19/5/22

Wants

→ slow rpm, 3 phase, 6 blde motor.

Have

→ high rpm, bemf sensing, positing arduino, performs well.

Dij

→ assume with low rpm that BEMF will be too noisy.

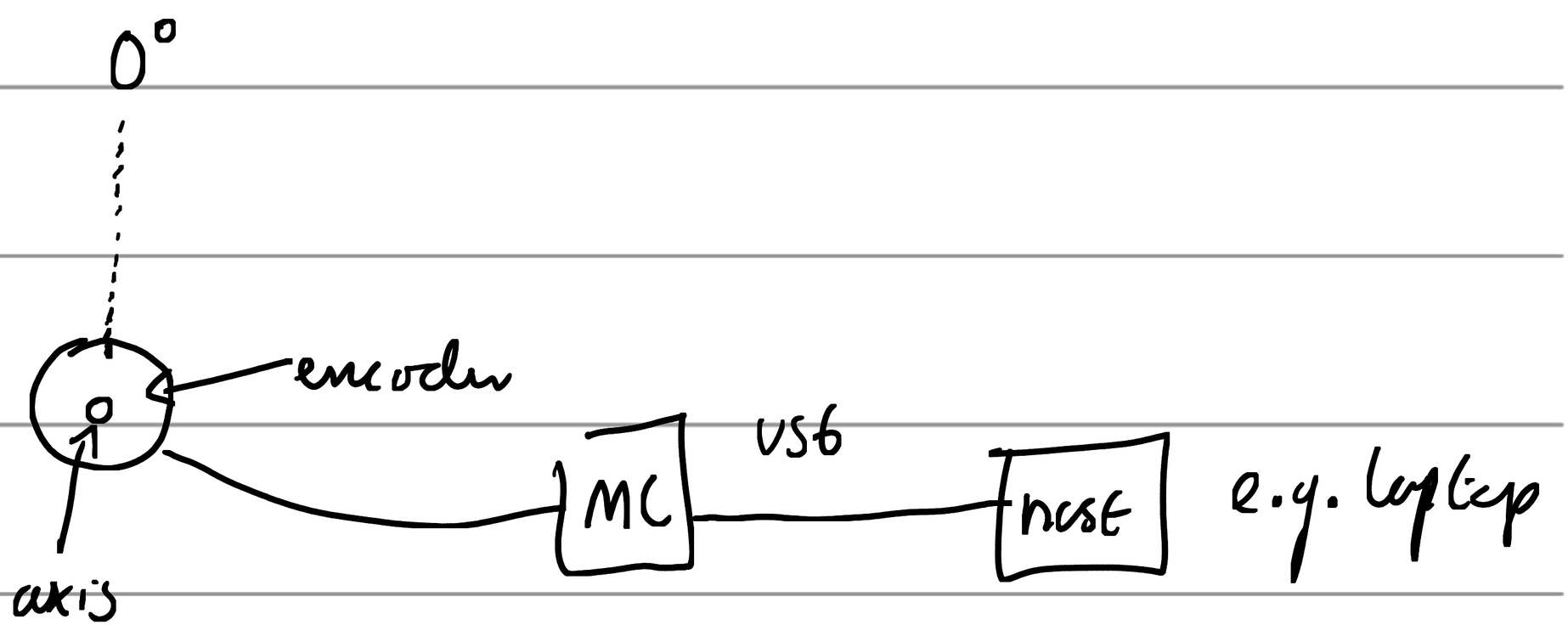
→ Arduino works very capebuly

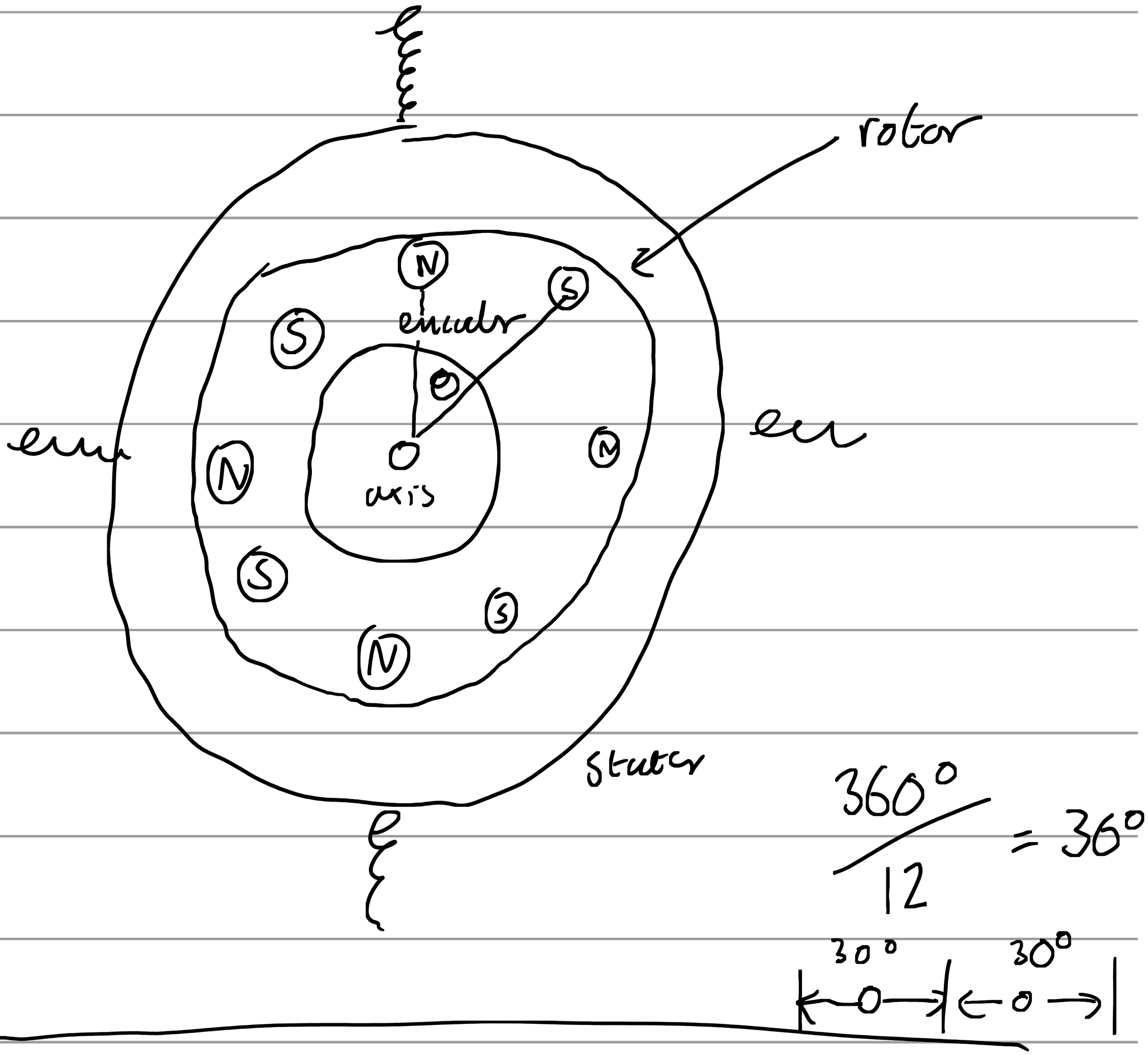
→ Teensy 4.0 work very marginal, 100/255 duty

What we should consider.

→ BEMF strategy will not work for low rpm, therefore we will not know precise information velocity.

→ Rotary encoder (absolute)

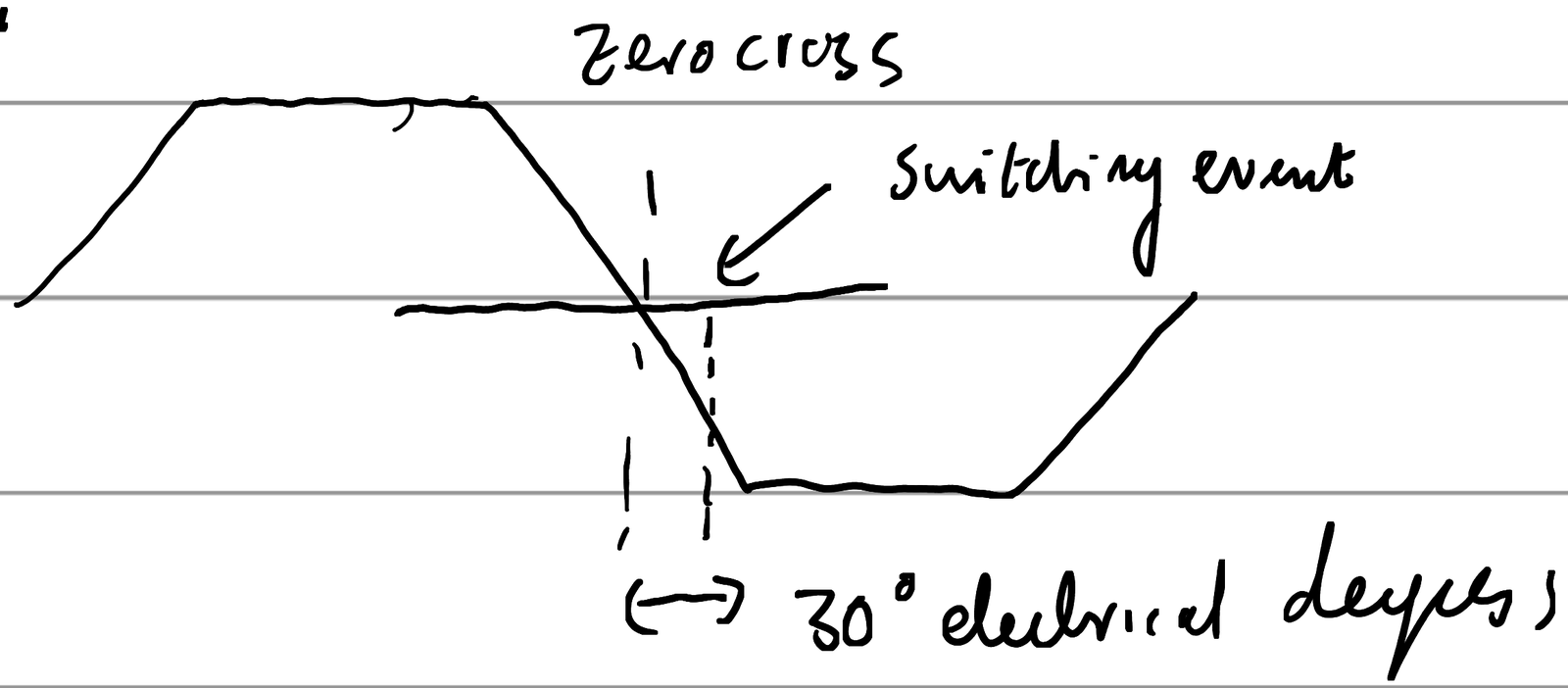




$$\frac{360^\circ}{12} = 36^\circ$$

30° 30°

Phase x:



BldC 6 electrical steps per electrical revolution

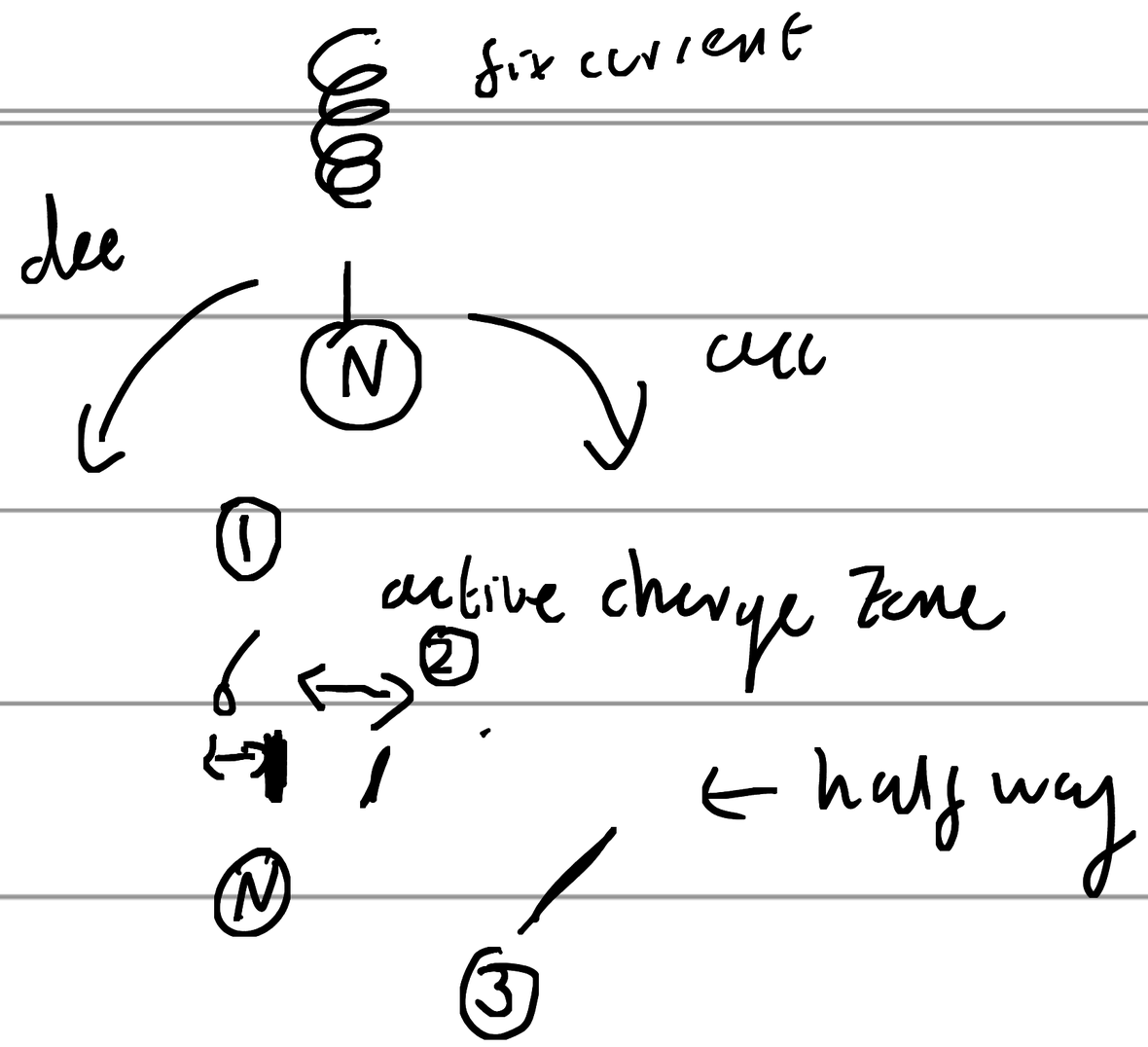
1 cycle will have n - electrical cycles per mechanical cycle.

Mechanical steps per mechanical cycle = $\frac{N \text{ poles}}{2} \cdot 6$

$= \frac{12}{2} \cdot 6 = 36$

6.6

9 coils 12 poles



step 1 charge coils, step 2 discharge coils, step 3 charge commutation step A → B

processing that we charge the relevant coil when the encoder has indicated precession beyond the central point of the pole (where max flux), then we have an interrupt each time the encoder senses a rotational displacement change, such that we disable the coil charge after some time well before we enter the next zone

swap phase & + charge coil, wait for displacement discharge coil. [for each mechanical step.]

Duty cycle resolution, (impulse of a single pulse of a single electrical / mechanical step)

Arduino 256

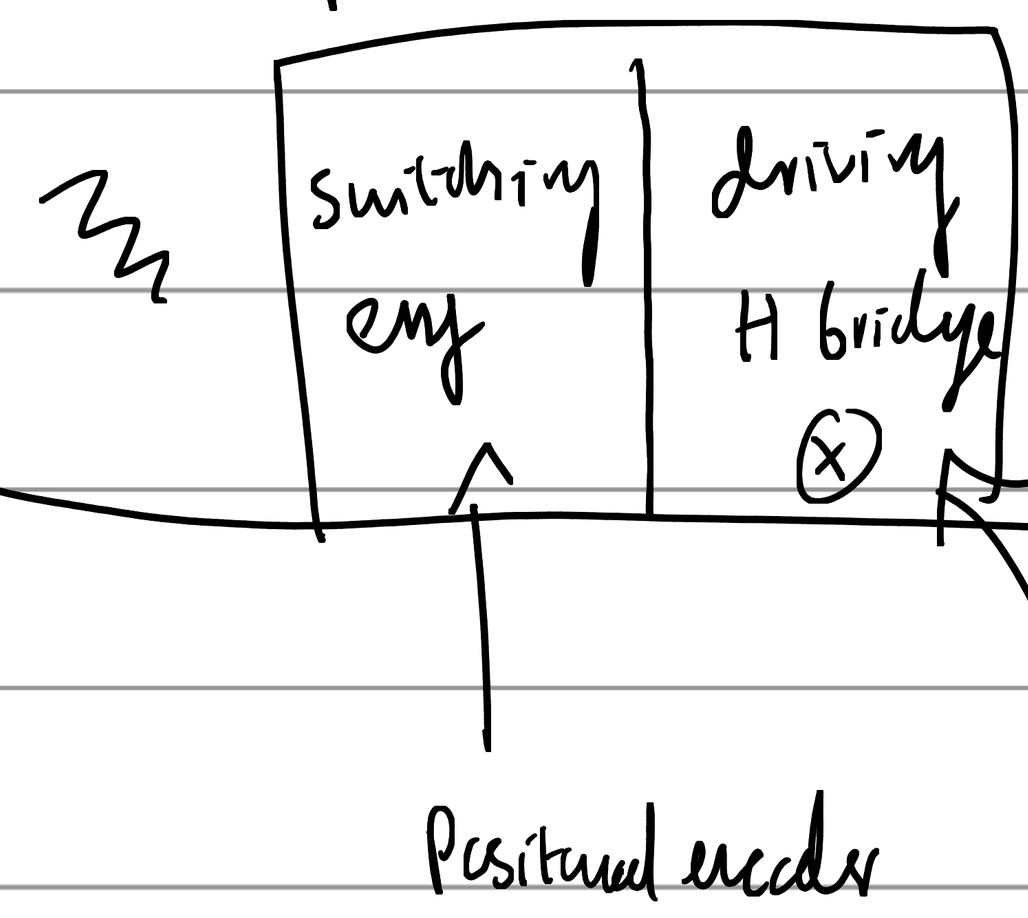
revert

Teensy 40 256, 512, 1024, ~~8192~~ 2048

4096

8192

problems



Positional encoder

might still be an issue

removes this problem

360° → 36 steps

10° per pr step.

↑

mechanical

10 samples per step 1° angular resolution

maybe we want 100 samples per step (0.1°)

what we want from an rotary encoder

→ absolute position

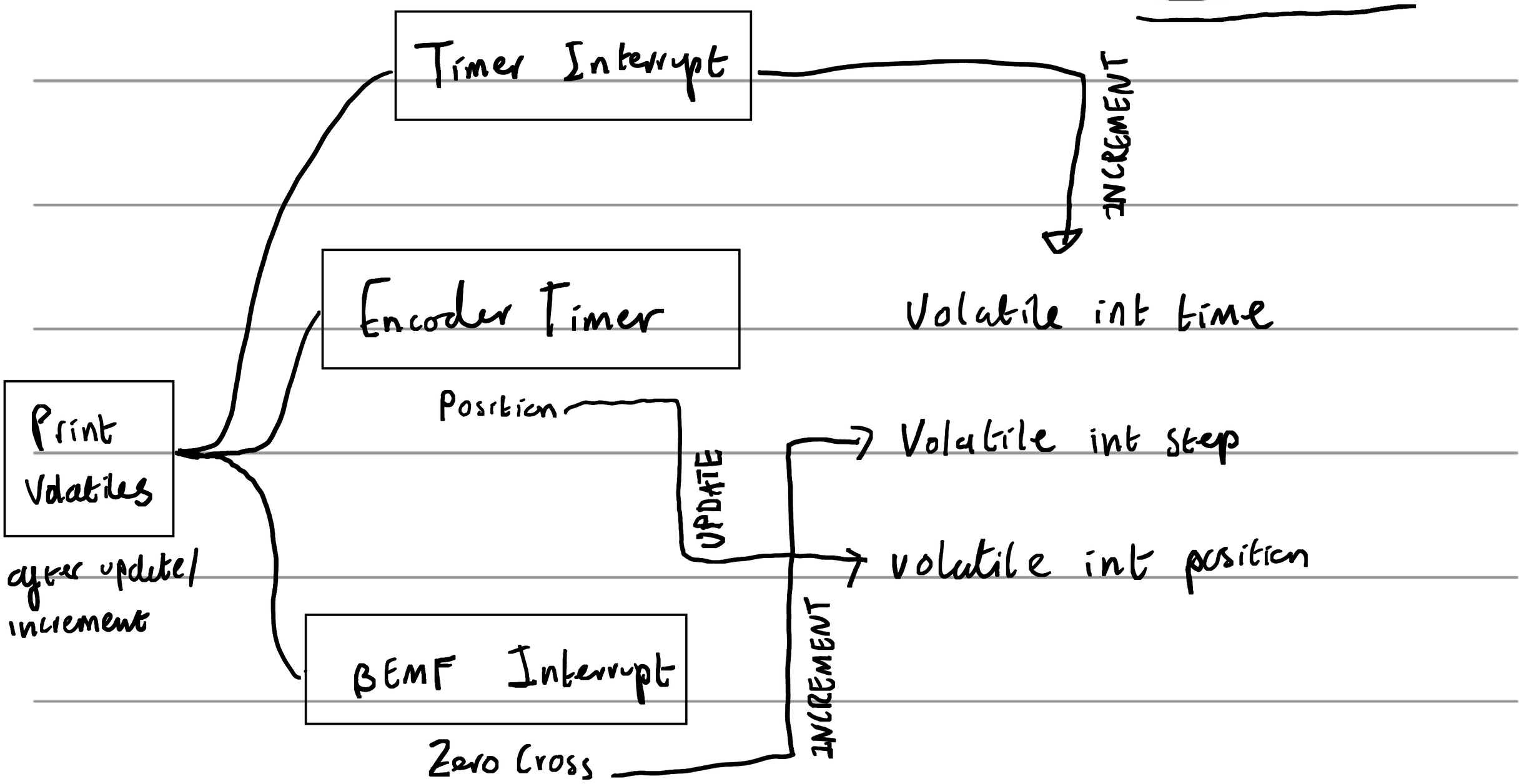
→ radial resolution (0.1 → 1°)

Nice to have

optical (no drag), no influence the bearing, arduino + teensy 40 support.

CALIBRATION

18/5/22



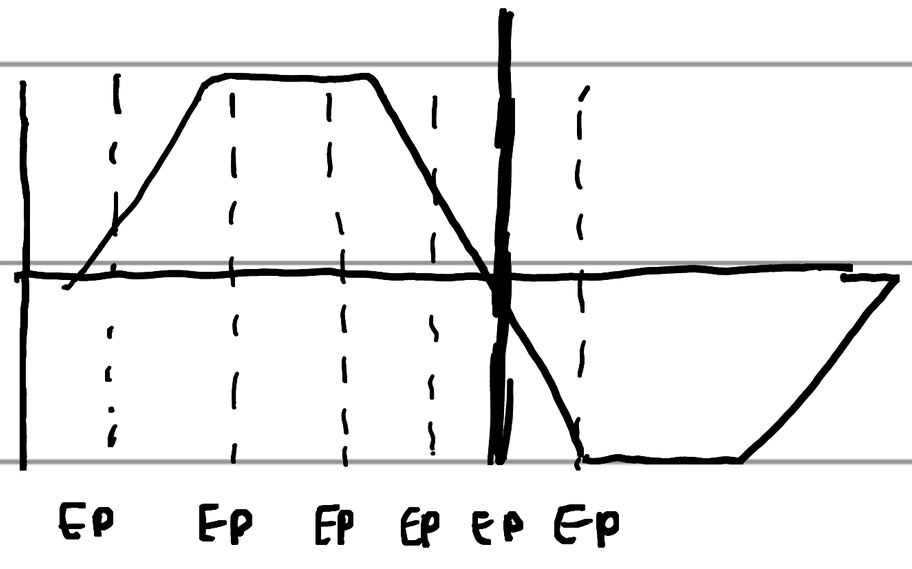
The the idea is to drive the assembly to a minimal speed

such that we get clear bemf signals while all the time

logging the bemf step, the rotational position via encoder

values, and maybe the time.

ZC



EP - encoder pulse

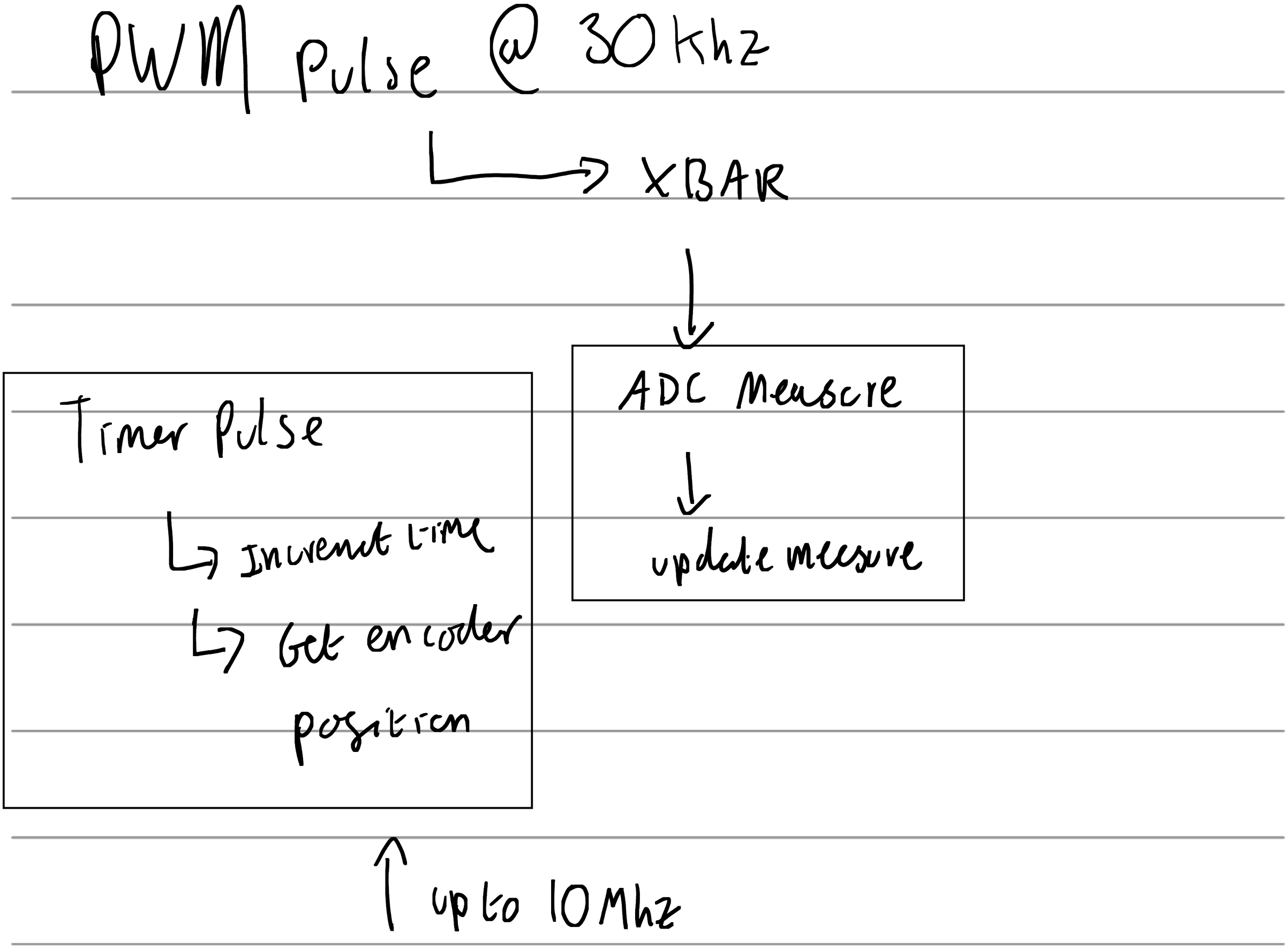
ZC - Zero cross

We can fin the data via the encoder value the position

I therefore get an average error for the Zero-cross angular positions. Ideal switching points are 30° (electrical) after the ZC positions.

One could use the ADC to confirm this.

The way this could work:



21/5/22

Chosen the ADC method as the primary method.

Found 2 good magnetic encoders: Both 3.3 → 5v

→ AS5600 12-bit I^2C Not for high RPM

↓

4096

→ AS5147p 14-bit SPI 28krpm

↘

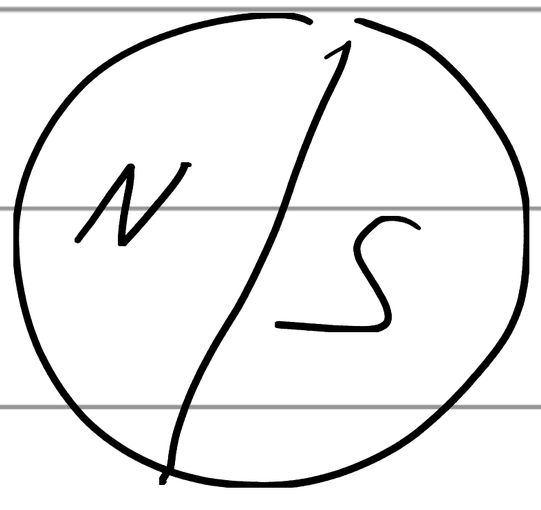
16,384

↑ This one is dedicated to motor control applications like BLDC.

AS5147p

With 16,384 angular steps , 9 coils 12 poles thus 36 steps.

$\frac{16,384}{36}$ = 455.11 samples per zone.



2^{12} 4096

3601

4096

0.088°

0.2 → 0.7

optical

Jerk model

We will be making position & time measurements

$P_i = [S_i, T_i]$ S - displacement angular

T - time interval

Euler method

$$W_{i+1} = \frac{S_{i+1} - S_i}{T_{i+1} - T_i} \qquad W_{i+2} = \frac{S_{i+2} - S_{i+1}}{T_{i+2} - T_{i+1}}$$

$$\alpha_{i+2} = \dot{\omega}_{i+2} = \frac{\omega_{i+2} - \omega_{i+1}}{t_{i+2} - t_{i+1}}$$

$$= \frac{1}{T_{i+2} - T_{i+1}} \left(\frac{S_{i+2} - S_{i+1}}{T_{i+2} - T_{i+1}} - \frac{S_{i+1} - S_i}{T_{i+1} - T_i} \right)$$

$$= \frac{1}{a-b} \left(\frac{c-d}{a-b} - \frac{d-e}{b-f} \right)$$

$$= \frac{c-d}{(a-b)^2} \quad \frac{d-e}{(a-b)(f-b)} = \frac{S_{i+2} - S_{i+1}}{(T_{i+2} - T_{i+1})^2} \quad \frac{S_{i+1} - S_i}{(T_{i+2} - T_{i+1})(T_i - T_{i+1})}$$

what are we measuring θ_i, T_i $\theta_i(t)$

$$\omega_i = \dot{\theta}_i = \frac{d\theta_i}{dT_i}$$ angular velocity

$$\alpha_i = \dot{\omega}_i = \frac{d\omega_i}{dT_i} = \frac{d^2\theta_i}{dt^2}$$ angular acceleration

$$\dot{j}_i = \dot{\alpha}_i = \frac{d\alpha_i}{dT_i} = \frac{d^3\theta_i}{dt^3}$$ angular jerk.

Jerk model $X_i = [\theta, \dot{\theta}, \ddot{\theta}, \dddot{\theta}]$ in 1D

White noise $w(t)$ applying to jerk

$$X = AX_i + Bw(t)$$ a is a constant

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot{\theta}\\ \ddot{\theta}\\ \dddot{\theta}\end{bmatrix} = \begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&-a\end{bmatrix}\begin{bmatrix}\theta\\ \dot{\theta}\\ \ddot{\theta}\\ \dddot{\theta}\end{bmatrix} + \begin{bmatrix}0\\0\\0\\1\end{bmatrix} w(t)$$

$$\frac{d}{dt}\begin{bmatrix}\theta\\ \dot\theta\\ \ddot\theta\\ \dddot\theta\end{bmatrix} = \begin{bmatrix}0&1&0&0\\ 0&0&1&0\\ 0&0&0&1\\ 0&0&0&-a\end{bmatrix}\begin{bmatrix}\theta\\ \dot\theta\\ \ddot\theta\\ \dddot\theta\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0\\ 1\end{bmatrix} w(t)$$

$$\frac{d\theta}{dt} = \dot\theta \qquad \frac{d\dot\theta}{dt} = \ddot\theta \qquad \frac{d\ddot\theta}{dt} = \dddot\theta$$

$$\frac{d\dddot\theta}{dt} = -a\dddot\theta + w(t)$$

Measurement vector $Z(k+1) = H X(k+1) + V(k+1)$

H is the system measurement or observation matrix

V is the measurement noise vector.

1D tracking

$$X(k+1) = F(k+1, k)\, X(k) + u(k)$$

$$F(k+1, k) = e^{\underbrace{A}_{\text{matrix}}(t_{k+1}, t_k)}$$

$\Delta(k)$ is the discrete white noise

$$F(t) = \begin{bmatrix} 1 & T & T^2/2 & p_1 \\ 0 & 1 & T & q_1 \\ 0 & 0 & 1 & r_1 \\ 0 & 0 & 0 & s_1 \end{bmatrix}$$

$T = t_{k+1} - t_k$

$p_1 = (2 - 2aT + a^2T^2 - 2e^{-aT}) / 2a^3$

$q_1 = (e^{-at} - 1 + at) / a^2$

$r_1 = (1 - e^{-at}) / a$

$s_1 \sim e^{-a}t = 1 - at + \frac{a^2T^2}{2!} - \frac{a^3T^3}{3!} \cdots$

where ΔT is small

$$F(T)\ \substack{\lim \\ a \to 0} = \begin{bmatrix} 1 & T & T^2/2 & T^3/6 \\ 0 & 1 & T & T^2/2 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kalman algorithm

Prediction

1) Predict next frame

$$X_t = f(X_{t-1}, u_t) + q_1$$

2) Calculate prediction covariance matrix

$$Z_t = h(X_t, u_t) + q_2$$

Correction

3) calculate EKF gain

$$K_t = P_{t|t-1} H_t^T \left[H_T P_{t|t-1} \cdot H_t^T + R_t \right]^{-1}$$

4) Update the observation estimated value

$$X_t = X_{t|t-1} + K_t [Z_t - Z_{t|t-1}]$$

5) update error in covariance matrix

$$P_{t|t} = [Z_t - Z_{t|t-1} - K_t H_t] P_{t|t-1}$$

✓ ✓ ✓ ? Q? ✓ ✓

we need f, h, Z, K_t, X_+, H, P

Initialisation, 3 measurements to estimate $\hat{\theta}$, $\hat{\dot{\theta}}$ & $\hat{\ddot{\theta}}$

$\hat{\theta}$ taken to be zero initially.

$$\hat{\theta}_3 = M_x(3) \qquad \hat{\dot{\theta}}_3 = \frac{M_x(3) - M_x(2)}{T}$$

$$\hat{\ddot{\theta}}_3 = \left[\frac{M_x(3) - 2M_x(2) + M_x(1)}{T^2}\right] \qquad \hat{\ddot{\theta}}_3 = 0$$

constant time interval T

Measurement noise

$$M_x(1) = x(1) + v(1)$$

$$M_x(2) = x(2) + v(2)$$

$$M_x(3) = x(3) + v(3)$$

Error in $\theta, \dot{\theta}, \ddot{\theta}, \dddot{\theta}$ $\quad \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$

$$\varepsilon_1[3|3] = x(3) - \hat{x}(3|3) = x(3) - x(3) - v(3)$$

$$= -v(3)$$

$$\varepsilon_2(3|3) = \dot{x}(3) - \hat{\dot{x}}(3|3) =$$

Equations

$$X_j(k+1) = F_j X_j(k) + w_j(k)$$

$$Z_j(k+1) = H_j X_j(k+1) + v_j(k+1)$$

$$X_j = \begin{bmatrix} x & \dot{x} & \ddot{x} & \dddot{x} \end{bmatrix}^T$$

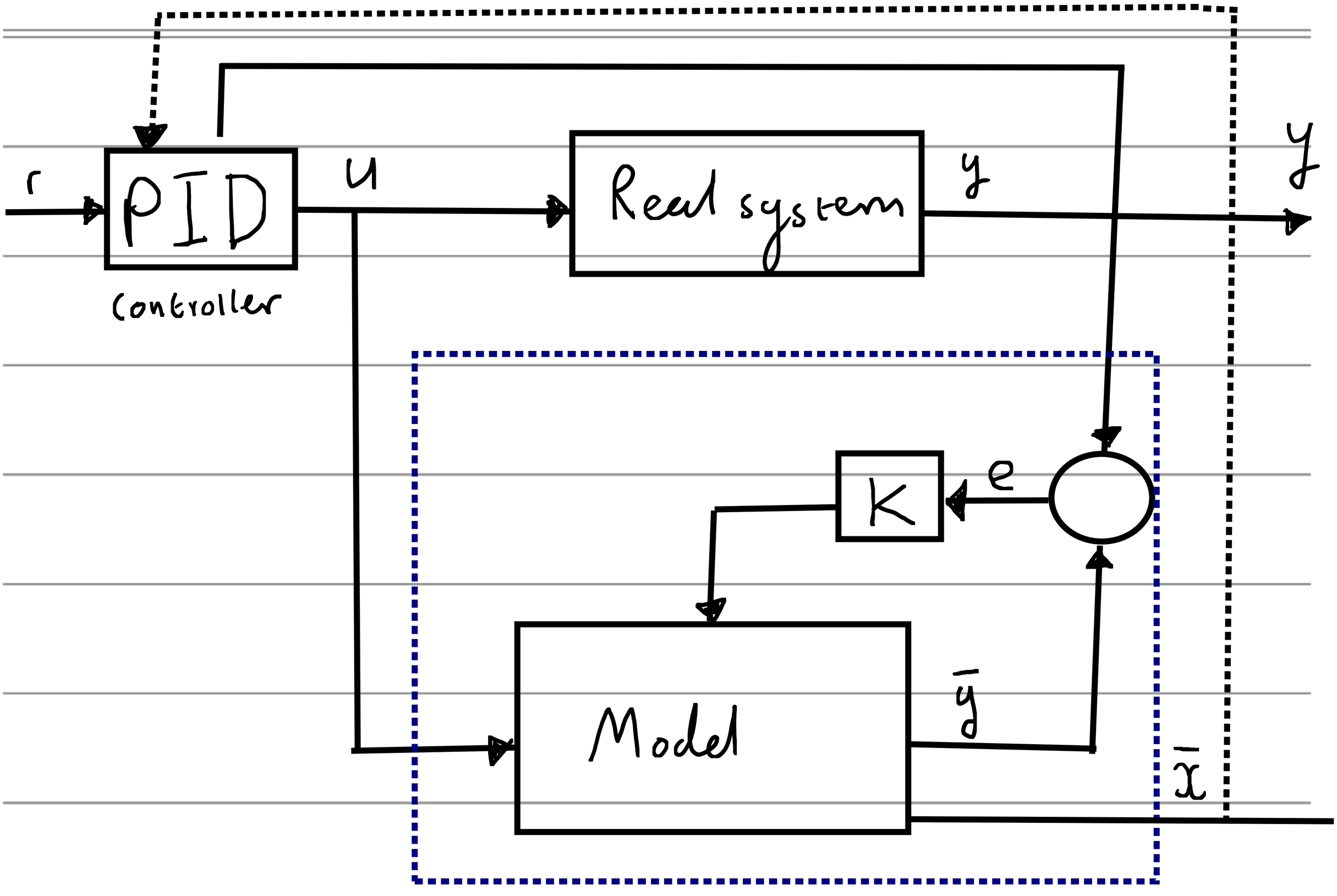
$$w_j = \begin{bmatrix} u_{1j} & u_{2j} & u_{3j} & u_{4j} \end{bmatrix}^T$$

$$H_j = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Z_j = \begin{bmatrix} M_{sc} & \end{bmatrix}^T$$

$$v_j = \begin{bmatrix} noise\ u_x \end{bmatrix}^T$$

$$F_j = \begin{bmatrix} 1 & T & T^2/2 & P_1 \\ 0 & 1 & T & q_1 \\ 0 & 0 & 1 & r_1 \\ 0 & 0 & 0 & s_1 \end{bmatrix}$$



Kalman Filter