Integration Demonstration

Sunday, April 27, 2025

$$I = \int_{-\infty}^{\infty} e^{-x^2}\,dx$$

$$I^2 = \left(\int_{-\infty}^{\infty} e^{-x^2}\,dx\right) \cdot \left(\int_{-\infty}^{\infty} e^{-y^2}\,dy\right)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-x^2} e^{-y^2}\,dx\,dy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\,dx\,dy$$

Add Derivation for Jacobian

$$= \int_{0}^{2\pi}\int_{0}^{\infty} e^{-r^2} r\,dr\,d\theta$$

$$I^2 = \mathbb{R}\pi\left(-\frac{1}{2}e^{-r^2}\Big|_0^{\infty}\right)$$

$$I^1 = \pi$$

$$I = \sqrt{\pi}$$