

## Cahn-Hilliard model

The Cahn-Hilliard model can be formulated as a parabolic equation which is typically used to model phase separation in binary mixtures.

$$\partial_t c = \Delta(c^3 - c - \gamma\Delta c)$$

The order of this equation can be, however, lowered, yielding two coupled second-order partial differential evolution equations in terms of the so-called phase fraction  $\phi$ , which represents the concentration of one of the components in the binary mixture and the chemical potential  $\mu$

$$\frac{\partial\phi}{\partial t} = \nabla \cdot (b(\phi)\nabla\mu) \quad \text{and} \quad \mu = -\gamma\Delta\phi + f'(\phi). \quad (1)$$

The Time evolution  $\partial_t\phi$  was approximated using Crank-Nicholson scheme. We studied the above system in a unit square  $\Omega = (0, 1)^2$  with periodic boundary conditions, in other words, we were solving the system on a torus. The initial data for the phase fraction  $\phi$  were given as<sup>1</sup>

$$\phi_0(x, y) = 0.3^{0.1} \sin(4\pi x) \sin(2\pi y) + 0.1^{0.6}. \quad (2)$$

The interface parameter  $\gamma$  was set to  $\gamma = 0.003$ , the double well potential  $f(\phi)$  and the mobility functions were defined as

$$f(\phi) = 0.3(\phi - 0.99)^2(\phi + 0.99)^2 \quad \text{and} \quad b(\phi) = (1 - \phi)^2(1 + \phi)^2 + 10^{-3}. \quad (3)$$

In order to solve the posed problem, we need to reformulate it first into the weak form

$$\langle \partial_t\phi, v \rangle + \langle b(\phi)\nabla\mu, \nabla v \rangle = 0 \quad \text{and} \quad \langle \mu - f'(\phi(t)), w \rangle - \langle \gamma\nabla\phi, \nabla w \rangle. \quad (4)$$

As per the original article [1], the model was computed with mixed elements and the Newton method was used to solve the corresponding **NonlinearVariationalProblem**.

Below we present the snapshots from the time evolution on the square as well as one instance of the model on a torus.

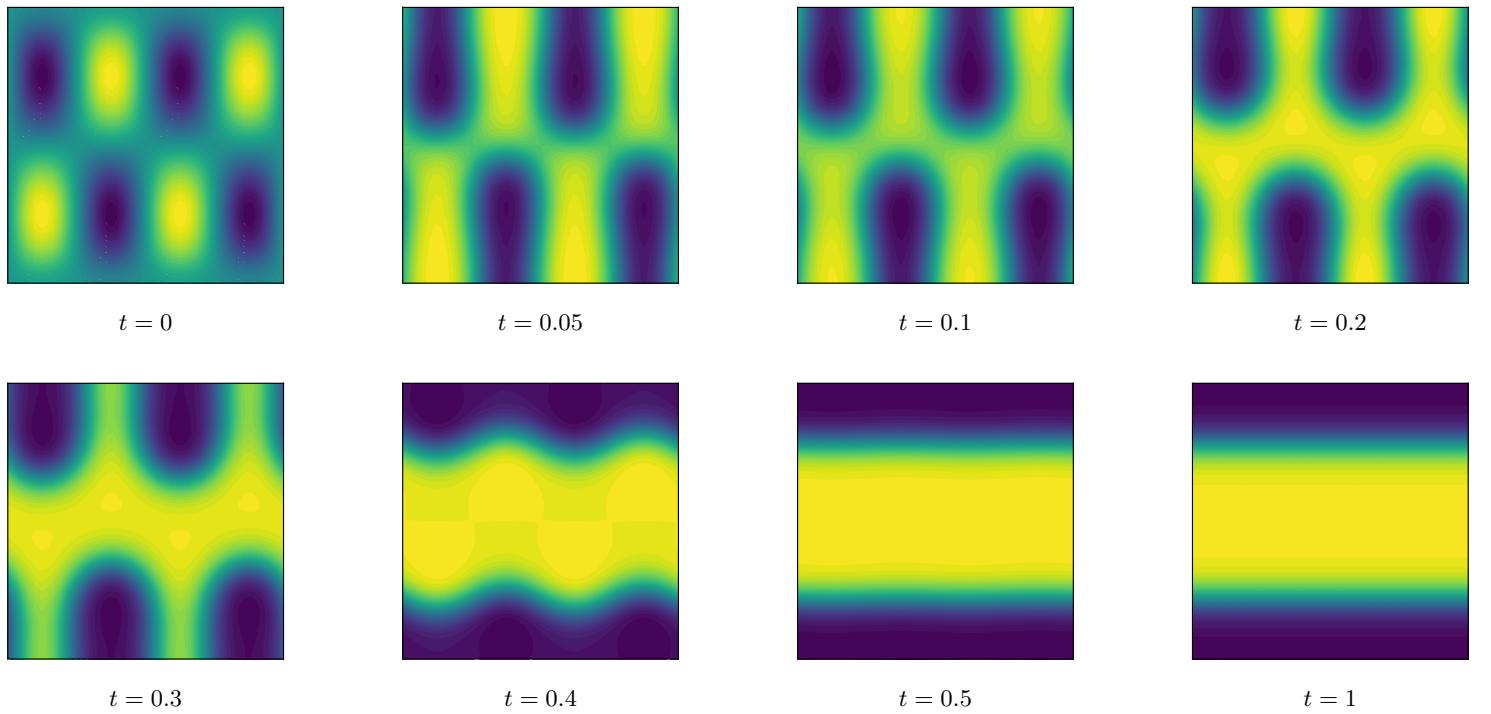


Figure 1: Snapshots from time evolution of the Cahn-Hilliard model on a unit square with periodic boundary conditions.

## References

- [1] Aaron Brunk et al. “Stability and discretization error analysis for the Cahn-Hilliard system via relative energy estimates”. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 57.3 (May 2023), pp. 1297–1322. ISSN: 2804-7214. DOI: 10.1051/m2an/2023017. URL: <http://dx.doi.org/10.1051/m2an/2023017>.

<sup>1</sup>The values stated in the paper did not seem to correspond to the presented figures and so were adjusted in our simulations.