AALBORG UNIVERSITY

Link Analysis – PageRank & HITS

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Based (in part) on Stanford slides by Christopher Manning & Pandu Nayak and on the 'Introduction to Information Retrieval ' book (Chap. 21) by Christopher Manning, Prabhakar Raghavan & Hinrich Schütze. Also, some slides are provided by Bo Thiesson, Manfred Jaeger, Thomas D. Nielsen, AAU.

Outline

Link-based techniques for ranking

- Motivation
- Anchor text
- Structural measures
  - PageRank (query-independent)
  - HITS (query-dependent)

Today's lecture – hypertext and links (for ranking)

We look beyond the *content* of documents

- We begin to look at the hyperlinks between them

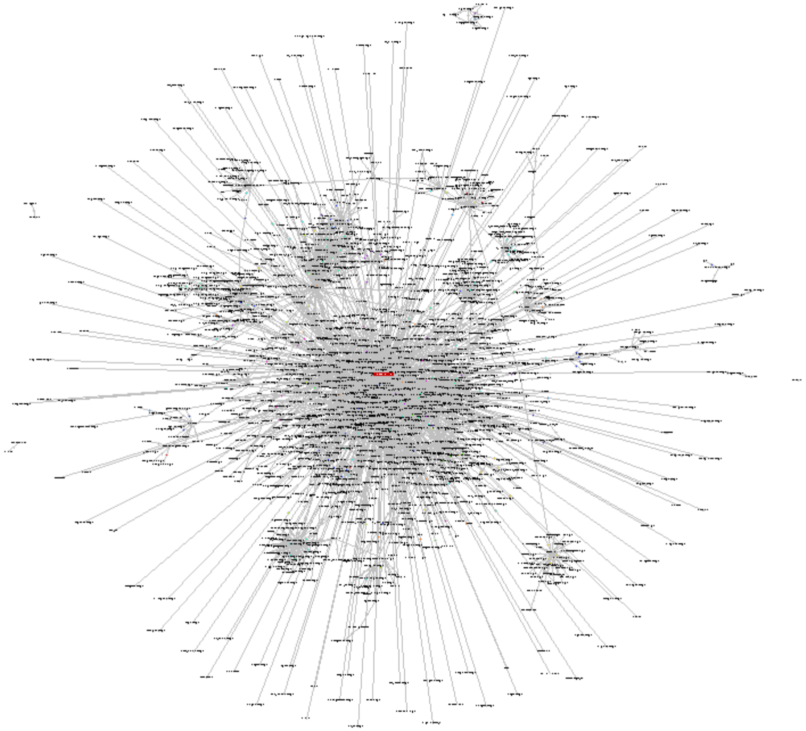
Links are everywhere

- The Web (today)
- Email
- Social networks
- Phone call logs
- Usage (connectivity) logs
- …

Web Structure – Social Network

- **A** node **for each** person
- **A** (directed) edge **for each (directed)** relationship

ENRON social network



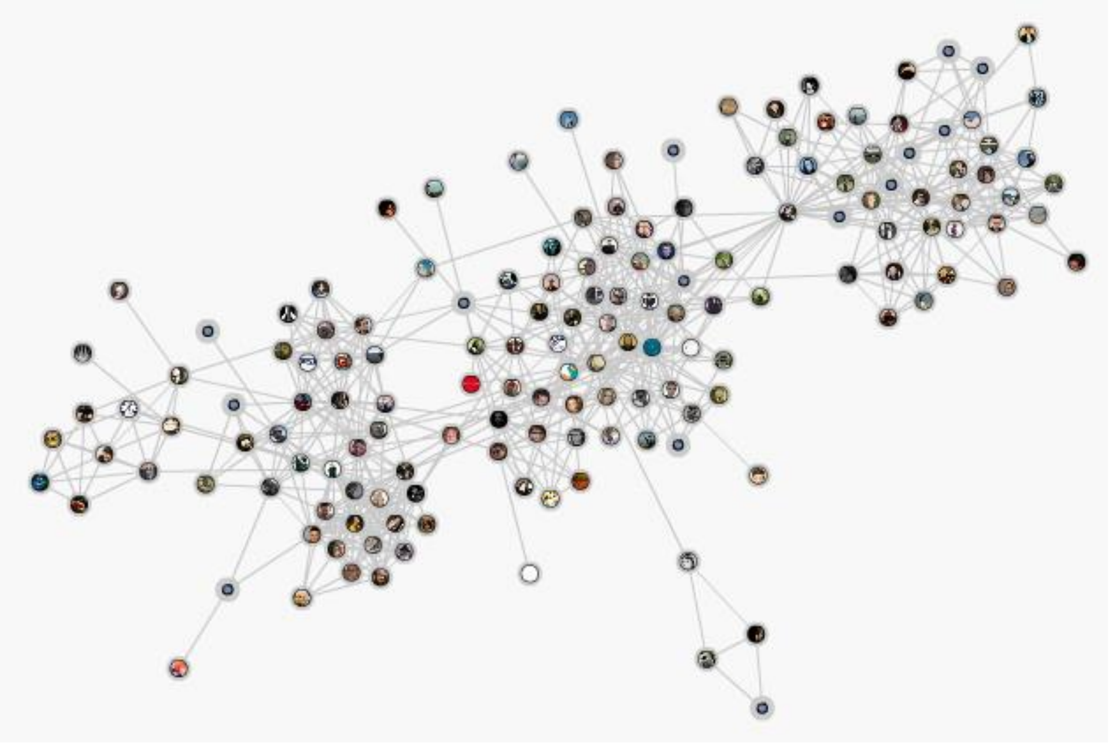
Nodes: ENRON employees

(Directed) link: "sent email to"

Web Structure – Social Network

- A node for each person
- A (directed) edge for each (directed) relationship

Facebook – Social graph



Nodes: Facebook profile & friends

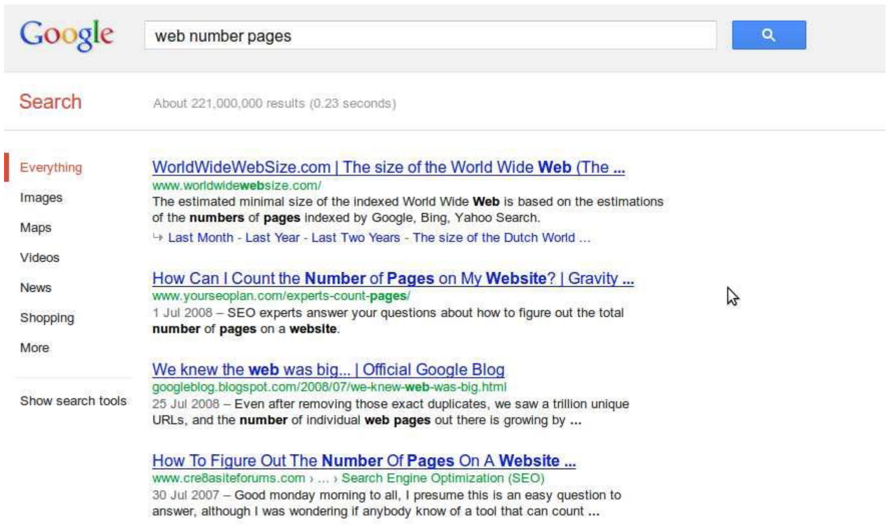
(Directed) link: "friend of"

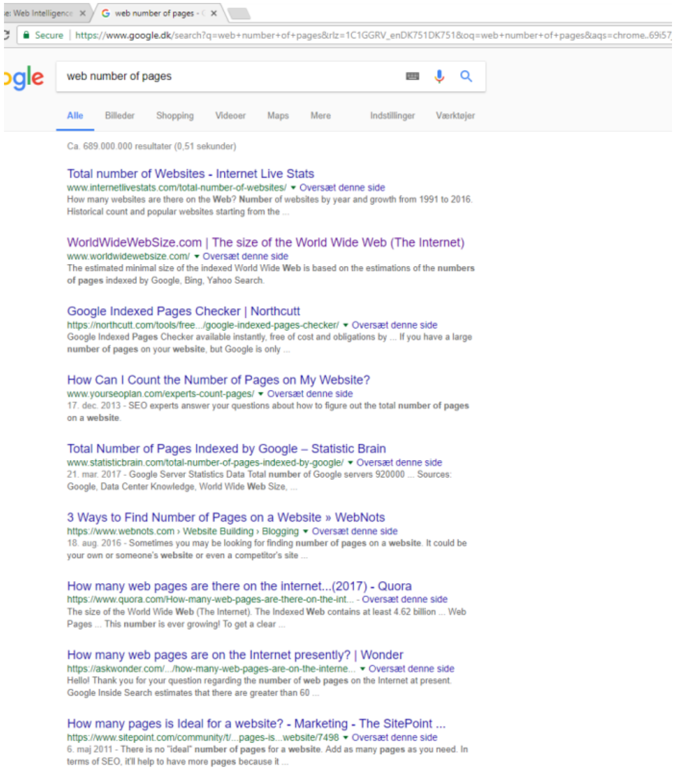
Ranking motivation

Google search for "web number pages":

2017

2015



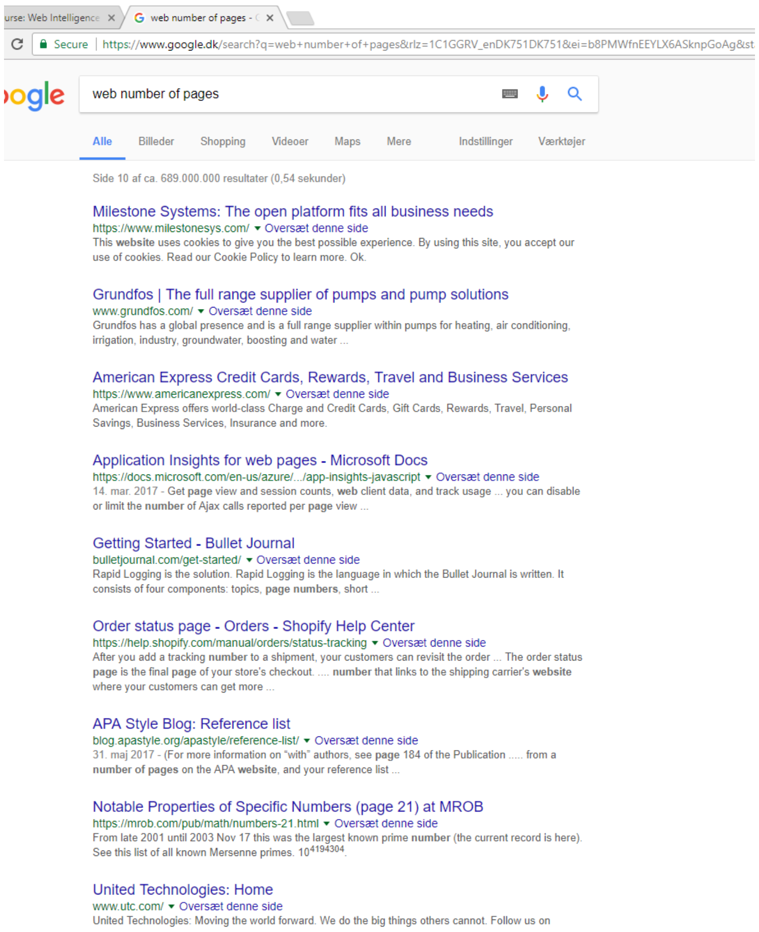


- 221.000.000 hits in 2015, 689.000.000 in 2017
- Top 4 results contains useful and trustworthy information

Ranking Motivation cntd.

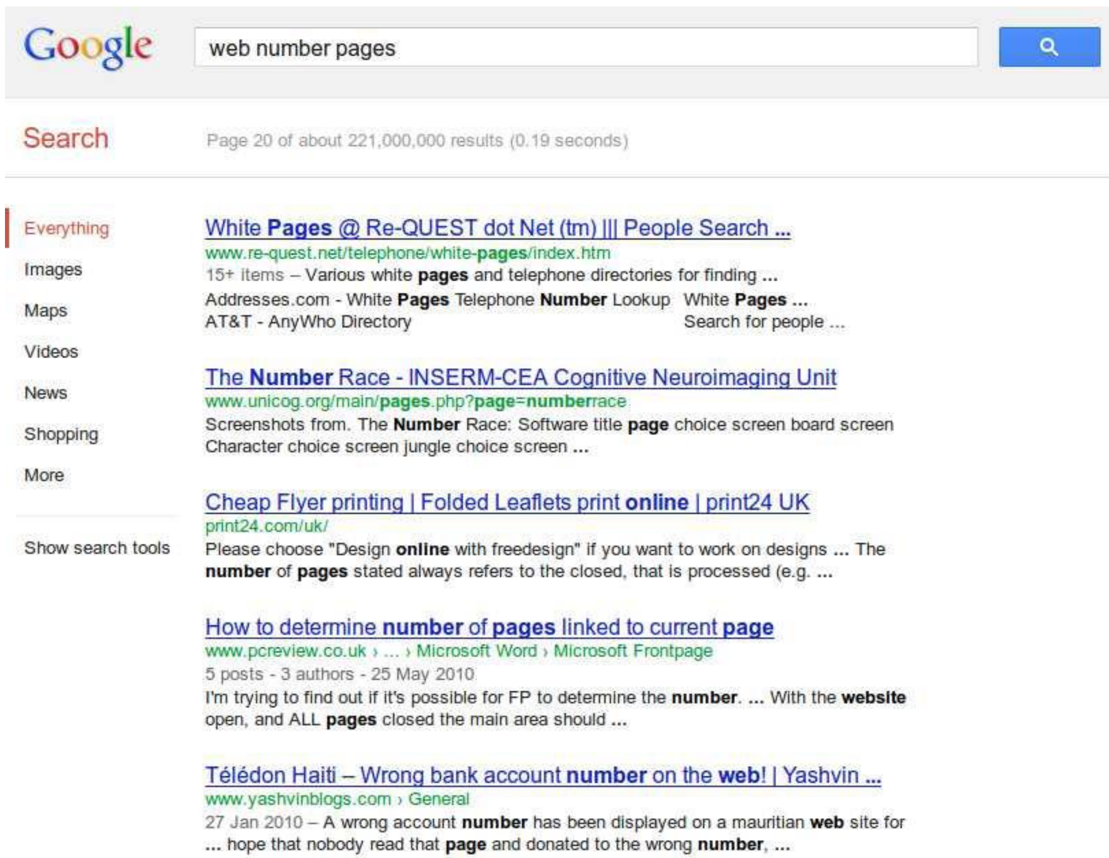
Look at 10th result page
(top 90-100)

Not so useful anylonger



Ranking motivation (cont.)

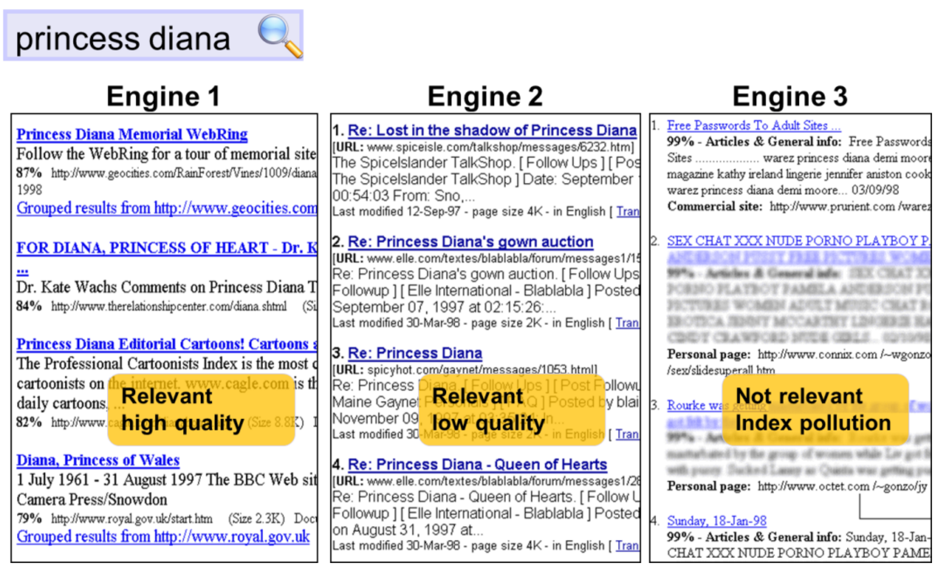
And results 200-205 cnt. less useful:



Ranking

Goal: order the answers to a query in decreasing order of value

- Just show top k (≈10) results (at a time)
- User is not overwhelmed



Some Ranking Criteria

- Content-based techniques (e.g. vector space model) – query-dependent
- Ad-hoc factors (anti-porn heuristics, location on page, publication/location data, length …) – mostly query-independent
- Human annotations
- Structure-based techniques (this lecture)
  - PageRank – query-independent
  - HITS – query-dependent

Ranking criteria defines a **ranking score** that measures how well a document and query "match".

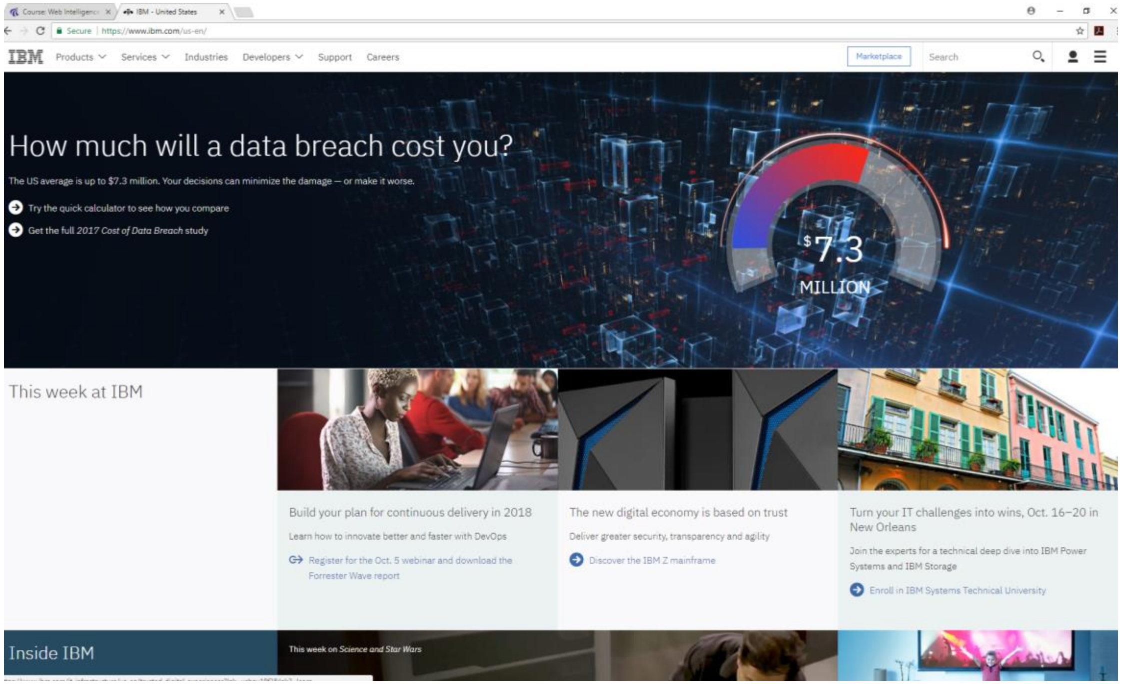
- E.g., ranking score $\in [0,1]$

IBM.com is first hit for search "IBM" – How?

IBM.com mostly graphical – not much mentioning of IBM in text-content

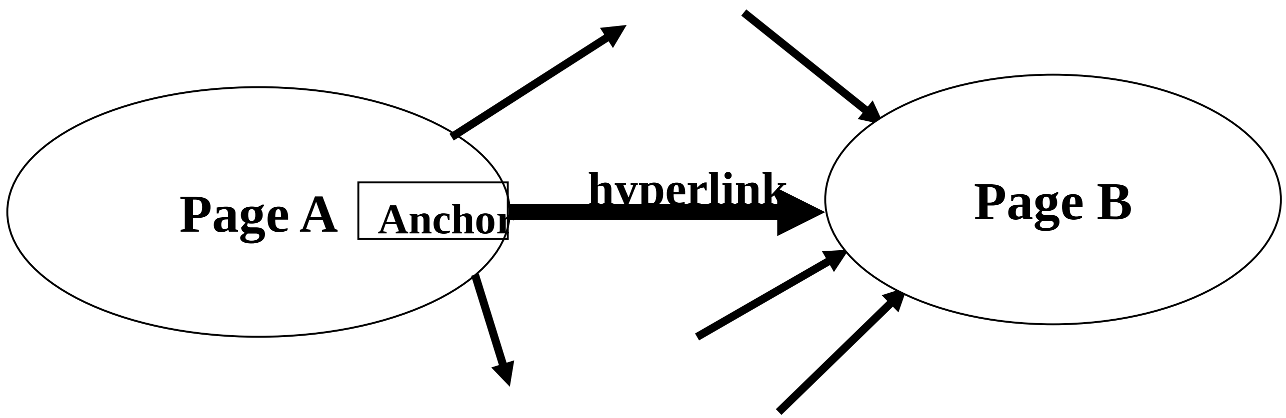
Why not:

- **IBM's copyright page – has high term frequency for "IBM"**
- **Spam page – with high term frequency for "IBM"**



The Web as a Directed Graph

- A node for each page
- A directed edge for each hyperlink



Hypothesis 1: A hyperlink between pages is a conferral of authority (quality signal)

Hypothesis 2: The text in the anchor of the hyperlink on page A describes the target page B

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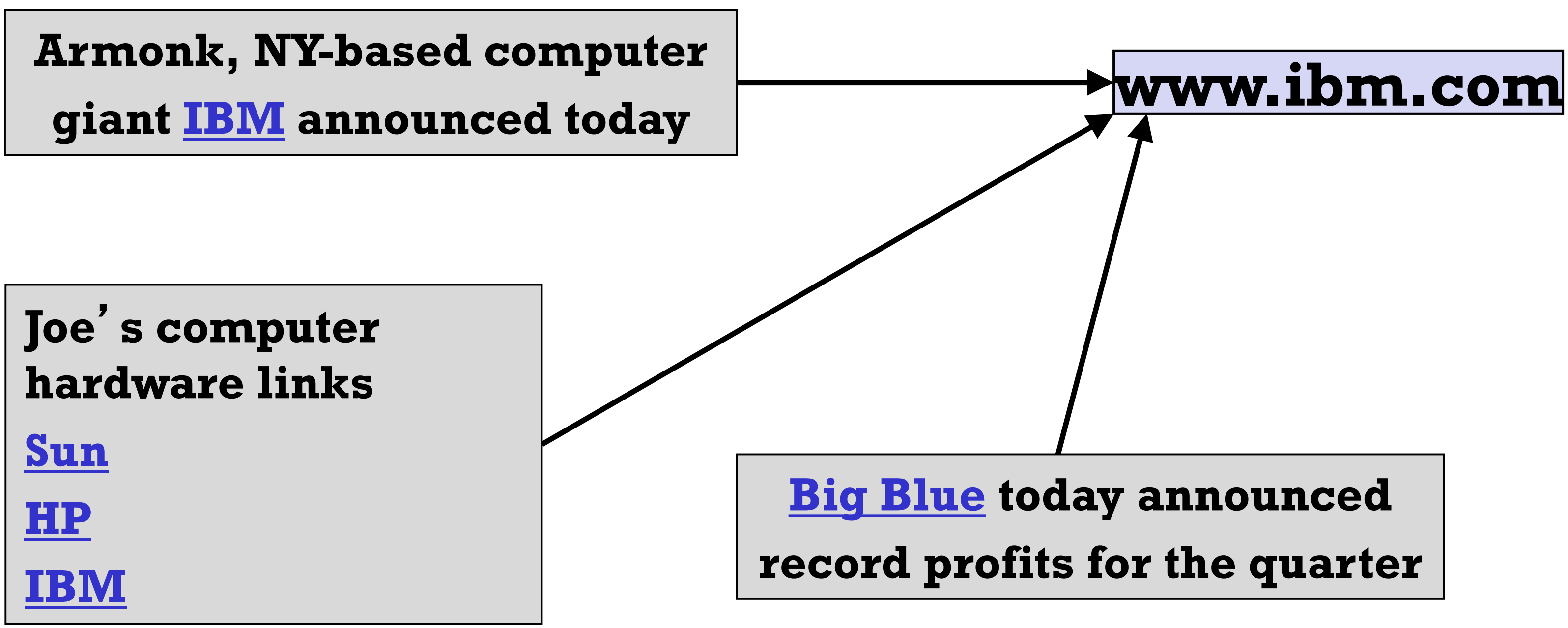
Indexing anchor text

When indexing a document *D*, include (with some weight) anchor text from links pointing to *D*.



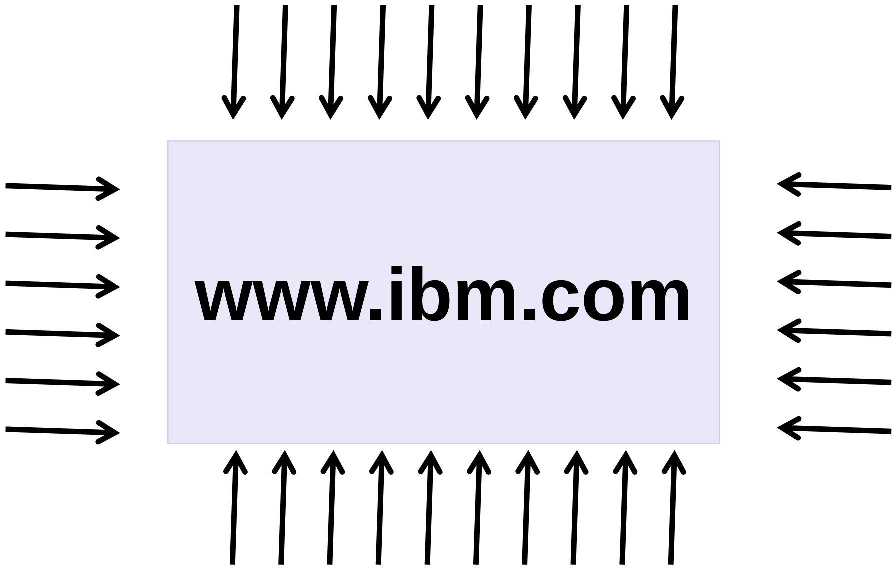
Use anchor text for indexing!

- Anchor is often a useful descriptor of page that it links to
  - (e.g., …the computer giant IBM today announces…)
- Anchor might be different from the text on the page → broader view of the page that better reflect "the crowds" multiple views
  - (e.g., …Big Blue today announced…)
  - Can be exploited by spammers (link spam) to generate anchors with text dishonesting some web sites
  - Weight anchor according to authority of referring page!
- Some undescriptive anchors: here, click, this…
  - (e.g., Information about IBM can be found here.)
  - Extended Anchor Text (i.e., neighbourhood of a link anchor may help!

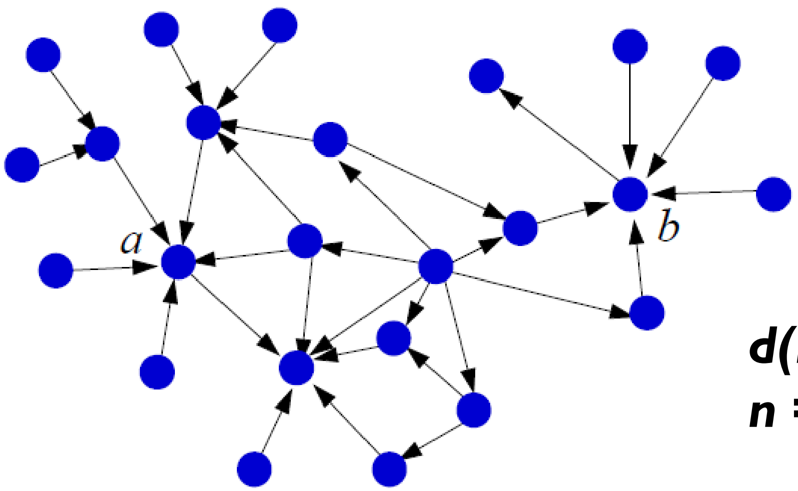
Hypothesis 1: A hyperlink between pages is a conferral of authority (quality signal)

Wisdom of the crowd!

A million links send a strong signal of prestige



Prestige as a simple local measure



d(i) **= in-degree (# incoming links)**
n **= # nodes**

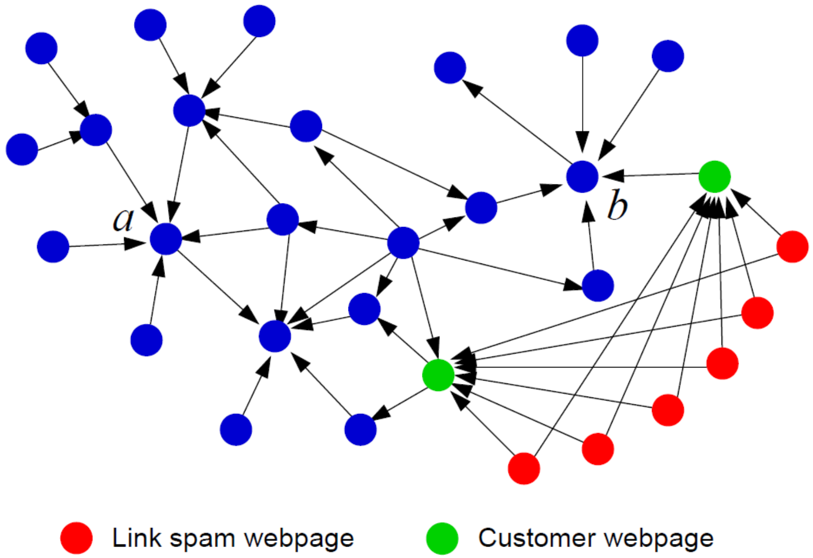
Prestige

$$P(i) = d(i) \,/\, (n-1)$$

Example: P(a) = P(b) = 5/23

Prestige (cont.)

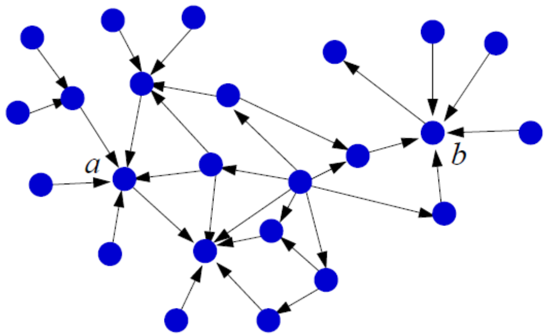
Local measures can be easily manipulated by link spamming:



On the other hand, more difficult to manipulate global measures (though still possible)

A global prestige measure

Rank prestige:



$$P(j) = \sum_{i:i \to j} P(i)$$

Recursion: Prestige of page depends on

- Its in-degree, and
- The prestige of pages linking to it
- This does not directly defines prestige – only a mutual relationship between values

Can we find a prestige measure that satisfies these relationships?

PageRank

Definition of PageRank
(Brin and Page '98)

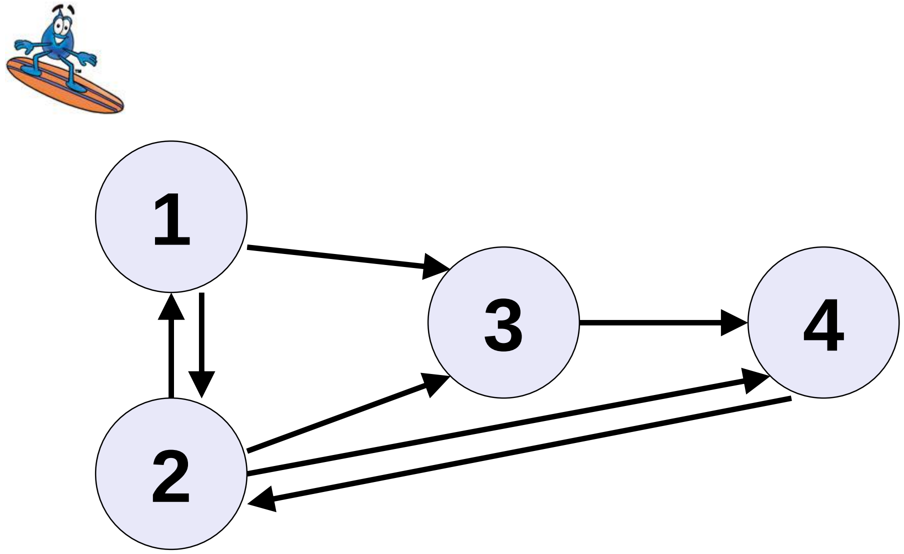
Consider the following infinite random walk (surf):

- Initially the surfer is at a random page
- At each step, the surfer proceeds
  - to a randomly chosen web page with probability α
  - to a randomly chosen successor of the current page with probability $1 - \alpha$

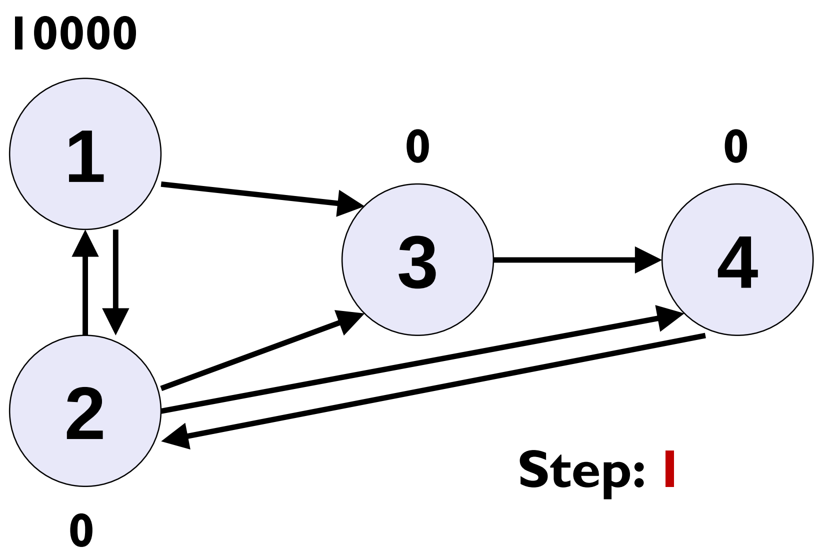
The PageRank of a page p is the fraction of steps the surfer spends at p in the limit. (A score between 0 and 1)

Notice: Using link structure only!

The Random Web Surfer



10.000 Random Surfers



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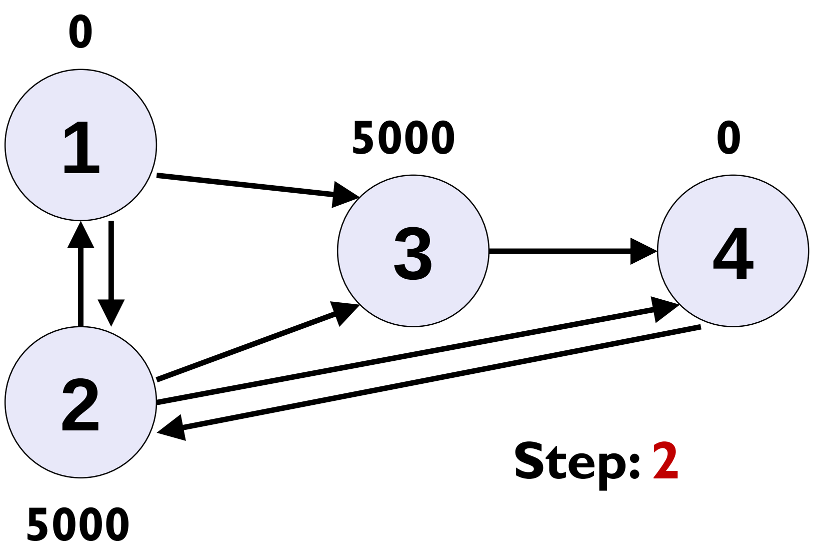

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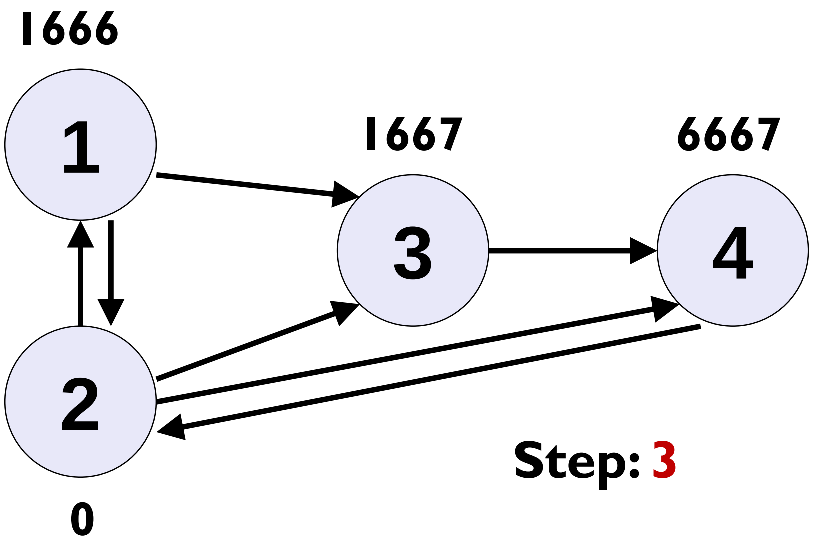

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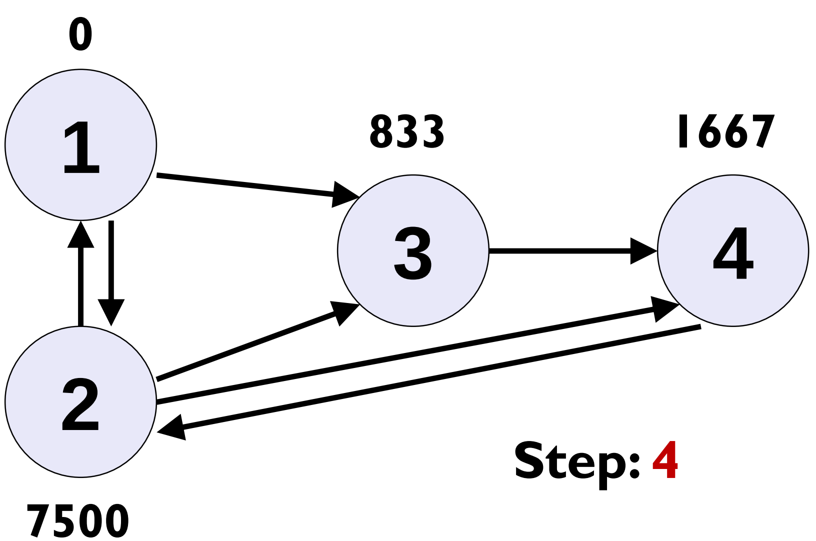

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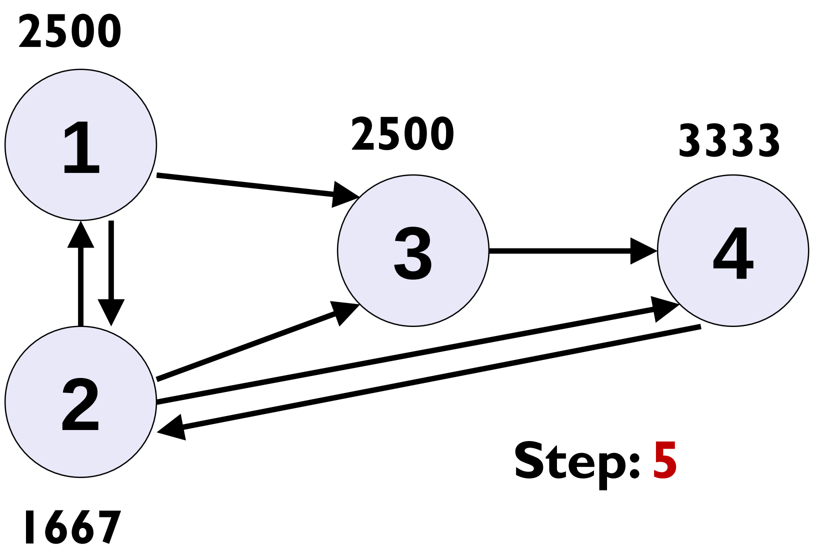

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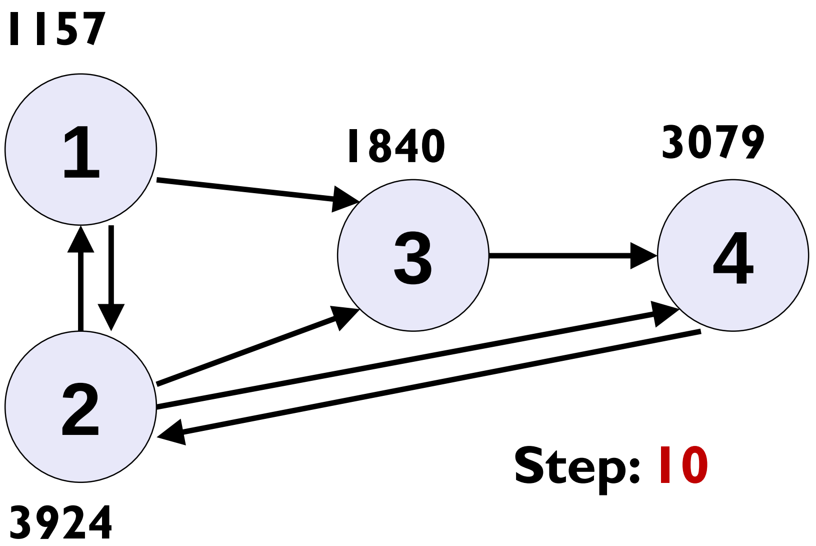
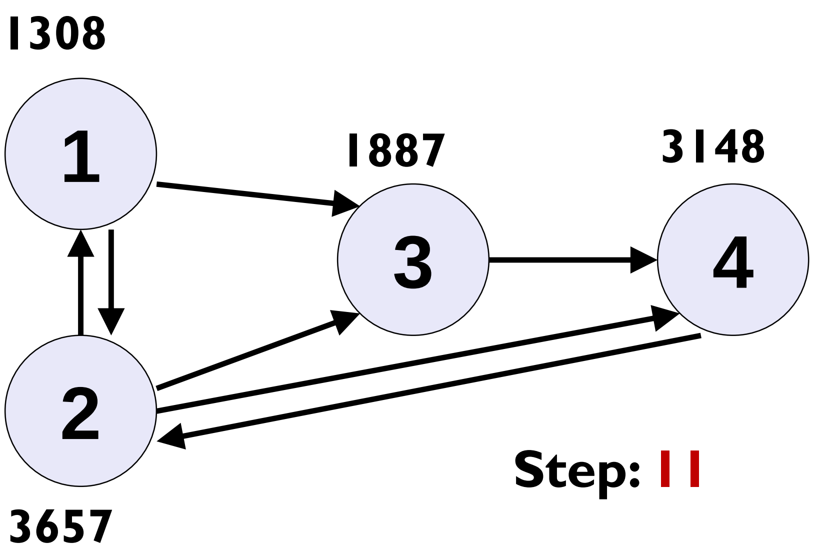
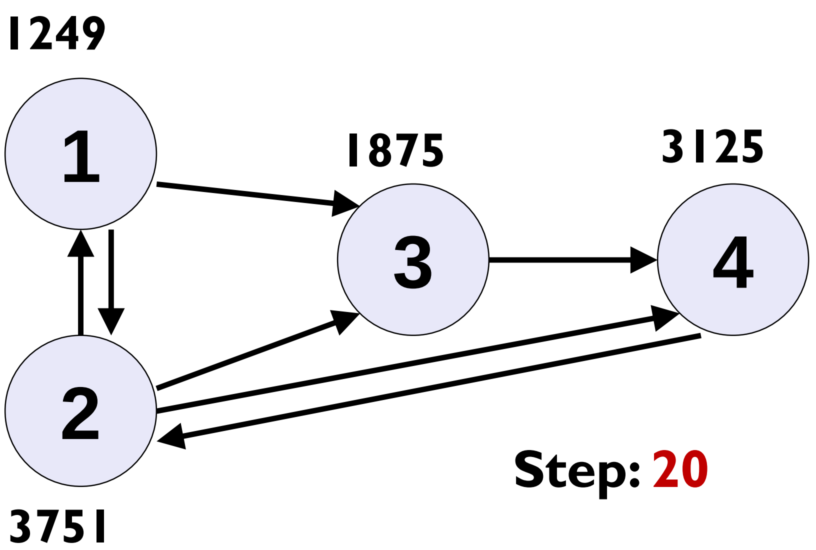


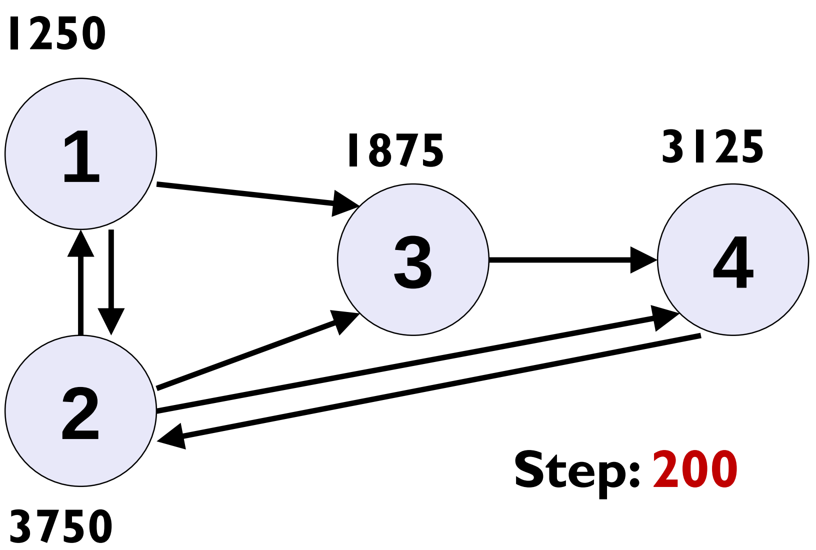
10.000 Random Surfers



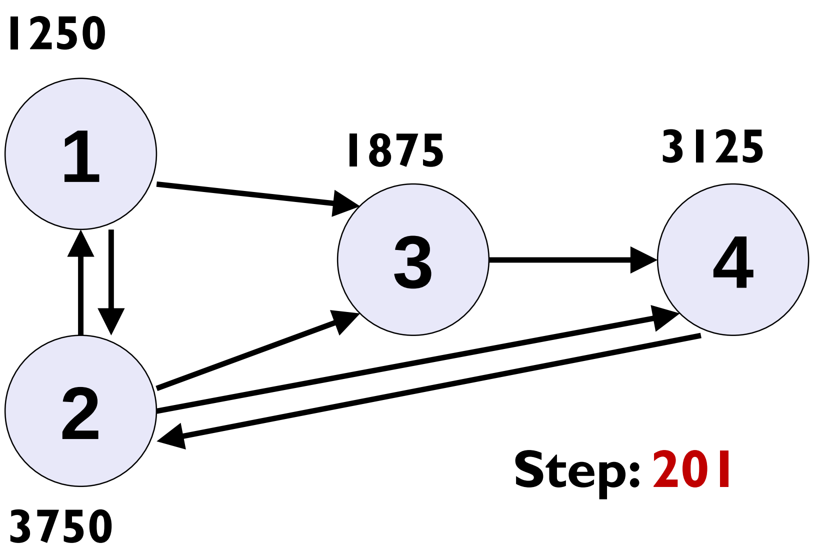
10.000 Random Surfers



10.000 Random Surfers



10.000 Random Surfers

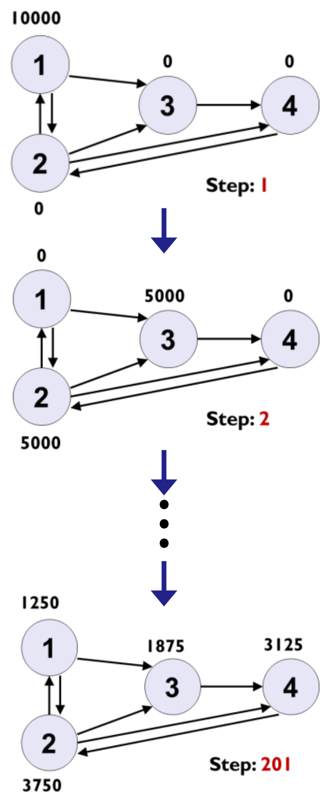


Surfers visit some nodes more often than others.

These are those with many in-links from other frequently visited pages.

Pages visited more often has higher prestige (rank)

PageRank Intuition



Imagine users doing a random walk on web pages:

- Start on a **random** page
- At each step, continue to next page along one of the links on the current page, **equiprobably**

PageRank idea: Run to convergence; the rank (prestige) of a web-page is then proportional to:

- the **proportion** of random web-surfers that will be visiting the page at a given point in time
- = the **probability** that a random web-surfer is at this page at any point in time

$$(P(1), P(2), P(3), P(4)) \;=\; (0.1250, 0.3750, 0.1875, 0.3125)$$

PageRank – The Markov Chain Model
(Markov chains are abstractions of random walks)

The random surfer is described by

- An initial (step 0) probability distribution over all (n) web-pages

$$\boldsymbol{q}^{(0)} = \left(q_1^{(0)}, \dots, q_n^{(0)}\right)$$

- A transition probability matrix

$$\boldsymbol{P} = \begin{pmatrix} P_{1,1} & \cdots & P_{1,j} & \cdots & P_{1,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{i,1} & \cdots & P_{i,j} & \cdots & P_{i,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{n,1} & \cdots & P_{n,j} & \cdots & P_{n,n} \end{pmatrix}$$

Worth noticing:

- $P_{i,j} = 0$, if no link from i to j
  Happens many, many times!!!
- Each row sums to 1

$P_{i,j}$ = probability of moving from page $\boldsymbol{i}$ to page $\boldsymbol{j}$

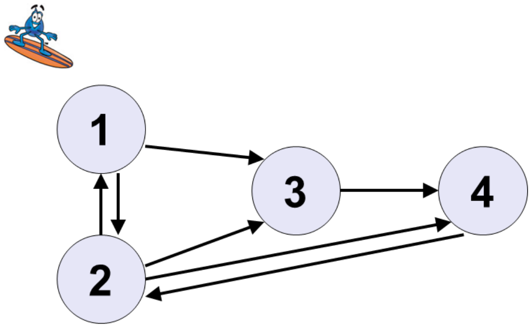
$$= \frac{1}{out\!-\!degree(i)}$$

Example (from before)

Initial probability distribution

(Here, surfer starts on page 1, but could have started anywhere!)

$$\boldsymbol{q}^{(0)} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$



Transition probability matrix

$$\boldsymbol{P} = \begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Markov Chain Transitions

$P_{i,j} = 0$ when $i \not\to j$

The way we compute transitions:

$$q_j^{(t)} = \sum_{i=1}^{n} q_i^{(t-1)} P_{i,j} = \sum_{i:i \to j}^{n} q_i^{(t-1)} P_{i,j}$$

computed for all pages $j \in \{1, \dots n\}$

Matrix notation, the way we talk about it:

$$\boldsymbol{q}^{(t)} = \boldsymbol{q}^{(t-1)} \boldsymbol{P}$$

$$\left(q_1^{(t)} \dots q_j^{(t)} \dots q_n^{(t)}\right) = \left(q_1^{(t-1)} \dots q_j^{(t-1)} \dots q_n^{(t-1)}\right) \begin{pmatrix} P_{1,1} & \cdots & P_{1,j} & \cdots & P_{1,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{i,1} & \cdots & P_{i,j} & \cdots & P_{i,n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ P_{n,1} & \cdots & P_{n,j} & \cdots & P_{n,n} \end{pmatrix}$$

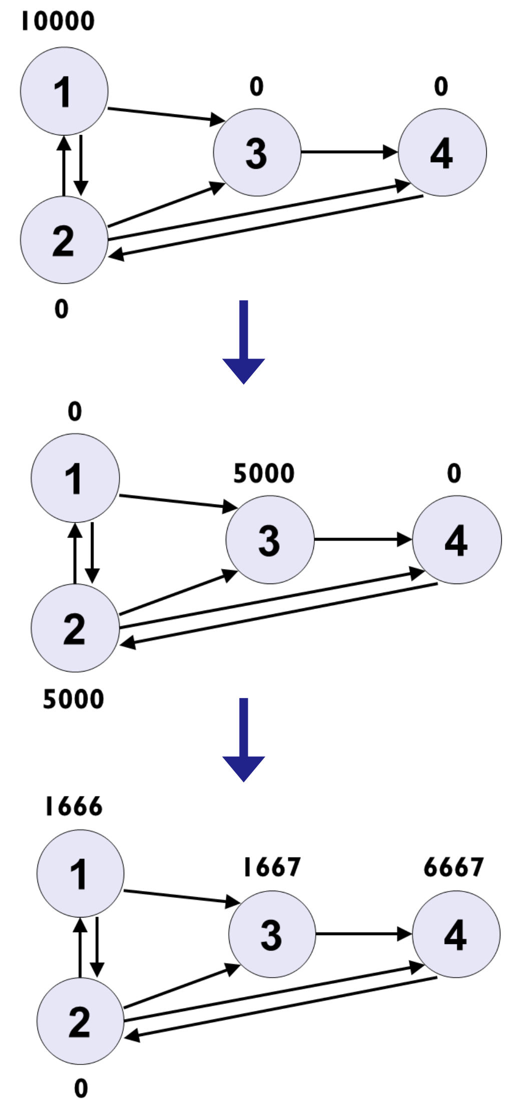
Example (cont.)

$$q^{(1)} = q^{(0)} P$$

$$\begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$q^{(2)} = q^{(1)} P$$

$$\begin{pmatrix} \frac{1}{6} & 0 & \frac{1}{6} & \frac{4}{6} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$



…and for the Bayesian network folks!

PageRank is just a (first order) Markov chain represented by a Bayesian network as follows

$$q^0 \xrightarrow{P} q^1 \xrightarrow{P} q^2 \xrightarrow{P} q^3 \xrightarrow{P} \dots$$

PageRank – Stationary Distribution

$\boldsymbol{q}$ is stationary, if

$$\boldsymbol{q} = \boldsymbol{q}\boldsymbol{P}$$

Example

$$\begin{pmatrix} \frac{1}{8} & \frac{3}{8} & \frac{3}{16} & \frac{5}{16} \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{3}{8} & \frac{3}{16} & \frac{5}{16} \end{pmatrix} \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Under *some conditions* (more detail on next slide)

- For any $\boldsymbol{q}^{(0)}$
- a Markov chain has a *unique* stationary distribution $\boldsymbol{q}^*$

$$\lim_{t \to \infty} \boldsymbol{q}^{(t)} = \boldsymbol{q}^*$$

"Eigen-Stuff"

$\boldsymbol{q}$ with $\boldsymbol{q} = \boldsymbol{q}\boldsymbol{P}$ is also called an Eigen-vector of $\boldsymbol{P}$ with Eigen-value 1.
In fact, $\boldsymbol{q}$ is the principal Eigen-vector, because $\boldsymbol{P}$ is a transision probalility matrix.

Many ways to find the principal Eigen-vector in practice:

- SVD
- Power iteration – PageRank algorithm

Stationarity conditions

The PageRank iterations $\boldsymbol{q}^{(t)} = \boldsymbol{q}^{(t-1)}\boldsymbol{P}$ must be representable by an **ergodic** Markov chain.

A Markov chain is ergodic if it is

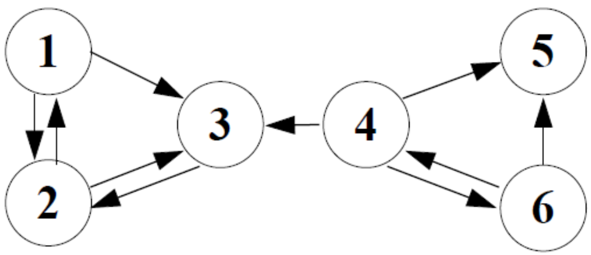
- *Irreducible*: "every node is reachable from every other node"
- *Aperiodic*: "nodes are not partitioned into sets such that all transitions occur cyclically from one set to another

Sufficient condition:

- A Markov chain is ergodic, if there is a strictly positive probability to pass from any state to any other state in one step.

We will be using this one

Complication: Dangling Pages



Ups! – A dangling page
⇒ ~~irreducible~~

"Transition matrix":

$$P = \begin{pmatrix} 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 \end{pmatrix}$$

Problem: Not a proper transition matrix, because dangling pages (Ex.: page **5**) have no defined transitions

Q: What do we do now – our theory breaks down!

Teleporting

At a dangling node, jump to a random web page.

At any non-dangling node, with probability α (e.g.,10%), jump to a random web page.

- With remaining probability (1-α) (e.g., 90%), go out on a random link.
- α - a parameter. Should mirror our belief that a random web surfer would leave a page in another way than following a link

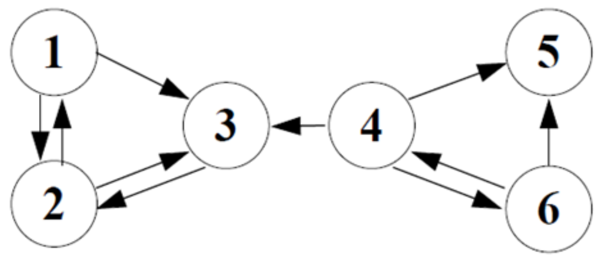
Result of teleporting:

- Now we cannot get stuck locally.
- there is a strictly positive probability to pass from any state to any other state in one step $\Rightarrow$ guaranteed ergodicity
- The power iteration converges to a unique $\boldsymbol{p}^*$ $\leftarrow$PageRank

Teleporting …in math

New transition matrix: $P_{PageRank} = (1-\alpha)P + \alpha U$

Example:



$$P_{pageRank} = (1-\alpha)\begin{pmatrix} 0 & 1/2 & 1/2 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1/3 & 0 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/2 & 1/2 & 0 \end{pmatrix} + \alpha \begin{pmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \end{pmatrix}$$

Definition of PageRank

(Brin and Page '98)

Consider the following infinite random walk (surf):

- Initially the surfer is at a random page
- At each step, the surfer proceeds
  - to a randomly chosen web page with probability α
  - to a randomly chosen successor of the current page with probability $1 - \alpha$

The PageRank of a page p is the fraction of steps the surfer spends at p in the limit. (A score between 0 and 1)

Notice: Using link structure only!

PageRank -- Summary

- The page rank of webpage *i* is

$$n \cdot q_i^*$$

where q_i^* is the (unique) limit distribution of the Markov chain defined by $\boldsymbol{P}_{PageRank}$

- We use Power Iterations to approximate the PageRank by iterating

$$\boldsymbol{q}^{(t)} = \boldsymbol{q}^{(t-1)}\boldsymbol{P}_{PageRank}$$

until $\boldsymbol{q}^{(t)}$ does not change very much

- Used as one of the ranking criteria in Google, Bing,…

Scalability issues

Large matrix – 50bil x 50 bil

With teleport fix dense

Hard to compute in main memory

Fixes:

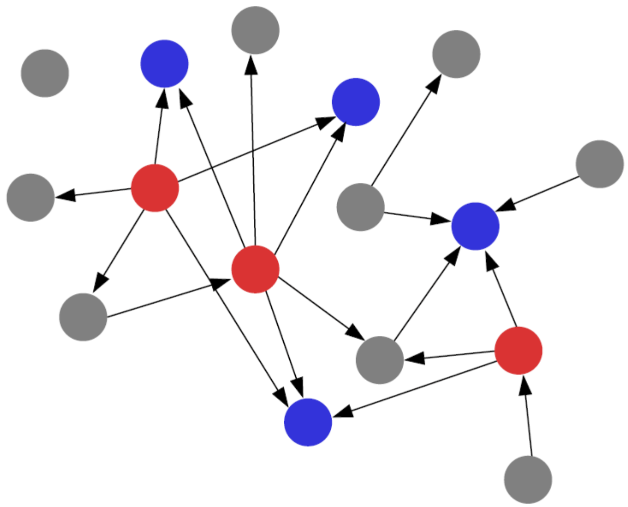
- Compression
- I/O operations and swaps
- Splitting computation on dangling and non dangling parts
- Looking just at some values from each column/raw
- Only computing few (say 10?) iterations to get order right but not necessarily the rank values
- …

More in:

Amy N. Langville and Carl D. Meyer: Deeper Inside PageRank. Internet Mathematics Vol. 1, No. 3: 335-380. http://www.internetmathematicsjournal.com/article/1388-deeper-inside-pagerank

HITS
(Hyperlink-Induced Topic Search)

Authorities and Hubs



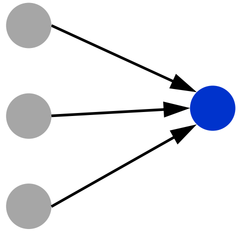
Authorities: Web pages linked to by many other pages.
Example: important company homepages

Hubs: Web pages pointing to many (relevant) pages.
Example: Business listings (yellow pages)

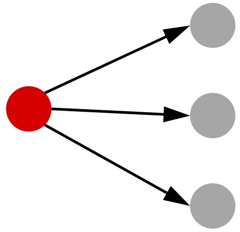
HITS – The Goal

Given a query, find

- Good sources of content (authorities)

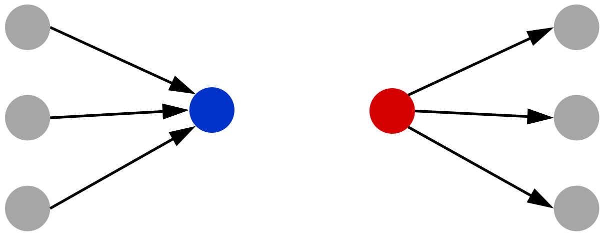


- Good sources of links (hubs)

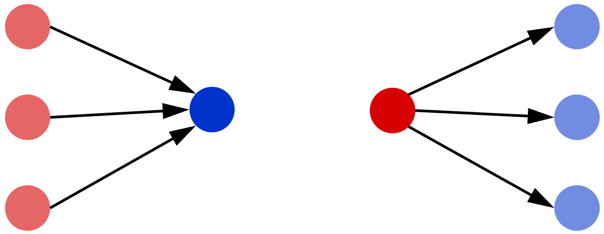


Intuition

Authority **comes from** in-edges.
Being a good hub **comes from** out-edges.



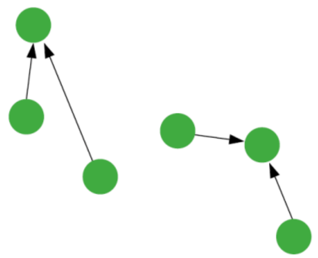
Better Authority **comes from** in-edges from good hubs.
Being a better hub **comes from** out-edges to good authorities.



HITS Algorithm

Step 1 (of 3)

Retrieve top *t* webpages for query (mostly content-based):

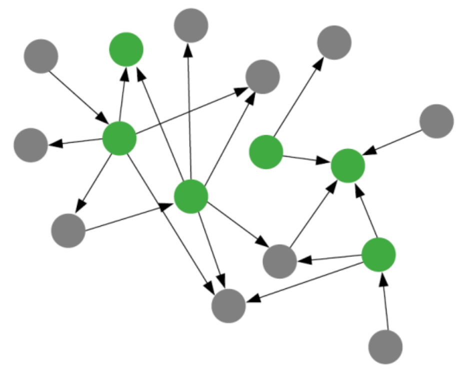


Result: the **root** set

HITS Algorithm (cont.)

Step 2 (of 3)

Add all neighbors of the pages in the root set:



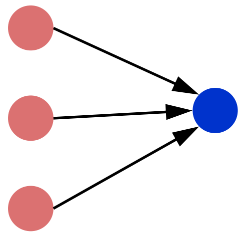
Result: the **base** set

HITS Algorithm (cont.)

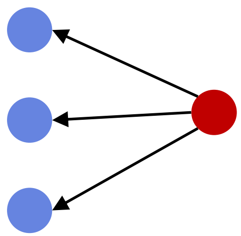
Step 3 (of 3)

Iteratively update (normalized) **authority** and **hub** **scores** for all pages in the base set – *until convergence*:

$$a(j) = \sum_{i:i \to j} h(i)$$



$$h(j) = \sum_{i:j \to i} a(i)$$



Notice: Each page both have an authority and a hub score

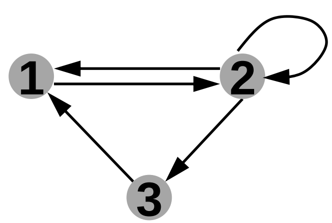
Math…

$n \times n$ <u>link-matrix</u> $\boldsymbol{L}$:

- each of the n pages in the base set has a row and column in the matrix.
- Entry $L_{ij} = 1$ if page i links to page j, else = 0.

$n \times n$ <u>*transposed* link-matrix</u> $\boldsymbol{L}^T$:

- Entry $L^T_{ij} = 1$ if page i is linked to from page j, else = 0.



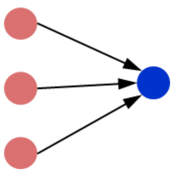
$$\begin{array}{ccc} & & j \\ & \boldsymbol{L} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ i & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{ccc} & & j \\ & \boldsymbol{L}^T & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ i & \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

HITS Algorithm (cont.)

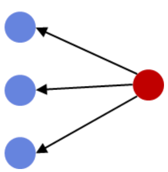
Step 3 (of 3) – the way we compute it

Iteratively update (normalized) **authority** and **hub scores** for all pages in the base set – *until convergence*:

$$a(j) = \sum_{i:i \rightarrow j} h(i)$$



$$h(j) = \sum_{i:j \rightarrow i} a(i)$$



Matrix notation – the way we talk about it

$$\boldsymbol{a} = \boldsymbol{L}^T \boldsymbol{h}$$

$$\boldsymbol{h} = \boldsymbol{L}\boldsymbol{a}$$

Eigen-stuff

HITS - Step 3 (of 3)

$$\boldsymbol{a} = \boldsymbol{L}^T \boldsymbol{h}$$

$$\boldsymbol{h} = \boldsymbol{L}\,\boldsymbol{a}$$

Substituting $\Rightarrow$

$$\boldsymbol{a} = \boldsymbol{L}^T \boldsymbol{L}\,\boldsymbol{a}$$

$$\boldsymbol{h} = \boldsymbol{L}\boldsymbol{L}^T \boldsymbol{h}$$

What does this look like?

- $\boldsymbol{a}$ is an eigenvector of $\boldsymbol{L}^T \boldsymbol{L}$
- $\boldsymbol{h}$ is an eigenvector of $\boldsymbol{L}\boldsymbol{L}^T$

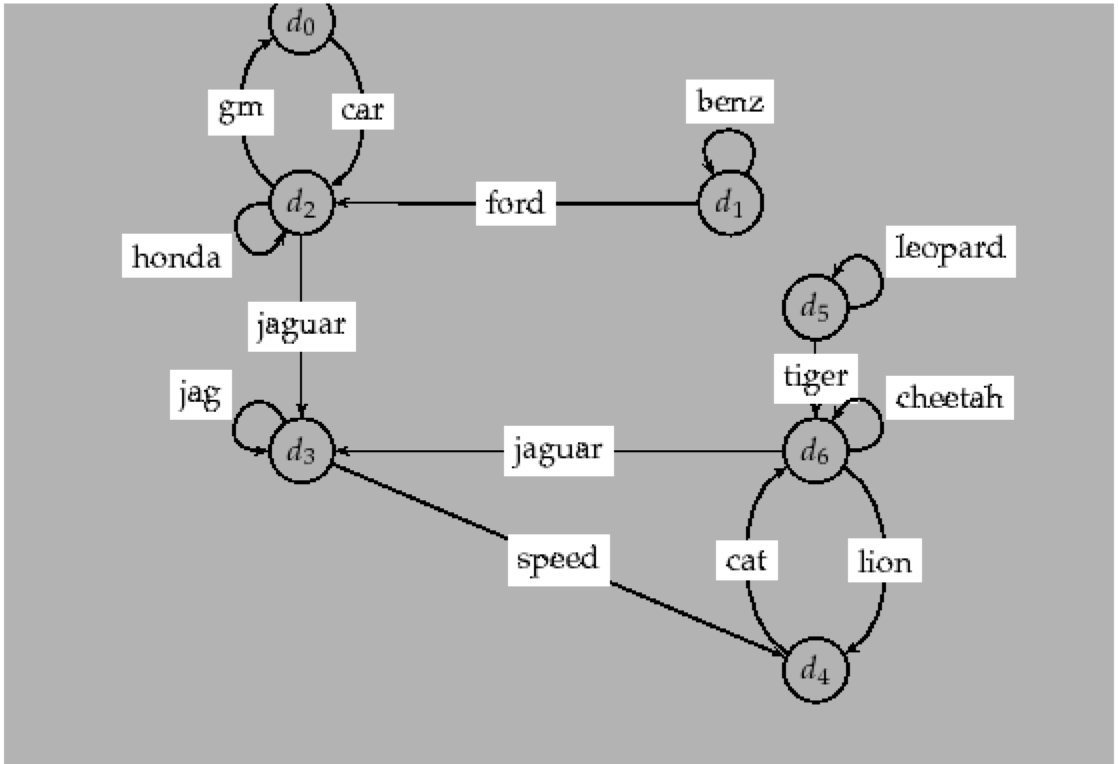
The HITS algorithm is a particular, known algorithm for computing eigenvectors: the *power iteration* method.

Guaranteed to converge

HITS -- Recommendations

- 200 or so pages in the root set is sufficient
- 5 iterations are sufficient to get good results
- Matrix operations are not necessary, just additive updates as the web graph is quite sparse (average 10 links per page)
- Experiments has shown that HITS works to certain extent multi-lingualy

PageRank vs. HITS -- Example



► **Figure 21.1** A small web graph. Arcs are annotated with the word that occurs in the anchor text of the corresponding link.

http://nlp.stanford.edu/IR-book/html/htmledition/the-pagerank-computation-1.html

PageRank vs. HITS – Example (cont.)

$\boldsymbol{P}_{PageRank}$ from PageRank:

0.02	0.02	0.88	0.02	0.02	0.02	0.02
0.02	0.45	0.45	0.02	0.02	0.02	0.02
0.31	0.02	0.31	0.31	0.02	0.02	0.02
0.02	0.02	0.02	0.45	0.45	0.02	0.02
0.02	0.02	0.02	0.02	0.02	0.02	0.88
0.02	0.02	0.02	0.02	0.02	0.45	0.45
0.02	0.02	0.02	0.31	0.31	0.02	0.31

$\boldsymbol{L}$ from HITS:

0	0	1	0	0	0	0
0	1	1	0	0	0	0
1	0	1	2	0	0	0
0	0	0	1	1	0	0
0	0	0	0	0	0	1
0	0	0	0	0	1	1
0	0	0	2	1	0	1

PageRank scores:

$$\vec{x} = (0.05 \quad 0.04 \quad 0.11 \quad 0.25 \quad 0.21 \quad 0.04 \quad 0.31)$$

Hub & Authority scores:

$$\vec{h} = (0.03 \quad 0.04 \quad 0.33 \quad 0.18 \quad 0.04 \quad 0.04 \quad 0.35)$$

$$\vec{a} = (0.10 \quad 0.01 \quad 0.12 \quad 0.47 \quad 0.16 \quad 0.01 \quad 0.13)$$

http://nlp.stanford.edu/IR-book/html/htmledition/hubs-and-authorities-1.html
http://nlp.stanford.edu/IR-book/html/htmledition/the-pagerank-computation-1.html

PageRank vs. HITS

PageRank	HITS
Computation: • Expensive • Once for all documents and queries (offline)	Computation: • Expensive • Requires computation for each query (online)
Query-independent • requires combination with query-dependent criteria	Query-dependent
Hard to spam	Relatively easy to spam • partly local
	Quality depends on quality of start set Gives hubs as well as authorities

Outline

Link-based techniques for ranking

- Motivation
- Anchor text
- Structural measures
  - PageRank (query-independent)
  - HITS (query-dependent)