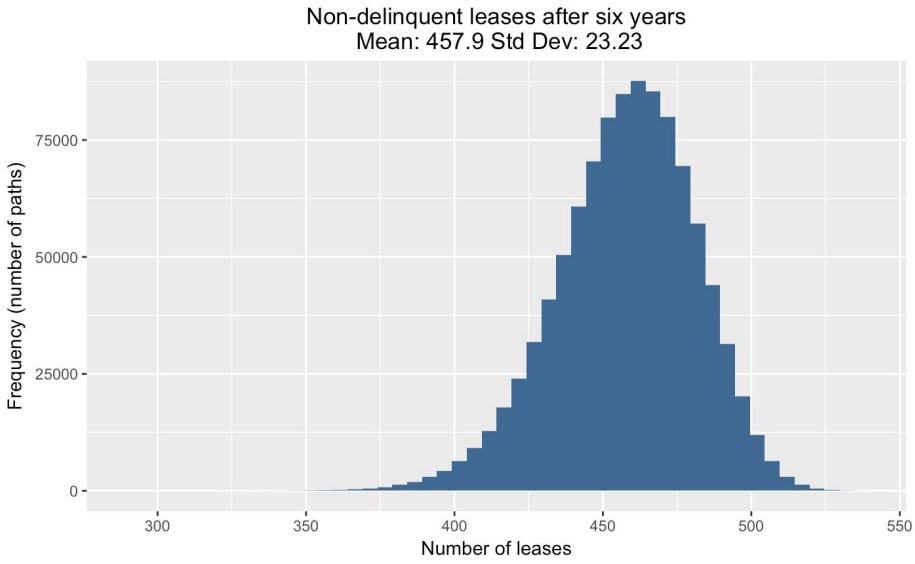
On the other hand, for the gathered payments, we further need to split them into two parts, for one the leases that have not defaulted yet and the other for the defaulted leases. Here, we needed to predict the default rates for each period based on the LTV at that time. Given the four sets of default data in Exhibit 3c, we calibrated the default rate based on a parametric regression regime—Generalized Linear Model.

In order understand the mechanics of the evolvement of delinquency risks, we built a regression model for semi-annual delinquency rates to LTV ratio and years until maturity.

We started with linear models, building 12 linear regression models of delinquency rate to LTV ratio at each maturity respectively. However, according to the scatter plot of delinquency rate to LTV ratio, their relationship is clearly not explainable by linear regression. As seen below, the relationship between default rates and LTV was not linear by visual inspection. This was also non linear for default rates and maturity.



This motivated us to use the generalized linear model to capture the relationship between the factors affecting the default rates. The graph below shows the distribution of the number of non-delinquent (after the six years) after simulation using the default rates given by the generalized linear model above.



Green Book Prices:

Further, based on the underlying assets price process, we simulated one million paths to generate the Green Book Price process under 1 million scenarios, which provides the market price for phoropters. We cannot use physical discount factors when pricing because under the actual measure, money market account and various assets generate different rates of return and different people have different views of discount factors themselves. However, if we use risk-neutral measure, which is equivalent to actual measure, all assets generate the same mean rate of return, so we could directly use that risk-free rate as the discount rate of the asset that we are pricing.

$$dS_t = (\mu - \delta_t)S_t dt + \sigma S_t dW$$

We first use change of measure to switch to risk neutral pricing:

$$dW_t = d\tilde{W}_t + \frac{r-(\mu-\delta_t)}{\sigma}dt$$

Then we have:

$$dS_t = r_t S_t dt + \sigma S_t d\tilde{W}_t$$

Use Euler discretization, we have:

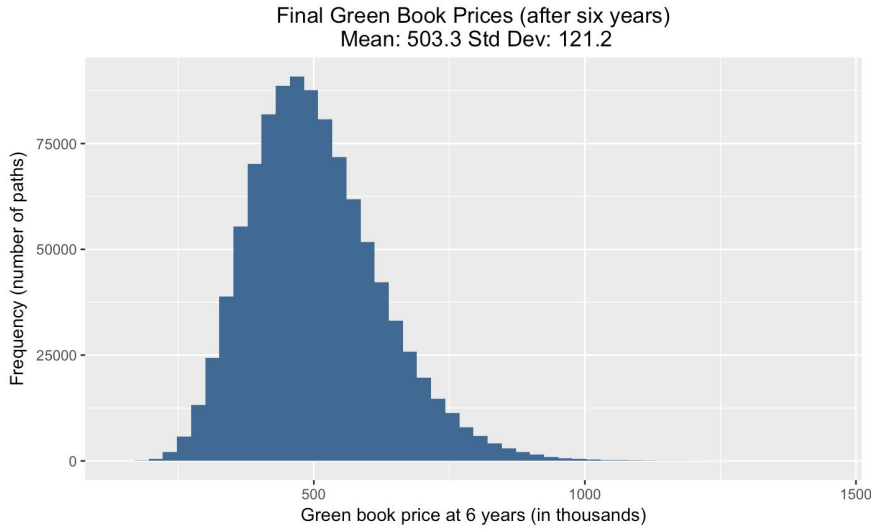
$$S_{t+1} = S_t + dt * rS_t + \sqrt{dt} * \sigma S_t * Z_1$$

where Z_1 is $N(0,1)$.

It is given that Z_1, Z_2 are independent random variables since no evidence has been found of any correlation between the phoropter and interest rate Brownian motion dw and dz.

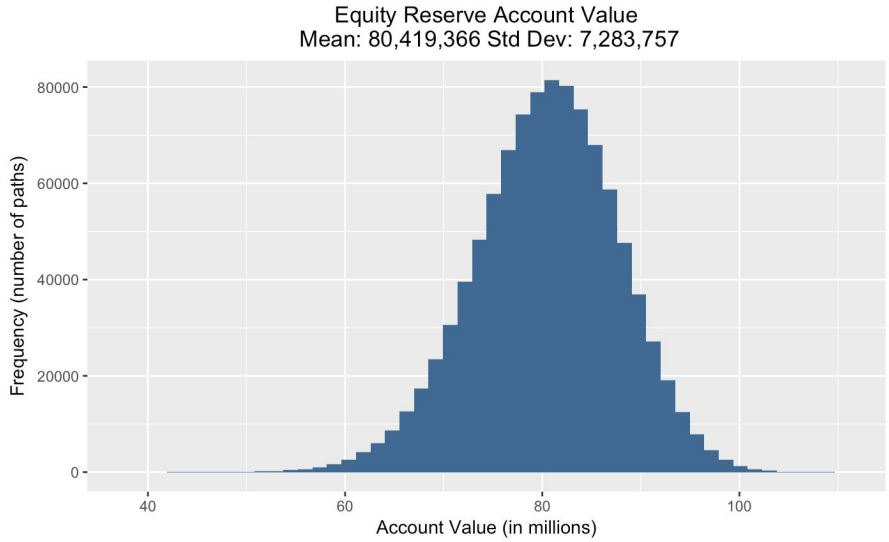
The number of leases remaining in the pool depends on the default leases per period. From the predicted default values from the LTV for each period for each path, we calculated the payments from selling the default leases under different paths and which gives the one-time payments from the defaulted leases.

As for the non-delinquent leases, it is simply the remaining leases paying the scheduled principal and interest payments each period. We sum up the results we get for defaulted and non-defaulted leases, and this gives the actual payments we can collect from the leases every period.



Option for equity value of reserve account:

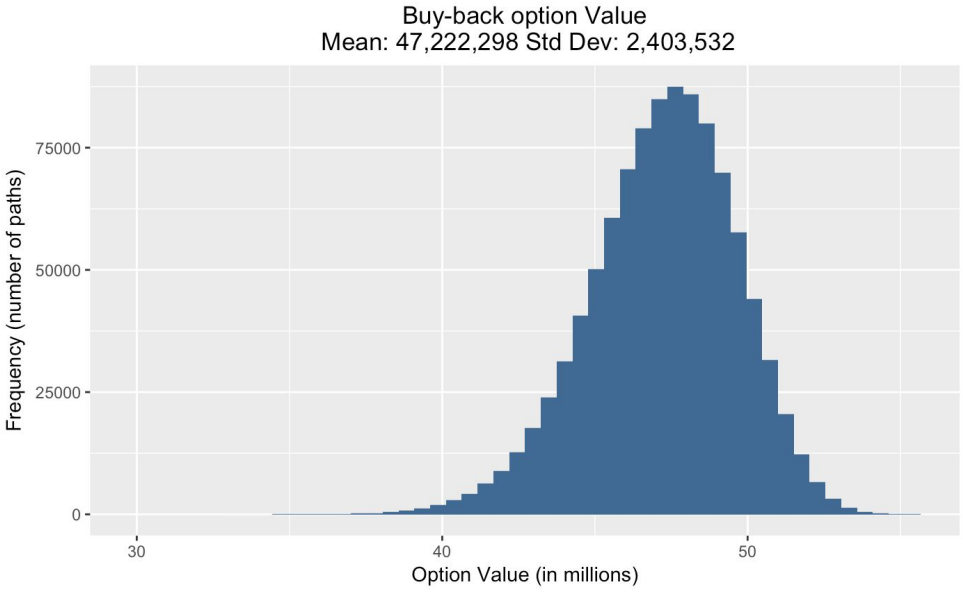
The reserve account initially starts with no value and then cumulates through the 6 years, whenever we have residual after paying out the principal and interest for each tranche for each maturity period. This deposit in the reserve account is then invested in 6-month t-bills. Whenever there is a gap between the collected payments and the scheduled payments, we use the reserve account as a backup for payments.



Total Option Value	$80,419,366
Delta (**δ**) (per $1000 change)	140,060
Gamma (Γ)	2.692e-4
Vega	182,325
Rho (ρ)	2,745,000

Residual purchase option:

At the end of the lease, the obligors have the option to decide if they would like to buy the used phoropter for $150,000. If the green book price for the used phoropter is much higher, they would feel incentivised to buy the phoropter instead of returning it. If the value of the phoropter is lesser than $150,000, they would prefer to return the phoropter and buy it outside from the market if they need it. From our simulations we compute the number of final non-delinquent leases and calculate if they would buy the equipment at $150,000 or return the equipment and we would then sell it at the then green book price. The average of a million such simulated paths is the option value.



Total Option Value	$47,222,298
Delta (δ) (per $1000 change)	36,390
Gamma (Γ)	6.395e-3
Vega	8,971,500
Rho (ρ)	2,082,300

Payments we get from the hedge fund: Kitzstein Partners L.P., a New Jersey based hedge fund has expressed an interest in exotic business options. Kitzstein has said that they will take a 50% position in some combination of the default and residual purchase option cash flows, provided that they can buy them at a 10% discount relative to their theoretical value. Upon Valuation we get the below values:

50% Equity Reserve Sell Price : $ 36,188,715
50% Buy-back Option Sell Price:$ 21,250,034

Competitor Model	
ABS Total Principal:	$183,251,840
Total Deal Value:	$183,251,840

DCJ Square Derivatives Model	
ABS Total Principal:	$190,875,880
50% Equity Reserve Sell Price :	$ 36,188,715
50% Buy-back Option Sell Price:	$ 21,250,034
Total Deal Value:	**$248,314,629**

Deal VaR: