ECO 373: Financial Econometrics Homework

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Answer: 1(a) Time Plot of the Series

The time plot of the series 'Y3' provides a visual representation of the data over time, allowing us to observe any trends or patterns.

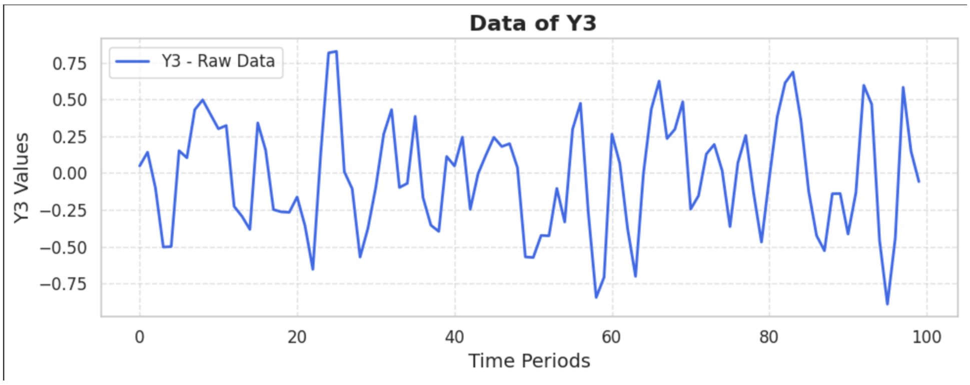


Figure 1: Time Plot of Series Y3

Interpretation of the timeplot: It is evident from the time plot of 'Y3' that the series appears stationary, as it oscillates around a constant mean and displays no visible trends. This makes it suitable for Box-Jenkins ARMA modeling.

(b) Correlograms of the Series

To analyze the dependency structure of the series 'Y3', the sample autocorrelation function (ACF) and partial autocorrelation function (PACF) are computed.

Autocorrelation Function (ACF)

The autocorrelation function measures the correlation of the time path of the variable separated by k lags:

$$\rho_k = \frac{\text{Cov}(Y_t, Y_{t-k})}{\sqrt{\text{Var}(Y_t) \cdot \text{Var}(Y_{t-k})}},$$

where k is the lag.

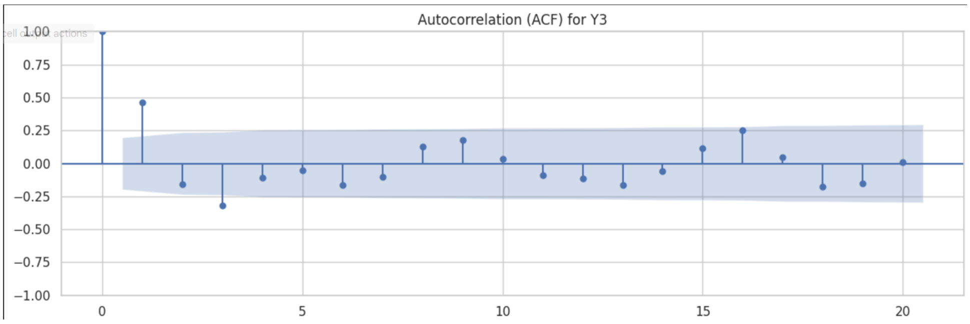


Figure 2: ACF Plot from Python for Series Y3

Interpretation of ACF

The autocorrelation function decays gradually with lag, suggesting potential long-term dependence. For comparison, below is the corresponding plot generated from EViews. It can be seen from the graph that there are two significant spikes in the ACF plot, hinting at an MA(2) Component in the data.

Sample: 1 100
Included observations: 100

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.466	0.466	22.326	0.000
		2	-0.161	-0.482	25.015	0.000
		3	-0.322	0.023	35.893	0.000
		4	-0.108	0.045	37.126	0.000
		5	-0.052	-0.253	37.415	0.000
		6	-0.165	-0.121	40.368	0.000
		7	-0.100	0.101	41.456	0.000
		8	0.128	0.037	43.282	0.000
		9	0.180	-0.076	46.897	0.000
		10	0.034	0.023	47.030	0.000
		11	-0.087	-0.020	47.897	0.000
		12	-0.113	-0.139	49.388	0.000
		13	-0.164	-0.167	52.540	0.000
		14	-0.058	0.207	52.937	0.000
		15	0.115	0.007	54.527	0.000
		16	0.254	0.085	62.362	0.000
		17	0.046	-0.216	62.622	0.000
		18	-0.175	0.013	66.412	0.000
		19	-0.150	-0.022	69.258	0.000
		20	0.010	-0.032	69.271	0.000
		21	0.032	0.015	69.402	0.000
		22	-0.087	-0.061	70.390	0.000
		23	-0.046	0.037	70.666	0.000
		24	0.052	-0.184	71.024	0.000

Figure 3: ACF Plot from EViews for Series Y3

Partial Autocorrelation Function (PACF)

The partial autocorrelation function measures the correlation between Y_t and Y_{t-k} while removing the influence of intermediate lags. Mathematically, the PACF at lag k is computed as:

$$\phi_{kk} = \text{Coefficient of } Y_{t-k} \text{ in the regression of } Y_t \text{ on } (Y_{t-1}, \ldots, Y_{t-k}).$$

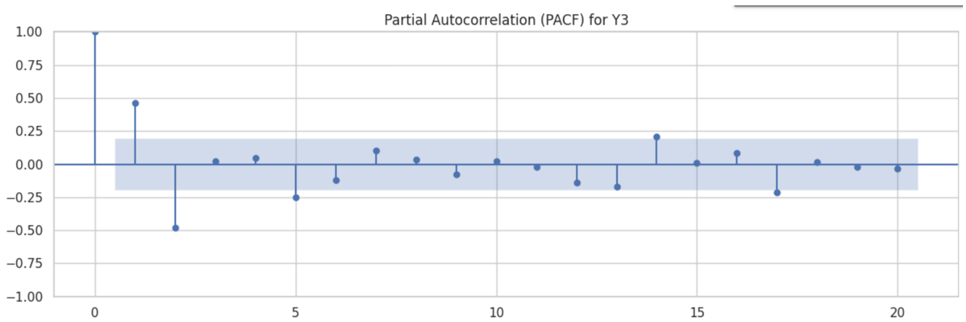
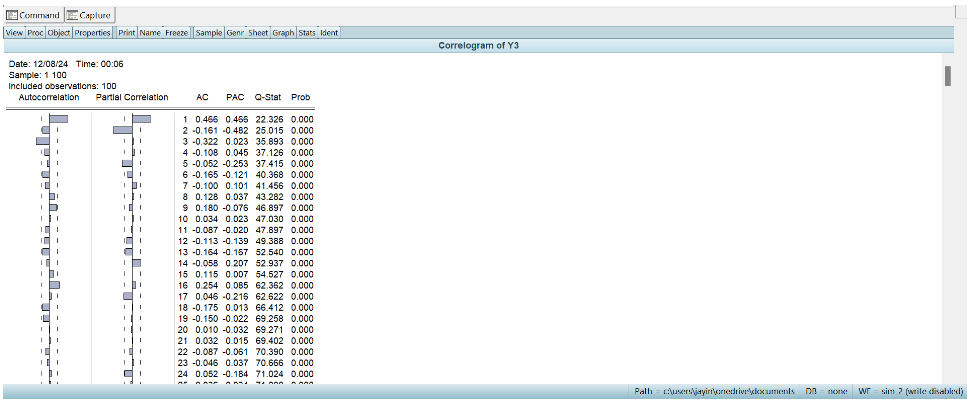


Figure 4: PACF Plot from Python for Series Y3

Interpretation of PACF

The PACF shows significant spikes at lag 1, lag 2 and possibly at lag 3 also, followed by a sharp cutoff, suggesting that the series may follow an AR(3)/AR(2) process. Below is the corresponding PACF plot from EViews for further validation:



Date: 12/08/24 Time: 00:06
Sample: 1 100
Included observations: 100

Lag	AC	PAC	Q-Stat	Prob
1	0.466	0.466	22.326	0.000
2	-0.161	-0.482	25.015	0.000
3	-0.322	0.023	35.893	0.000
4	-0.108	0.045	37.126	0.000
5	-0.052	-0.253	37.415	0.000
6	-0.165	-0.121	40.368	0.000
7	-0.100	0.101	41.456	0.000
8	0.128	0.037	43.282	0.000
9	0.180	-0.076	46.897	0.000
10	0.034	0.023	47.030	0.000
11	-0.087	-0.020	47.897	0.000
12	-0.113	-0.139	49.388	0.000
13	-0.164	-0.167	52.540	0.000
14	-0.058	0.207	52.937	0.000
15	0.115	0.007	54.527	0.000
16	0.254	0.085	62.362	0.000
17	0.046	-0.216	62.622	0.000
18	-0.175	0.013	66.412	0.000
19	-0.150	-0.022	69.258	0.000
20	0.010	-0.032	69.271	0.000
21	0.032	0.015	69.402	0.000
22	-0.087	-0.061	70.390	0.000
23	-0.046	0.037	70.666	0.000
24	0.052	-0.184	71.024	0.000

Figure 5: PACF Plot from EViews for Series Y3

Comparison with Theoretical AR(2) Process

For a theoretical AR(2) process:

- **ACF**: Decays gradually, often with oscillatory behavior if ϕ_1 and ϕ_2 have opposite signs.
- **PACF**: Shows significant spikes at lags 1 and 2, followed by a sharp cutoff.

The ACF and PACF plots from both Python and EViews indicate:

- **ACF**: The gradual decay in the sample ACF suggests long-term dependence, consistent with an AR(2) process.
- **PACF**: The significant spikes at lags 1 and 2 in the sample PACF suggest the appropriateness of an AR(2) model.

0.1 AR(1) Model Estimation for Series $Y3$

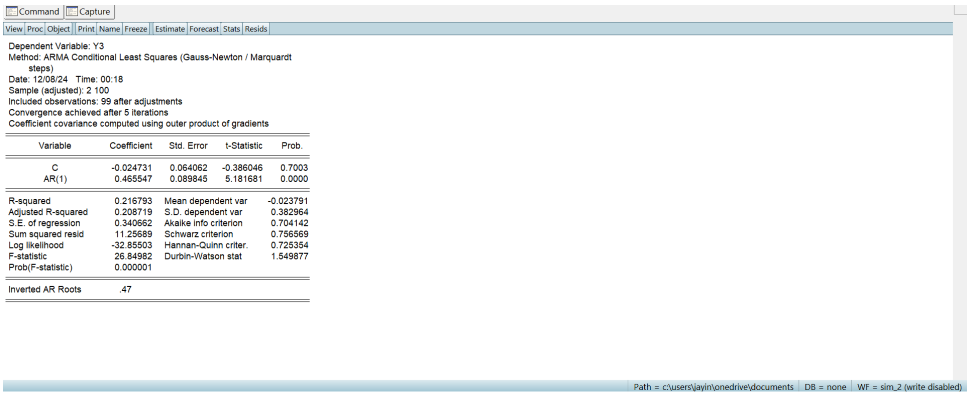
This section discusses the estimation of an AR(1) process for the time series $Y3$, its adequacy checks, and interpretation of the results.

0.1.1 Model Estimation

The AR(1) model was estimated using the Conditional Least Squares method. The model output is summarized in Table 1.

Variable	**Coefficient**	**Std. Error**	**t-Statistic**	**p-Value**
Constant (C)	-0.0247	0.0641	-0.3860	0.7003
AR(1)	0.4655	0.0898	5.1817	0.0000

Table 1: AR(1) Model Results for Series $Y3$.


Command Capture
View Proc Object Print Name Freeze Estimate Forecast Stats Resids

Dependent Variable: Y3
Method: ARMA Conditional Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/08/24 Time: 00:18
Sample (adjusted): 2 100
Included observations: 99 after adjustments
Convergence achieved after 5 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.024731	0.064062	-0.386046	0.7003
AR(1)	0.465547	0.089845	5.181681	0.0000

R-squared	0.216793	Mean dependent var	-0.023791
Adjusted R-squared	0.208719	S.D. dependent var	0.382964
S.E. of regression	0.340662	Akaike info criterion	0.704142
Sum squared resid	11.25689	Schwarz criterion	0.756569
Log likelihood	-32.85503	Hannan-Quinn criter.	0.725354
F-statistic	26.84982	Durbin-Watson stat	1.549877
Prob(F-statistic)	0.000001		

Inverted AR Roots .47

Path = c:\users\jayin\onedrive\documents DB = none WF = sim_2 (write disabled)

Figure 6: Correlogram of Residuals for the AR(1) Model.

Interpretation: The AR(1) coefficient is statistically significant ($p = 0.0000$), indicating that the past value of $Y3$ significantly predicts the current value. However, the constant term (C) is not statistically significant ($p = 0.7003$).

The goodness-of-fit statistics are as follows:

- $R^2 = 0.2168$: Indicates that approximately 21.68% of the variation in $Y3$ is explained by the model.
- Durbin-Watson statistic: 1.5499: Suggests potential autocorrelation in residuals.

Model Adequacy Check

To check the adequacy of the AR(1) model, the following steps were undertaken:

1. **Residual Analysis:** The correlogram of residuals is provided in Figure 7. Significant autocorrelation was observed in the residuals at various lags, suggesting that the AR(1) model does not fully capture the time series dynamics.

2. **Residual and Fitted Plots:** Figure 8 shows the residuals, actual, and fitted values. The residuals exhibit variability and patterns, indicating possible misspecification of the model or the need for a higher-order AR process.

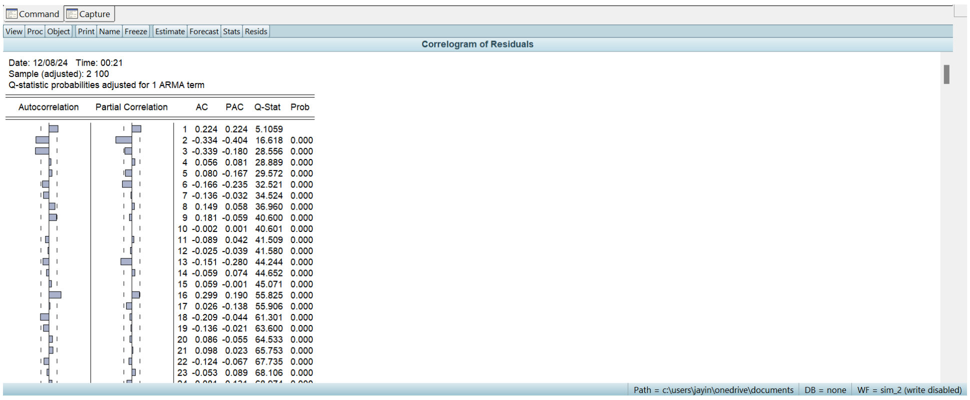


Figure 7: Correlogram of Residuals for the AR(1) Model.

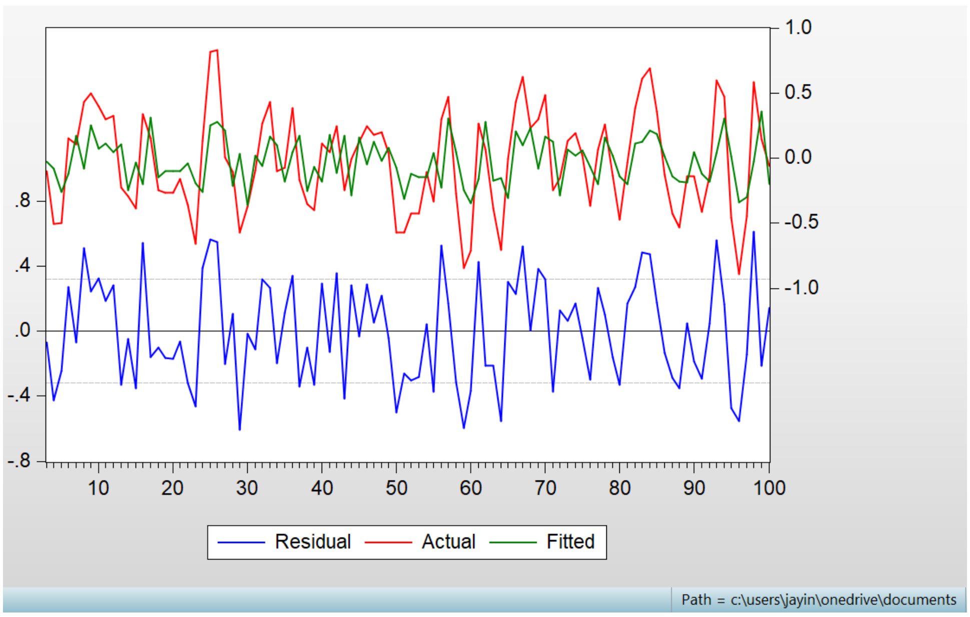


Figure 8: Actual, Fitted, and Residual Plot for the AR(1) Model.

0.1.2 Conclusion

While the AR(1) model provides a reasonable starting point, the significant autocorrelation in the residuals indicates that the model is inadequate. Higher-order AR terms or alternative models (e.g., ARMA or ARIMA) should be explored to improve the fit. Hence, I shall Investigate higher-order autoregressive models (e.g., AR(2) or AR(3)) while also accounting for the MA terms to fit a better model.

Ans 1(C)

An ARMA(1,1) process could potentially generate the type of sample ACF and PACF observed in part (a) under the following considerations:

- For an ARMA(1,1) process:
  - The ACF typically decays exponentially after the first lag.
  - The PACF shows a significant spike at lag 1 and decays thereafter.
- In part (a), the observed ACF and PACF indicate patterns consistent with these characteristics. Thus, it is plausible that an ARMA(1,1) process could generate the observed data.

Model Coefficients

- **Constant Term (C):** The coefficient of the constant term is -0.023291, with a P-value of 0.6915, indicating it is not statistically significant ($p > 0.05$).
- **AR(1):** The autoregressive term has a coefficient of 0.180522 and a P-value of 0.262, which suggests it is not significant in predicting $Y3$.
- **MA(1):** The moving average term has a coefficient of 0.510119 and is statistically significant ($p = 0.0005$), indicating that the previous error term significantly affects the current value of $Y3$.

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: Y3
Method: ARMA Conditional Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/08/24 Time: 00:54
Sample (adjusted): 2 100
Included observations: 99 after adjustments
Failure to improve likelihood (non-zero gradients) after 20 iterations
Coefficient covariance computed using outer product of gradients
MA Backcast: 1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.023291	0.058521	-0.397998	0.6915
AR(1)	0.180522	0.160000	1.128261	0.2620
MA(1)	0.510119	0.140804	3.622909	0.0005

R-squared	0.330358	Mean dependent var	-0.023791
Adjusted R-squared	0.316407	S.D. dependent var	0.382964
S.E. of regression	0.316634	Akaike info criterion	0.567691
Sum squared resid	9.624653	Schwarz criterion	0.646331
Log likelihood	-25.10071	Hannan-Quinn criter.	0.599509
F-statistic	23.68007	Durbin-Watson stat	1.939645
Prob(F-statistic)	0.000000		

Inverted AR Roots	.18
Inverted MA Roots	-.51

Figure 9: EViews Statistics for ARMA(1,1) Model Fitting

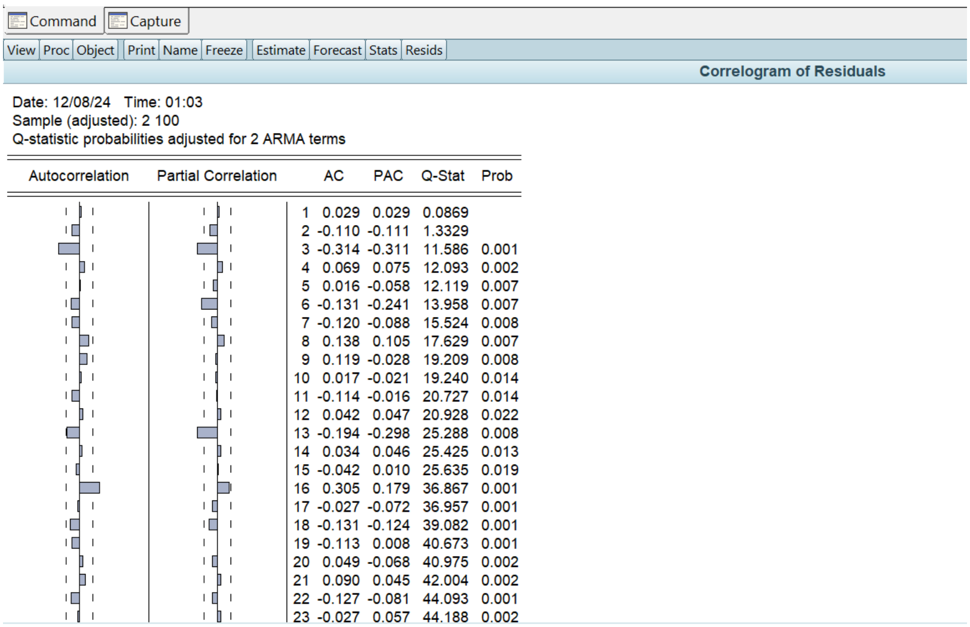
Model Performance Metrics

- **R-squared:** 0.330358, indicating that approximately 33% of the variability in $Y3$ is explained by the model.

- **AIC and SBC:** After fitting other ARMA models to the data, I concluded that the AIC and SBC values for the ARMA(1,1) model are moderate. However, better models may exist for predicting this data.

- **Residual Analysis:** After fitting the ARMA(1,1) model, correlograms of the residuals were plotted:

  - The first three significant spikes in the ACF and the first two in the PACF, which were observed before fitting the model, are no longer present. This indicates that the ARMA(1,1) model has captured those components well.
  - However, the residuals are not entirely a white noise process. Significant spikes remain at lags 3, 13, and 16, suggesting that further improvements can be made to the model.
  - Therefore, In the D part I use these insights to incorporate into my analysis.



Date: 12/08/24 Time: 01:03
Sample (adjusted): 2 100
Q-statistic probabilities adjusted for 2 ARMA terms

	AC	PAC	Q-Stat	Prob
1	0.029	0.029	0.0869	
2	-0.110	-0.111	1.3329	
3	-0.314	-0.311	11.586	0.001
4	0.069	0.075	12.093	0.002
5	0.016	-0.058	12.119	0.007
6	-0.131	-0.241	13.958	0.007
7	-0.120	-0.088	15.524	0.008
8	0.138	0.105	17.629	0.007
9	0.119	-0.028	19.209	0.008
10	0.017	-0.021	19.240	0.014
11	-0.114	-0.016	20.727	0.014
12	0.042	0.047	20.928	0.022
13	-0.194	-0.298	25.288	0.008
14	0.034	0.046	25.425	0.013
15	-0.042	0.010	25.635	0.019
16	0.305	0.179	36.867	0.001
17	-0.027	-0.072	36.957	0.001
18	-0.131	-0.124	39.082	0.001
19	-0.113	0.008	40.673	0.001
20	0.049	-0.068	40.975	0.002
21	0.090	0.045	42.004	0.002
22	-0.127	-0.081	44.093	0.001
23	-0.027	0.057	44.188	0.002

Figure 10: EViews Residual analysis for ARMA(1,1) Model Fitting

- **Residual and Fitted Plots:** The plots for the Residuals and the ARMA(1,1) model fitted against the actual data is also plotted below. It can be inferred from the image that the residuals seem to follow a white noise process, which suggests that the ARMA(1,1) model does a considerably good job in 'absorbing' all the important dynamic structure of the data, leaving out just noise.

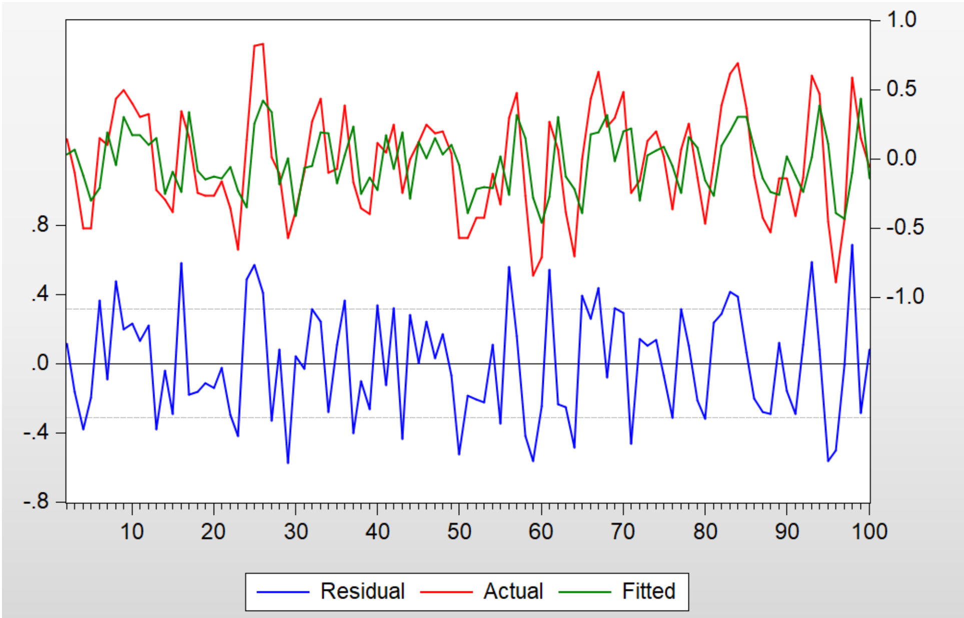


Figure 11: EViews Residual analysis for ARMA(1,1) Model Fitting

Ans 1 (d) Analysis of Models

For the best model selection, I went through various models. They are:

Model	**AIC**	**SBC**	**R^2**
AR(2)	-	-	-
ARMA(1,1)	0.56	0.64	0.33
ARMA(2,1)	0.48	0.59	0.39
ARMA(2,2)	0.42	0.55	0.44
ARMA(3,2)	0.42	0.57	0.46

Table 2: Model selection with AIC, SBC, and R^2 values for each model.

Interpretation of models:

AR(2):

No AIC, SBC, or R^2 values are provided for comparison. Based on the other models, AR(2) likely performs worse than ARMA models since AR processes often struggle to capture moving average terms.

ARMA(1,1):

- AIC = 0.56
- SBC = 0.64
- $R^2 = 0.33$

This model is better than AR(2), as shown by a moderate R^2 and lower AIC and SBC values. However, the relatively low R^2 indicates that it does not explain a large portion of the variability in $Y3$.

ARMA(2,1):

- AIC = 0.48
- SBC = 0.59
- $R^2 = 0.39$

ARMA(2,1) improves over ARMA(1,1), with a lower AIC and SBC and a higher R^2. This suggests that adding one more autoregressive term captures the dynamics of $Y3$ better.

ARMA(2,2):

- AIC = 0.42
- SBC = 0.55
- $R^2 = 0.44$

This model further improves on ARMA(2,1), showing the lowest AIC and SBC among all tested models up to this point. Its R^2 is also higher, indicating that it captures more variance in $Y3$.

ARMA(3,2):

- AIC = 0.42
- SBC = 0.57
- $R^2 = 0.46$

ARMA(3,2) achieves the same AIC as ARMA(2,2) but has a slightly higher SBC due to the addition of an extra autoregressive term. This suggests the added complexity does not significantly improve the model.

Final Observation

Model Fit vs Complexity:

ARMA(2,2) and ARMA(3,2) both achieve the lowest AIC, but ARMA(3,2) has a higher SBC, suggesting it may overfit the data. ARMA(2,2) balances complexity and fit better. Hence **ARMA(2,2) is a good Parsimonious model** balancing complexity and fit most efficiently.

Residual Analysis:

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: Y3
Method: ARMA Conditional Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/08/24 Time: 00:54
Sample (adjusted): 2 100
Included observations: 99 after adjustments
Failure to improve likelihood (non-zero gradients) after 20 iterations
Coefficient covariance computed using outer product of gradients
MA Backcast: 1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.023291	0.058521	-0.397998	0.6915
AR(1)	0.180522	0.160000	1.128261	0.2620
MA(1)	0.510119	0.140804	3.622909	0.0005

R-squared	0.330358	Mean dependent var	-0.023791
Adjusted R-squared	0.316407	S.D. dependent var	0.382964
S.E. of regression	0.316634	Akaike info criterion	0.567691
Sum squared resid	9.624653	Schwarz criterion	0.646331
Log likelihood	-25.10071	Hannan-Quinn criter.	0.599509
F-statistic	23.68007	Durbin-Watson stat	1.939645
Prob(F-statistic)	0.000000		

Inverted AR Roots	.18
Inverted MA Roots	-.51

Figure 12: EViews Residual Analysis for ARMA(2,2) Model Fitting

Fitted plot analysis:

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: Y3
Method: ARMA Conditional Least Squares (Gauss-Newton / Marquardt steps)
Date: 12/08/24 Time: 00:54
Sample (adjusted): 2 100
Included observations: 99 after adjustments
Failure to improve likelihood (non-zero gradients) after 20 iterations
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MA Backcast: 1

Variable	Coefficient	Std. Error	t-Statistic	Prob.
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Sum squared resid	9.624653	Schwarz criterion	0.646331
Log likelihood	-25.10071	Hannan-Quinn criter.	0.599509
F-statistic	23.68007	Durbin-Watson stat	1.939645
Prob(F-statistic)	0.000000		

Inverted AR Roots	.18
Inverted MA Roots	-.51

Figure 13: EViews Fitted plots for ARMA(2,2) Model Fitting

Ans 2 (a) Time plots of PPI and Growth rate of PPI

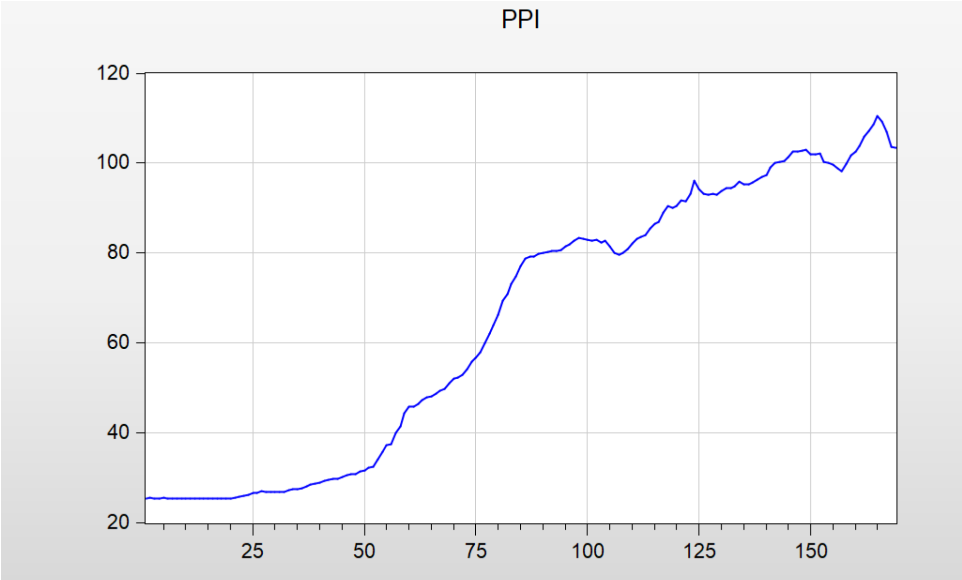


Figure 14: Time Plot of PPI

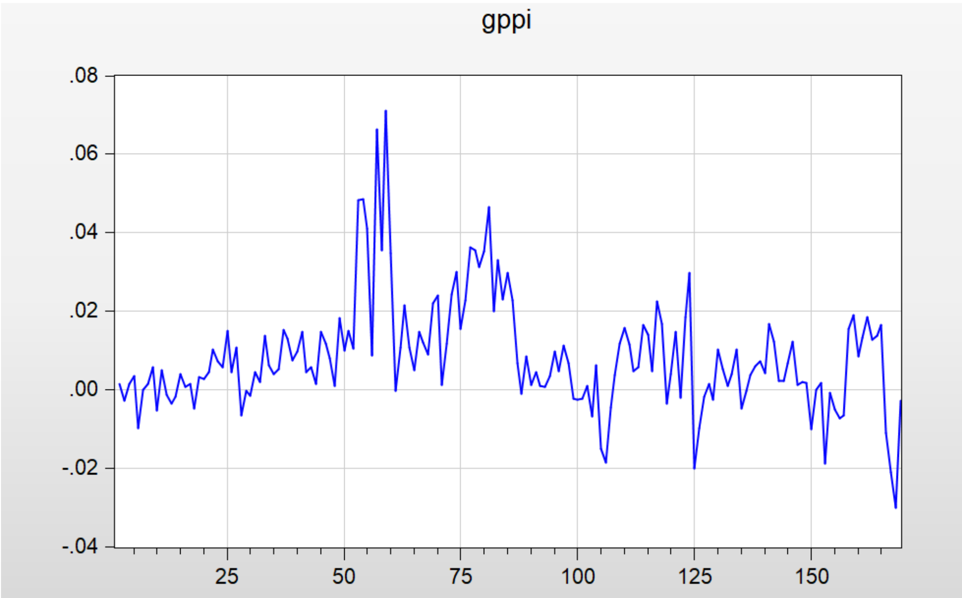


Figure 15: Time Plot of Growth rate of PPI

Ans 2(b)

Analysis of Time Plots

The time plot of the PPI reveals that it is not a stationary process since the mean and variance seem to be growing as time progresses. However, this can only be confirmed once the growth rate of PPI also captures the 1st difference of the series. The series has

a trend component and an intercept at approx. 25. Additionally, there seems to be a sudden change in slope in 1974
From analysing the Time plot of the Growth rate of PPI, it can be inferred that the process seems to be stationary till 1974, after which there is a sudden shift in the series, which might have been caused by a structural break. The series tends to follow a random walk or an MA process with a high persistent parameter since the Variance does not seem to die down quickly.

0.2 Analysis of Models for Growth PPI

Model	**AIC**	**SBC**	R^2	**LBQ Stat**
ARMA(1,1)	-6.14	-6.08	0.40	0.11
ARMA(1,2)	-6.13	-6.05	0.40	0.59
AR(1)	-6.08	-6.04	0.36	0.005
AR(2)	-6.11	-6.05	0.39	0.035
MA(1)	-5.91	-5.87	0.24	0.01

Table 3: Comparison of Models for Growth PPI with AIC, SBC, R^2, and LBQ Statistic

Below is the interpretation of each model along with a comparison of key metrics: AIC (Akaike Information Criterion), SBC (Schwarz Bayesian Criterion), and R^2 (Coefficient of Determination). These metrics are used to evaluate and compare the goodness of fit and complexity of the models.

1. ARMA(1,1)

- AIC = -6.14
- SBC = -6.08
- LBQ Stat = 0.11

This model achieves the lowest AIC among all the models, indicating the best goodness of fit. The LBQ statistic (0.11) suggests that the residuals of this model are not significantly autocorrelated, which is desirable. This model also balances fit and simplicity effectively.

2. ARMA(1,2)

- AIC = -6.13
- SBC = -6.05
- LBQ Stat = 0.59

The ARMA(1,2) model performs similarly to ARMA(1,1) in terms of AIC and SBC, but its LBQ statistic (0.59) suggests better residual independence compared to ARMA(1,1). However, the slight increase in complexity might not justify the marginal improvement in LBQ statistic over ARMA(1,1).

3. AR(1)

- AIC = -6.08
- SBC = -6.04
- LBQ Stat = 0.005

This model has a higher AIC and SBC compared to ARMA models and an LBQ statistic of 0.005, indicating significant autocorrelation in the residuals. This highlights poor residual diagnostics and suggests that this model is not suitable for the data.

4. AR(2)

- AIC = -6.11
- SBC = -6.05
- LBQ Stat = 0.035

The AR(2) model improves over AR(1) in terms of AIC and LBQ statistic (0.035 vs. 0.005), but the residual autocorrelation is still significant. While the model captures more data dynamics, it does not perform as well as ARMA models in terms of overall fit and diagnostics.

5. MA(1)

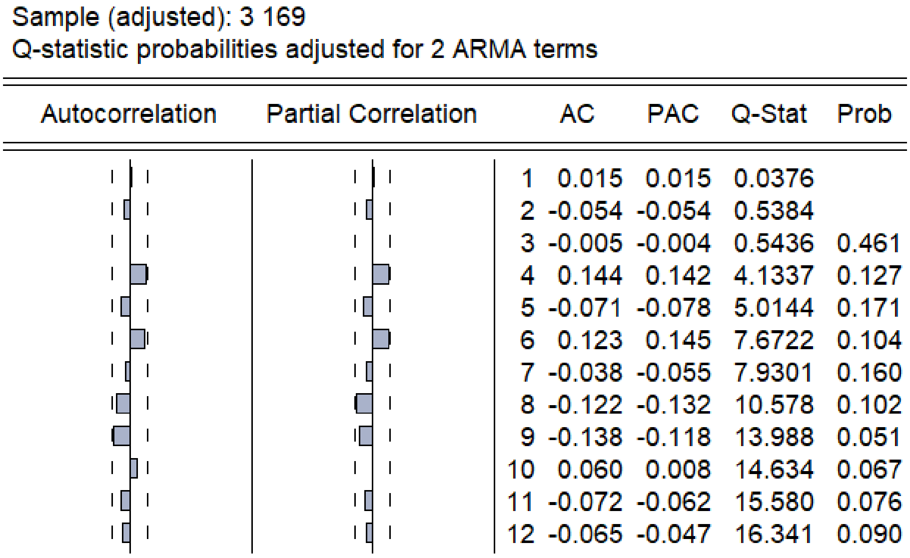
- AIC = -5.91
- SBC = -5.87
- LBQ Stat = 0.01

The MA(1) model has the poorest metrics among all the models, with the highest AIC and SBC and a low LBQ statistic (0.01), indicating significant residual autocorrelation. It explains less variability in the data and fails to adequately model the underlying dynamics.

Best Model Selection

The ARMA(1,1) model is the best choice for Growth PPI. It achieves the lowest AIC (-6.14) and the highest R^2 (0.40), indicating a good balance between model fit and simplicity. The ARMA(1,2) performs similarly in terms of R^2, but its slightly higher AIC suggests it is not as efficient in terms of goodness of fit relative to model complexity.

Residual plot of the Best Model Selected



Sample (adjusted): 3 169
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.015	0.015	0.0376	
		2	-0.054	-0.054	0.5384	
		3	-0.005	-0.004	0.5436	0.461
		4	0.144	0.142	4.1337	0.127
		5	-0.071	-0.078	5.0144	0.171
		6	0.123	0.145	7.6722	0.104
		7	-0.038	-0.055	7.9301	0.160
		8	-0.122	-0.132	10.578	0.102
		9	-0.138	-0.118	13.988	0.051
		10	0.060	0.008	14.634	0.067
		11	-0.072	-0.062	15.580	0.076
		12	-0.065	-0.047	16.341	0.090

Figure 16: Residual plot of the Fitted plot ARMA(1,1)

Fitted plot of the Best Model Selected

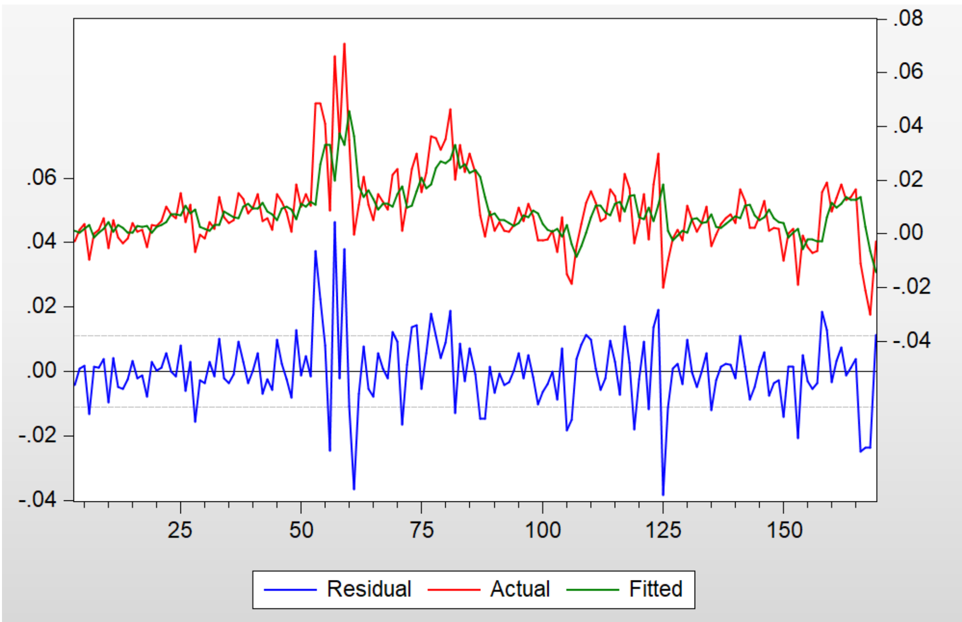


Figure 17: Fitted plot ARMA(1,1)

Ans 3(a)

Analysis of RGDP Series

The Real GDP (RGDP) series depicted in the plot appears to be non-stationary. Here's why and the type of non-stationarity expected:

Observations

- **Upward Trend:** The RGDP series shows a persistent upward trend over time. This indicates that the mean of the series is not constant, which is very important indicator of non-stationarity.
- **Increasing Variance:** While not very evident, there may be periods where the variability appears to change over time.
- **Absence of Mean Reversion:** The series does not oscillate around a fixed mean but instead grows over time, pointing to a unit root or deterministic trend.

Expected Type of Non-Stationarity

- **Trend Non-Stationarity:** The series exhibits a deterministic trend, meaning that removing the linear or exponential trend would render the series stationary.
- **Unit Root Non-Stationarity:** A potential unit root could exist, indicating stochastic trends. This would require differencing the data to achieve stationarity.

Intuitive Explanation

Real GDP measures the economic output of the U.S. economy, and it naturally grows over time due to factors like population growth, technological advancements. These growth factors introduce a deterministic trend into the data. Furthermore, short-term economic shocks or random fluctuations (e.g., recessions or booms) might add stochastic elements, requiring differencing to remove them(as I have done in the alter parts of the question).

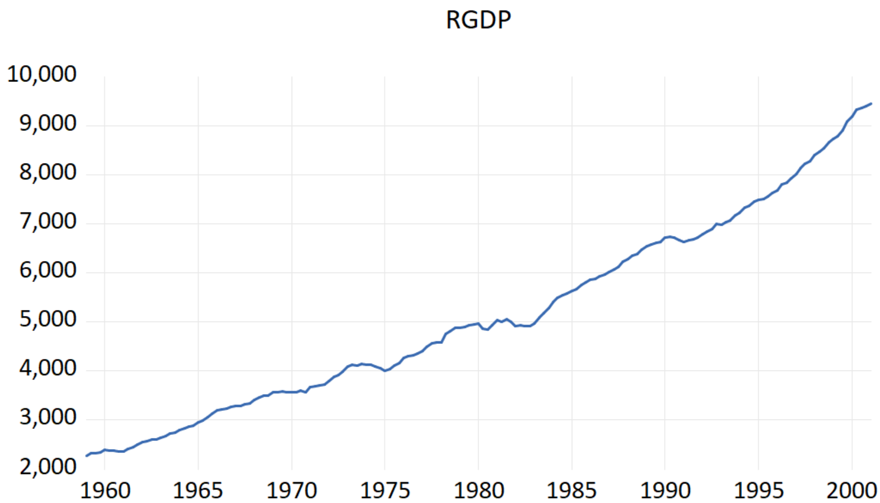


Figure 18: RGDP Time PLot

Ans 3(B)

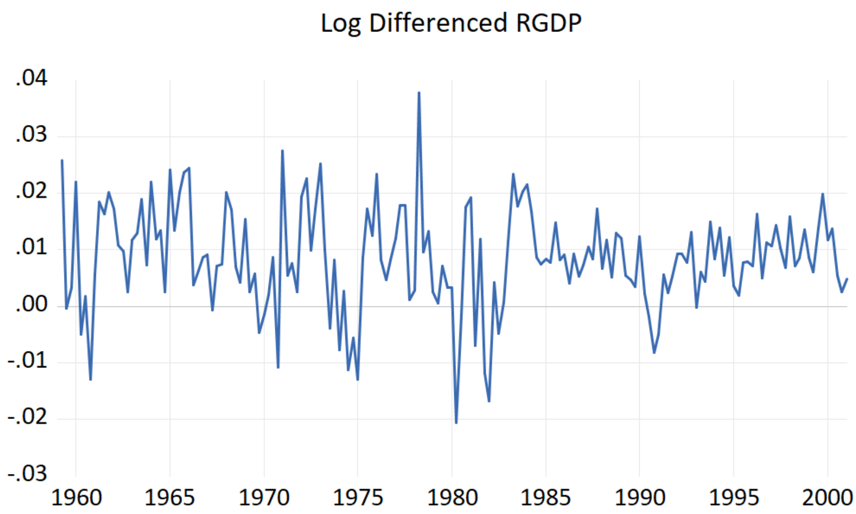


Figure 19: 1st difference log RGDP Time PLot

Interpretation of Model Selection Results for Log Real GDP

Sample: 1959Q1 2001Q1
Included observations: 169

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.980	0.980	165.18	0.000
		2	0.960	-0.003	324.69	0.000
		3	0.940	-0.023	478.44	0.000
		4	0.919	-0.017	626.42	0.000
		5	0.899	0.002	768.88	0.000
		6	0.879	-0.016	905.80	0.000
		7	0.859	-0.007	1037.3	0.000
		8	0.838	-0.021	1163.4	0.000
		9	0.817	-0.013	1284.1	0.000
		10	0.797	-0.003	1399.6	0.000
		11	0.777	-0.002	1510.1	0.000
		12	0.758	-0.002	1615.8	0.000
		13	0.738	-0.003	1716.8	0.000
		14	0.720	-0.003	1813.3	0.000
		15	0.701	-0.009	1905.5	0.000
		16	0.682	-0.011	1993.3	0.000
		17	0.663	-0.004	2076.9	0.000
		18	0.645	-0.004	2156.6	0.000
		19	0.627	-0.000	2232.4	0.000
		20	0.609	-0.012	2304.4	0.000
		21	0.592	0.005	2372.9	0.000
		22	0.575	-0.007	2438.0	0.000
		23	0.558	-0.006	2499.6	0.000

Figure 20: Corellograms of LRGDP:

0.3 Analysis of the Correlogram for Log Real GDP

The correlogram of the log of real GDP, as shown in Figure, exhibits a slow decay in both the autocorrelation (AC) and partial autocorrelation (PAC) functions. This suggests that the series may follow either a unit root process or a highly persistent autoregressive (AR) process.

The statistical significance of the autocorrelations, indicated by the Q-statistic and corresponding p-values, highlights the strong dependence structure in the data over time. Such behavior is characteristic of non-stationary time series or series with a near-unit root.

Model	**AIC**	**SBC**	$\mathbf{R}^2$
ARMA(1,1)	-8.33	-8.27	0.08
ARMA(1,2)	-8.34	-8.26	0.10
AR(2)	-8.35	-8.29	0.10
ARMA(2,1)	-8.34	-8.26	0.10

Table 4: Model Selection Results for Log Real GDP

The objective is to select the best model for the log of Real GDP based on the given metrics: Akaike Information Criterion (AIC), Schwarz Bayesian Criterion (SBC), and R^2. Here is a detailed analysis of the models:

Model Comparisons

(1) ARMA(1,1):

- AIC = -8.33, SBC = -8.27, $R^2 = 0.08$
- This model has the simplest structure with one autoregressive term (AR(1)) and one moving average term (MA(1)).
- While its AIC and SBC values are relatively low, the R^2 value is the smallest, meaning the model explains only 8% of the variance in the log of RGDP.

(2) ARMA(1,2):

- AIC = -8.34, SBC = -8.26, $R^2 = 0.10$
- This model adds an additional moving average term (MA(2)) compared to ARMA(1,1).
- It has slightly better AIC and SBC values compared to ARMA(1,1), and R^2 has improved to 10%, suggesting a better fit at the cost of slightly increased complexity.

(3) AR(2):

- AIC = -8.35, SBC = -8.29, $R^2 = 0.10$
- This model uses only autoregressive terms (AR(1) and AR(2)).

- It achieves the lowest AIC (-8.35) and SBC (-8.29), meaning it balances goodness-of-fit and model simplicity better than other models.
- It also has the same $R^2 = 0.10$ as ARMA(1,2), making it equally effective at explaining the variance in the log of RGDP.

(4) ARMA(2,1):

- AIC = -8.34, SBC = -8.26, $R^2 = 0.10$
- This model includes two autoregressive terms (AR(1) and AR(2)) and one moving average term (MA(1)).
- Its AIC and SBC values are slightly higher than AR(2), and it has no significant improvement in R^2. This suggests that adding the moving average term increases model complexity without improving fit substantially.

Best Model Based on AIC and SBC:

According to analysis AR(2) has the lowest AIC (-8.35) and SBC (-8.29), indicating it strikes the best balance between fit and simplicity. The additional moving average terms in ARMA(1,2) and ARMA(2,1) do not improve the model significantly and increase complexity.

All models except ARMA(1,1) explain 10% of the variance in log RGDP. While this value is not particularly high, it is consistent across ARMA(1,2), AR(2), and ARMA(2,1). This suggests that including more terms beyond AR(2) does not meaningfully enhance explanatory power.

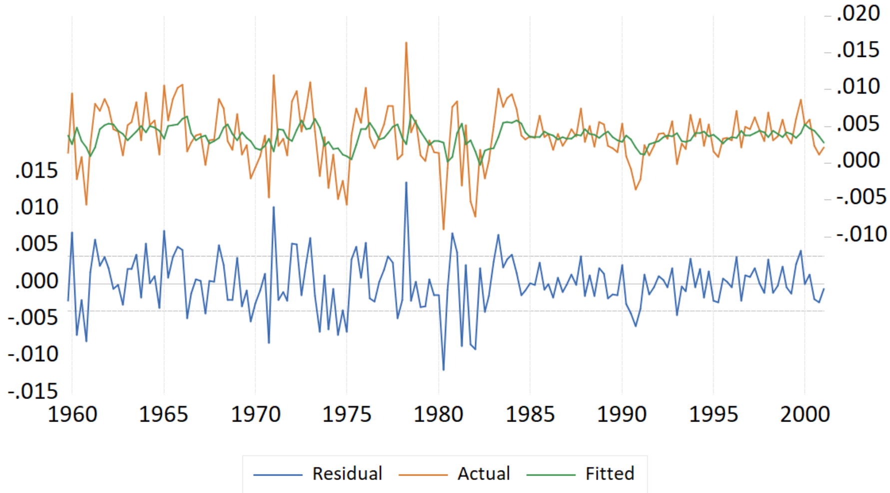


Figure 21: Fitted model of AR(2) to log RGDP

Ans 3(c)

Interpretation of ADF Test Results for Log Real GDP (with Intercept and Trend)

I have conducted the Augmented Dickey-Fuller (ADF) test for both intercept and trend in EViews, The reason for the same is that by visually inspecting the graphs it can be seen that the series has a clear growing trend with an intercept term and mild shocks added at each time step as we would theorize for an actual gdp graph. Hence I went with Augmented Dickey-Fuller (ADF) test for both intercept and trend

The interpretation of the results would be as follows:

ADF Test Results (with Intercept and Trend):

- **Test Statistic:** -2.85
- **Critical Values:**
  - 1%: -4.01
  - 5%: -3.43
  - 10%: -3.14

Interpretation:

Compare the Test Statistic with Critical Values: The test statistic (-2.85) is greater than the critical value at 1% (-4.01), greater than the critical value at 5% (-3.43), and greater than the critical value at 10% (-3.14).

Conclusion: Since the test statistic (-2.85) is not more negative than the critical values at the 1%, 5%, or 10% significance levels, we fail to reject the null hypothesis of the ADF test. This suggests that the log of Real GDP (lrgdp) does not exhibit stationarity when both intercept and trend are included in the test. Therefore, we conclude that lrgdp has a unit root and is non-stationary.

Implication:

The inclusion of both intercept and trend a *deterministic trend* (whether the series has a trend that should be accounted for in the stationarity test). Given the results, this confirms that the series follows a *stochastic trend*, meaning that its trend is random or unpredictable, not deterministic.

Based on the interpretation of the ADF test results for the log of Real GDP (lrgdp), the test suggests that the series has a unit root, which means it is non-stationary and follows a stochastic trend rather than a deterministic one.

Comparison with Expectations:

- I expected the series to have a unit root process, the test results confirm my expectation. A unit root implies that the series has a stochastic trend, where shocks to the series have permanent effects and the series does not revert to a long-term mean.

Conclusion: The test results confirm that the log of Real GDP (lrgdp) follows a unit root process, meaning it is non-stationary with a stochastic trend. This aligns with expectations of a random or unpredictable trend, not a deterministic one.