$$\underset{}{Q_{(k,k+1)}} = \overset{[4\times4]}{Q(dt)}$$ Variance matrix at step $k \to k+1$

$$F_{(k,k+1)} = \overset{[4\times4]}{F(dt)}$$ state transition matrix

Take last known state $X_k = \begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{pmatrix}$ & project it ahead in time using the transition matrix.

$$\underset{[4\times1]}{X_{(k+1)}} = \underset{[4\times4]}{F_{(k,k+1)}} \cdot \underset{[4\times1]}{X_{(k)}} + \overbrace{\underset{[4\times4]}{B} \cdot \underset{[4\times1]}{w_{(k)}}}^{\text{Ignore as this comes from the environment}}$$

Project the error covariance ahead note $P_{(k,k+1)}$ is the covariance matrix at step $k \to k+1$, $P_{(k-1,k)}$ same for $k-1 \to k$.

$$\underset{}{\overset{[4\times4]}{P_{(k,k+1)}}} = \overset{[4\times4]}{F_{(k,k+1)}} \cdot \underbrace{\overset{[4\times4]}{P_{(-k,k)}}}_{\text{? not } (k-1,k) \text{ seems wrong}} \cdot \overset{[4\times4]}{F_{(k,k+1)}}^{T} + \overset{[4\times4]}{Q_{(k,k+1)}}$$

Calculate uncertainty in measurement + uncertainty in estimate:

where H is the measurement matrix & R is the measurement uncertainty

$$S = H \cdot P_{(k,k+1)} \cdot H^T + R$$

$$[1\times4] \quad [4\times4] \quad [4\times1] \quad [1\times1]$$

$$[1\times4]$$

$$[1\times1] + [1\times1] = [1\times1]$$

Calculate uncertainty in estimate

$$C = P_{(k,k+1)} \cdot H^T$$

$$[4\times1] \quad [4\times4] \quad [4\times1]$$

Calculate the gain

$$K = C \cdot S^{-1}$$

$$[4\times1] \quad (4\times1) \quad [1\times1]$$

Update measurement via Z

Z = x.reshape(H.shape[0], 1) $[1\times1]$ in 1d

Calculate innovation

$$Y = Z - (H \cdot X_{(k+1)})$$

$[1 \times 1] - ([1 \times 4][4 \times 1]) = [1, 1]$

Calculate the filtered state

$$X_{(k+1)} = X_{(k+1)} + \overset{\text{kalm}}{K} \cdot Y$$

$[4 \times 1]$ $[4 \times 1]$ $[1 \times 1]$

Assign old to new state

$$X_{(k)} = X_{(k+1)} + (K \cdot Y)$$

$[4 \times 1]$ $[4 \times 1]$ $[4 \times 1]$ $[1 \times 1]$

$[4 \times 1]$ $[4 \times 1]$

Update error covariance

$$P_{(k,k)} = (\mathbb{1} - k \cdot H) \cdot P_{(k,k+1)}$$

$[4 \times 4]$ $[4 \times 4]$ $[4 \times 1][1 \times 4]$ $[4 \times 4]$

$[4 \times 4][4 \times 4] = [4 \times 4]$

K+1 then loop for next state.

So need a simple 1D non generic version which can be computed efficiently.

$$Q(dt) = \begin{bmatrix} \frac{dt^7}{256} & \frac{dt^6}{72} & \frac{dt^5}{30} & \frac{dt^4}{24} \\ \frac{dt^6}{72} & \frac{dt^5}{20} & \frac{dt^4}{8} & \frac{dt^3}{6} \\ \frac{dt^5}{30} & \frac{dt^4}{8} & \frac{dt^3}{3} & \frac{dt^2}{2} \\ \frac{dt^4}{24} & \frac{dt^3}{6} & \frac{dt^2}{2} & dt \end{bmatrix}$$

$$\sigma^2 = V$$

$$P(dt) = \begin{bmatrix} \sigma_\theta^2 & \frac{\sigma_\theta^2}{dt} & \frac{\sigma_\theta^2}{dt^2} & 0 \\ \sigma_\theta^2/dt & \frac{2\sigma_\theta^2}{dt^2} & \frac{3\sigma_\theta^2}{dt^3} & \frac{5\sigma_J^2}{6} dt^2 \\ \sigma_\theta^2/dt^2 & \frac{3\sigma_\theta^2}{dt^3} & \frac{6\sigma_\theta^2}{dt^4} & \sigma_J^2 dt \\ 0 & \frac{5\sigma_J^2}{6} dt^2 & \sigma_J^2 dt & \sigma_J^2 \end{bmatrix}$$

$$F(dt) = \begin{bmatrix} 1 & dt & \frac{dt^2}{2} & \frac{dt^3}{6} \\ 0 & 1 & dt & dt^2/2 \\ 0 & 0 & 1 & dt \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

To simplify this we could just name the components of P, F & Q as e.g. $P_{ij}(dt)$ & sub them in later.

Step by step: $\log k = \kappa$

$$[F_{ij}]_k = F(dt_k) = \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix}$$

$$[Q_{ij}]_k = Q_k = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} \end{bmatrix}$$

same with $[P_{ij}]_k$

We start with state $\overset{[4\times1]}{X_{(k)}} = [X_{11}^{k} \; X_{12}^{k} \; X_{13}^{k} \; X_{14}^{k}]$

1) Projects $\overset{[4\times1]}{X_{(k+1)}} = \overset{[4\times4]}{F} \overset{[4\times1]}{X_{(k)}}$

$$\Rightarrow [X_{11}^{k+1}, X_{12}^{k+1}, X_{13}^{k+1}, X_{14}^{k+1}] \begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} X_{11}^{k+1} F_{11}^{dt} + X_{12}^{k+1} F_{21}^{dt} + X_{13}^{k+1} F_{31}^{dt} + X_{14}^{k+1} F_{41} \\ X_{11}^{k+1} F_{12}^{dt} + X_{12}^{k+1} F_{22}^{dt} + X_{13}^{k+1} F_{32}^{dt} + X_{14}^{k+1} F_{42} \\ X_{11}^{k+1} F_{13}^{dt} + X_{12}^{k+1} F_{23}^{dt} + X_{13}^{k+1} F_{33}^{dt} + X_{14}^{k+1} F_{43} \\ X_{11}^{k+1} F_{14}^{dt} + X_{12}^{k+1} F_{24}^{dt} + X_{13}^{k+1} F_{34}^{dt} + X_{14}^{k+1} F_{44} \end{bmatrix}^{T} [4\times1]$$

$$F(dt_k)^T = \begin{bmatrix} F_{11} & F_{21} & F_{31} & F_{41} \\ F_{12} & F_{22} & F_{32} & F_{42} \\ F_{31} & F_{23} & F_{33} & F_{43} \\ F_{14} & F_{24} & F_{34} & F_{44} \end{bmatrix}$$

2) project error covar ahead.

$$P_{(k+1)} = F_{(k)} P_{(k-1)} F_{(k)}^T + Q_{(k)}$$

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ F_{21} & F_{22} & F_{23} & F_{24} \\ F_{31} & F_{32} & F_{33} & F_{34} \\ F_{41} & F_{42} & F_{43} & F_{44} \end{bmatrix} \begin{bmatrix} P_{11}^{k-1} & P_{12}^{k-1} & P_{13}^{k-1} & P_{14}^{k-1} \\ P_{21}^{k-1} & P_{22}^{k-1} & P_{23}^{k-1} & P_{24}^{k-1} \\ P_{31}^{k-1} & P_{32}^{k-1} & P_{33}^{k-1} & P_{34}^{k-1} \\ P_{41}^{k-1} & P_{42}^{k-1} & P_{43}^{k-1} & P_{44}^{k-1} \end{bmatrix} \begin{bmatrix} F_{11} & F_{21} & F_{31} & F_{41} \\ F_{12} & F_{22} & F_{32} & F_{42} \\ F_{31} & F_{23} & F_{33} & F_{43} \\ F_{14} & F_{24} & F_{34} & F_{44} \end{bmatrix} + \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} & Q_{14} \\ Q_{21} & Q_{22} & Q_{23} & Q_{24} \\ Q_{31} & Q_{32} & Q_{33} & Q_{34} \\ Q_{41} & Q_{42} & Q_{43} & Q_{44} \end{bmatrix}$$

Need a symbolic solver for this gonna be unwieldy!

2/4/23

Symp solver for kalmanprocess.

3/14/23

Need dependency graph.

$$P_{K+2} = (I - \underline{K} H) P_{\underline{K+1}}$$ * need for next loop

$$X^{KALMAN}_{K+1} = X_{K+1} + (\underline{K}\,\underline{Y})$$ * final state

↳ also taken as final state.

Dependencies...

NO

last state (Kalman?)

$$Y = Z - (\underline{H}\, X_{K+1})$$

ds

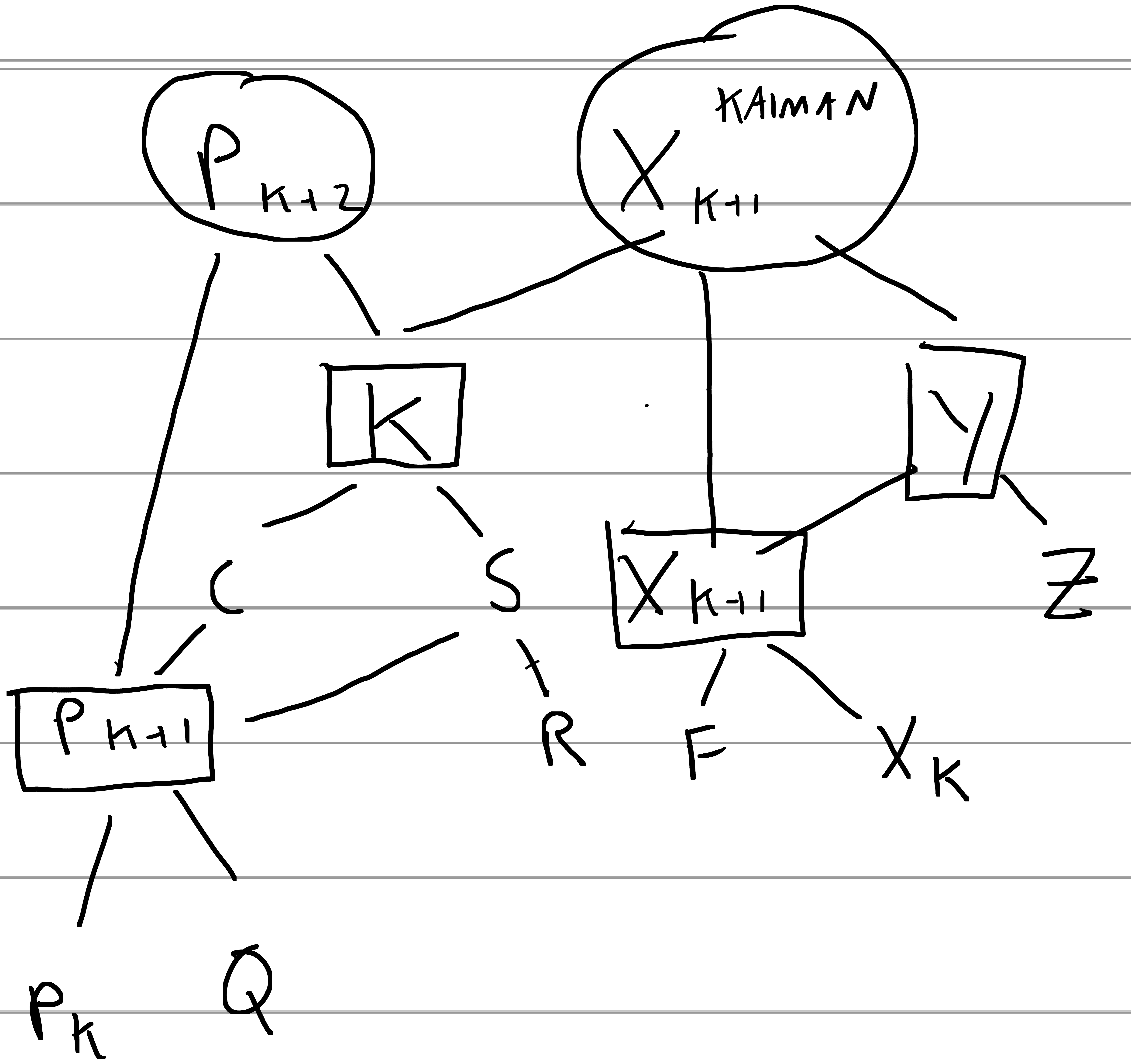
Z comes from new input reshaped via H

$$K = CS^{-1}$$

$$C = P_{K+1} H^T \qquad S = H P_{K+1} H^T + R$$

$$P_{K+1} = F P F^T + Q \qquad X_{K+1} = F X_K$$

H omitted



Each cycle we need P_{k+2} & X^{KALMAN}_{k+1}, what common computation can we leverage?

Need P_{k+1}, K, X_{k+1} & Y then compute the "rest" in terms of them $\begin{bmatrix} P_{KP1\_11} & P_{KP1\_12} & R_{KP1\_13} & \dots & \text{etc} \\ \vdots & \vdots & \vdots & & \vdots \end{bmatrix}$
etc

↳ $P_{(k+2)}$, X^{KALMAN}_{k+1}, $\underset{(P_{k+2})}{P_{k+1}}$, $\underset{(X^{KALMAN}_{k+1})}{X_{k+1}}$

↓

can generate error from this.

4/4/23

Missing error lunch state